SECTION 2

GENERAL INFORMATION, CONVERSION TABLES, AND MATHEMATICS

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2.1 GENERAL INFORMATION AND CONVERSION TABLES

TABLE 2.1 Fundamental Physical Constants

E. R. Cohen and B. N. Taylor, CODATA Bull. ***63****:1–49 (1986); J. Res. Nat. Bur. Standards,* ***92****:85 (1987).*

A. Defined values

Physical quantity	Name of SI unit	Symbol for SI unit	Definition
1. Base SI units			
Amount of substance	mole	mol	Amount of substance which contains as many specified entities as there are atoms of carbon-12 in exactly 0.012 kg of that nuclide. The elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles.
Electric current	ampere	A	Magnitude of the current that, when flowing through each of two straight parallel conductors of infinite length, of negligible cross-section, separated by 1 meter in a vacuum, results in a force between the two wires of 2×10^{-7} newton per meter of length.
Length	meter	m	Distance light travels in a vacuum during 1/299 792 458 of a second.
Luminous intensity	candela	cd	Luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian.
Mass	kilogram	kg	Mass of a cylinder of platinum-iridium alloy kept at Paris.
Temperature	kelvin	K	Defined as the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.
Time	second	s	Duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom.
2. Supplementary SI units			
Plane angle	radian	rad	The plane angle between two radii of a circle which cut off on the circumference an arc equal in length to the radius.
Solid angle	steradian	sr	The solid angle which, having its vertex in the center of a sphere, cuts off an area of the surface of the sphere equal to that of a square with sides of length equal to the radius of the sphere.

TABLE 2.1 Fundamental Physical Constants (*Continued*)

B. Derived SI units

Physical quantity	Name of SI unit	Symbol for SI unit	Expression in terms of SI base units
Absorbed dose (of radiation)	gray	Gy	$\mathrm{J \cdot kg^{-1}}$
Activity (radioactive)	becquerel	Bq	$\mathrm{s^{-1} = m^2 \cdot s^{-2}}$
Capacitance (electric)	farad	F	$\mathrm{C \cdot V^{-1} = m^{-2} \cdot kg^{-1} \cdot s^4 \cdot A^2}$
Charge (electric)	coulomb	C	$\mathrm{A \cdot s}$
Conductance (electric)	siemens	S	$\mathrm{\Omega^{-1} = m^{-2} \cdot kg^{-1} \cdot s^3 \cdot A^2}$
Dose equivalent (radiation)	sievert	Sv	$\mathrm{J \cdot kg^{-1} = m^2 \cdot s^{-2}}$
Energy, work, heat	joule	J	$\mathrm{N \cdot m = m^2 \cdot kg \cdot s^{-2}}$
Force	newton	N	$\mathrm{m \cdot kg \cdot s^{-2}}$
Frequency	hertz	Hz	$\mathrm{s^{-1}}$
Illuminance	lux	lx	$\mathrm{cd \cdot sr \cdot m^{-2}}$
Inductance	henry	H	$\mathrm{V \cdot A^{-1} \cdot s = m^2 \cdot kg \cdot s^{-2} \cdot A^{-2}}$
Luminous flux	lumen	Lm	$\mathrm{cd \cdot sr}$
Magnetic flux	weber	Wb	$\mathrm{V \cdot s = m^2 \cdot kg \cdot s^{-2} \cdot A^{-1}}$
Magnetic flux density	tesla	T	$\mathrm{V \cdot s \cdot m^{-2} = kg \cdot s^{-2} \cdot A^{-1}}$
Potential, electric (electromotive force)	volt	V	$\mathrm{J \cdot C^{-1} = m^2 \cdot kg \cdot s^{-3} \cdot A^{-1}}$
Power, radiant flux	watt	W	$\mathrm{J \cdot s^{-1} = m^2 \cdot kg \cdot s^{-3}}$
Pressure, stress	pascal	Pa	$\mathrm{N \cdot m^{-2} = m^{-1} \cdot kg \cdot s^{-2}}$
Resistance, electric	ohm	Ω	$\mathrm{V \cdot A^{-1} = m^2 \cdot kg \cdot s^{-3} \cdot A^{-2}}$
Temperature, Celsius	degree Celsius	°C	$\mathrm{{}^\circ C = (K - 273.15)}$

C. Recommended consistent values of constants

Quantity	Symbol	Value*
Anomalous electron moment correction	$\mu_e - 1$	0.001 159 615(15)
Atomic mass constant	$m_u = 1\ \mathrm{u}$	$1.660\ 540\ 2(10) \times 10^{-27}$ kg
Avogadro constant	L, N_A	$6.022\ 136\ 7(36) \times 10^{23}\ \mathrm{mol^{-1}}$
Bohr magneton $(= eh/4\pi m_e)$	μ_B	$9.274\ 015\ 4(31) \times 10^{-24}\ \mathrm{J \cdot T^{-1}}$
Bohr radius	a_0	$5.291\ 772\ 49(24) \times 10^{-11}$ m
Boltzmann constant	k	$1.380\ 658(12) \times 10^{-23}\ \mathrm{J \cdot K^{-1}}$
Charge-to-mass ratio for electron	e/m_e	$1.758\ 805(5) \times 10^{-11}\ \mathrm{C \cdot kg^{-1}}$
Compton wavelength of electron	λ_c	$2.426\ 309(4) \times 10^{-12}$ m
Compton wavelength of neutron	$\lambda_{c,n}$	$1.319\ 591(2) \times 10^{-15}$ m
Compton wavelength of proton	$\lambda_{c,p}$	$1.321\ 410(2) \times 10^{-15}$ m
Diamagnetic shielding factor, spherical water molecule	$1 + \sigma(H_2O)$	1.000 025 64(7)
Electron magnetic moment	μ_e	$9.284\ 770\ 1(31) \times 10^{-24}\ \mathrm{J \cdot T^{-1}}$
Electron radius (classical)	r_e	$2.817\ 938(7) \times 10^{-15}$ m
Electron rest mass	m_e	$9.109\ 389\ 7(54) \times 10^{-31}$ kg
Elementary charge	e	$1.602\ 177\ 33(49) \times 10^{-19}$ C
Energy equivalents:		
1 electron mass		0.511 003 4(14) MeV
1 electronvolt	1 eV/k	$1.160\ 450(36) \times 10^4$ K
	1 eV/hc	$8.065\ 479(21) \times 10^3\ \mathrm{cm^{-1}}$
	1 eV/h	$2.417\ 970(6) \times 10^{14}$ Hz
1 neutron mass		939.573 1(27) MeV
1 proton mass		938.279 6(27) MeV

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* The digits in parentheses following a numerical value represent the standard deviation of that value in terms of the final listed digits.

TABLE 2.1 Fundamental Physical Constants (*Continued*)

C. Recommended consistent values of constants (*continued*)

Quantity	Symbol	Value*
1 u		931.501 6(26) MeV
Faraday constant	F	96 485.309(29) C · mol^{-1}
Fine structure constant	α	0.007 297 353 08(33)
	α^{-1}	137.035 989 5(61)
First radiation constant	c_1	3.741 774 9(22) $\times 10^{-16}$ W · m^2
Gas constant	R	8.314 510(70) J · K^{-1} · mol^{-1}
g factor (Lande) for free electron	g_e	2.002 319 304 386(20)
Gravitational constant	G	6.672 59(85) $\times 10^{-11}$ m^3 · kg^{-1} · s^{-2}
Hartree energy	E_h	4.359 748 2(26) $\times 10^{-18}$ J
Josephson frequency-voltage ratio		4.835 939(13) $\times 10^{14}$ Hz · V^{-1}
Magnetic flux quantum	Φ_0	2.067 851(5) $\times 10^{-15}$ Wb
Magnetic moment of protons in water	μ_p/μ_B	1.520 993 129(17) $\times 10^{-3}$
Molar volume, ideal gas, $p = 1$ bar, $\theta = 0°\mathrm{C}$		22.711 08(19) L · mol^{-1}
Neutron rest mass	m_n	1.674 928 6(10) $\times 10^{-27}$ kg
Nuclear magneton	μ_N	5.050 786 6(17) $\times 10^{-27}$ J · T^{-1}
Permeability of vacuum	μ_0	$4\pi \times 10^{-7}$ H · m^{-1} exactly
Permittivity of vacuum	ϵ_0	8.854 187 816 $\times 10^{-12}$ F · m^{-1}
	$\hbar = h/2\pi$	1.054 572 66(63) $\times 10^{-34}$ J · s
Planck constant	h	6.626 0.75 5(40) $\times 10^{-34}$ J · s
Proton magnetic moment	μ_p	1.410 607 61(47) $\times 10^{-26}$ J · T^{-1}
Proton magnetogyric ratio	γ_p	2.675 221 28(81) $\times 10^{8}$ s^{-1} · T^{-1}
Proton resonance frequency per field in H_2O	$\gamma_p'/2\pi$	42.576 375(13) MHz · T^{-1}
Proton rest mass	m_p	1.672 623 1(10) $\times 10^{-27}$ kg
Quantum-charge ratio	h/e	4.135 701(11) $\times 10^{-15}$ J · Hz^{-1} · C^{-1}
Quantum of circulation	h/m_e	7.273 89(1) $\times 10^{-4}$ J · s · kg^{-1}
Ratio, electron-to-proton magnetic moments	μ_e/μ_p	6.582 106 88(7) $\times 10^{2}$
Rydberg constant	R_∞	1.097 373 153 4(13) $\times 10^{7}$ m^{-1}
Second radiation constant	c_2	1.438 769(12) $\times 10^{-2}$ m · K
Speed of light in vacuum	c_0	299 792 458 m · s^{-1} exactly
Standard acceleration of free fall	g_n	9.806 65 m · s^{-2} exactly
Standard atmosphere	atm	101 325 Pa exactly
Stefan-Boltzmann constant	σ	5.670 51(19) $\times 10^{-8}$ W · m^{-2} · K^{-4}
Thomson cross section	σ_e	6.652 448(33) $\times 10^{-29}$ m^2
Wien displacement constant	b	0.289 78(4) cm · K
Zeeman splitting constant	μ_B/hc	4.668 58(4) $\times 10^{-5}$ cm^{-1} · G^{-1}

D. Units in use together with SI units

Physical quantity	Name of unit	Symbol for unit	Value in SI units
Area	barn	b	10^{-28} m
Energy	electronvolt	eV ($e \times$ V)	≈1.60218 $\times 10^{-19}$ J
	megaelectronvolt[1]	MeV	
Length	ångström[2]	Å	10^{-10} m; 0.1 nm
Mass	tonne	t	10^{3} kg; Mg

*The digits in parentheses following a numerical value represent the standard deviation of that value in terms of the final listed digits.

[1] The term million electronvolts is frequently used in place of megaelectronvolts.

[2] The ångström and bar are approved for temporary use with SI units; however, they should not be introduced if not used at present.

TABLE 2.1 Fundamental Physical Constants (*Continued*)

D. Units in use together with SI units (*continued*)

Physical quantity	Name of unit	Symbol for unit	Value in SI units
	unified atomic mass unit	u[= $m_a(^{12}\mathrm{C})/12$]	$\approx 1.66054 \times 10^{-27}$ kg
	dalton[3]	Da	
Plane angle	degree	°	(π/180) rad
	minute	′	(π/10 800) rad
	second	″	(π/648 000) rad
Pressure	bar[2]	bar	10^5 Pa = 10^5 N m^{-2}
Time	minute	min	60 s
	hour	h	3600 s
	day	d	86 400 s
Volume	liter (litre)	L, l	dm^3 = 10^{-3} m^3
	milliliter	mL, ml	cm^3 = 10^{-6} m^3

[2] The ångström and bar are approved for temporary use with SI units; however, they should not be introduced if not used at present.

[3] The name dalton and symbol Da have not been approved although they are often used for large molecules.

TABLE 2.2 Physical and Chemical Symbols and Definitions

Symbols separated by commas represent equivalent recommendations. Symbols for physical and chemical quantities should be printed in *italic* type. Subscripts and superscripts which are themselves symbols for physical quantities should be italicized; all others should be in Roman type. Vectors and matrices should be printed in boldface italic type, e.g., ***B, b.*** Symbols for units should be printed in Roman type and should remain unaltered in the plural, and should not be followed by a full stop except at the end of a sentence. References: International Union of Pure and Applied Chemistry, *Quantities, Units and Symbols in Physical Chemistry,* Blackwell, Oxford, 1988; "Manual of Symbols and Terminology for Physicochemical Quantities and Units," *Pure Applied Chem.* **31:**577–638 (1972), **37:**499–516 (1974), **46:**71–90 (1976), **51:**1–41, 1213–1218 (1979); **53:**753–771 (1981), **54:**1239–1250 (1982), **55:**931–941 (1983); IUPAP-SUN, "Symbols, Units and Nomenclature in Physics," *Physica* **93A:** 1–60 (1978).

A. Atoms and molecules

Name	Symbol	SI unit	Definition
Activity (radioactivity)	A	Bq	$A = -dN_\mathrm{B}/dt$
Atomic mass constant	m_u	kg	$m_\mathrm{u} = m_\mathrm{a}(^{12}\mathrm{C})/12$
Bohr magneton	μ_B	J · T^{-1}	$\mu_\mathrm{B} = eh/4\pi m_\mathrm{e}$
Bohr radius	a_0	m	$a_0 = 2\epsilon_0 h^2/m_\mathrm{e} e^2$
Decay (rate) constant	λ	s^{-1}	$A = \lambda N_\mathrm{B}$
Dissociation energy	D, E_d	J	
From ground state	D_0	J	
From the potential minimum	D_e	J	
Electric dipole moment of a molecule	$\boldsymbol{p}, \boldsymbol{\mu}$	C · m	$E_\mathrm{p} = -\boldsymbol{p} \cdot \boldsymbol{E}$
Electric field gradient	$\boldsymbol{q}$	V · m^{-2}	$\mathrm{q}_{\alpha\beta} = -\partial^2 V/\partial\alpha\partial\beta$
Electric polarizability of a molecule	α	C · m^2 · V^{-1}	$p(\mathrm{induced}) = \alpha E$
Electron affinity	E_ea	J	
Electron rest mass	m_e	kg	
Elementary charge, proton charge	e	C	
Fine structure constant	α		$\alpha = e^2/2\epsilon_0 hc$
g factor	g		

TABLE 2.2 Physical and Chemical Symbols and Definitions (*Continued*)

A. Atoms and molecules (*continued*)

Name	Symbol	SI unit	Definition
Hartree energy	E_h	J	$E_h = h^2/4\pi^2 m_e a_0^2$
Ionization energy	E_i	J	
Larmor circular frequency	ω_L	s^{-1}	$\omega_L = (e/2m)B$
Larmor frequency	ν_L	Hz	$\nu_L = \omega_L/2\pi$
Longitudinal relaxation time	T_1	s	
Magnetogyric ratio	γ	$C \cdot kg^{-1}$	$\gamma = \mu/L$
Magnetic dipole moment of a molecule	$\boldsymbol{m}, \boldsymbol{\mu}$	$J \cdot T^{-1}$	$E_\mathbf{p} = -\boldsymbol{m} \cdot \boldsymbol{B}$
Magnetizability of a molecule	ξ	$J \cdot T^{-2}$	$\boldsymbol{m} = \boldsymbol{\xi} \boldsymbol{B}$
Mass of atom, atomic mass	m, m_a	kg	
Neutron number	N		$N = A - Z$
Nuclear magneton	μ_N	$J \cdot T^{-1}$	$\mu_N = (m_e/m_p)\mu_B$
Nucleon number, mass number	A		
Planck constant	h	$J \cdot s$	
Planck constant/2π	$\hbar$	$J \cdot s$	$\hbar = h/2\pi$
Principal quantum number (H atom)	n		$E = -hcR/n^2$
Proton number, atomic number	Z		
Quadrupole interaction	$\boldsymbol{\chi}$	J	$\chi_{\alpha\beta} = eQq_{\alpha\beta}$
Quadrupole moment of a molecule	$\boldsymbol{Q}; \boldsymbol{\Theta}$	$C \cdot m^2$	$E_p = 0.5\, \boldsymbol{Q}: \boldsymbol{V}'' = \frac{1}{3}\, \boldsymbol{\Theta} \boldsymbol{V}''$
Quadrupole moment	eQ	$C \cdot m^2$	$eQ = 2\langle \Theta_{zz} \rangle$
Rydberg constant	R_∞	m^{-1}	$R_\infty = E_h/2hc$
Transverse relaxation time	T_2	s	

B. Chemical reactions

Name	Symbol	SI unit	Definition
Amount (of substance)	n	mol	$n_B = N_B/L$
Atomic mass	m, m_a	kg	
Atomic mass constant[a]	m_u	kg	$m_u = m_a(^{12}C)/12$
Avogadro constant	L, N_A	mol^{-1}	
Concentration, amount (concentration)	c	$mol \cdot m^{-3}$	$c_B = n_B/V$
Degree of dissociation	α		
Density (mass)	ρ, γ	$kg \cdot m^{-3}$	$\rho = m_B/V$
Extent of reaction, advancement	ξ	mol	$\Delta\xi = \Delta n_B/\nu_{B_B}$
Mass (molecular or formula unit)	m, m_f	kg	
Mass fraction	w		$w_B = m_B/\Sigma m_i$
Molality (of a solute)	m	$mol \cdot kg^{-1}$	$m_B = n_B/m_A$
Molar mass	M	$kg \cdot mol^{-1}$	$M_B = m/n_B$
Molar volume	V_m	$m^3 \cdot mol^{-1}$	$V_{m,B} = V/n_B$
Molecular weight (relative molar mass)	M_r		$M_{r,B} = m_B/m_u$
Mole fraction[b], number fraction	x, y		$x_B = n_B/\Sigma n_i$
Number concentration	C, n	m^{-3}	$C_B = N_B/V$
Number of entities (e.g., molecules, atoms, ions, formula units)	N		
Pressure (partial)	p_B	Pa	$p_B = y_B p$
Pressure (total)	p, P	Pa	
Solubility	s	$mol \cdot m^{-3}$	$s_B = c_B$ (saturated solution)

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[a] In biochemistry this unit is called the dalton, with symbol Da.

[b] For condensed phases x is used, and for gaseous mixtures y may be used.

TABLE 2.2 Physical and Chemical Symbols and Definitions (*Continued*)

B. Chemical reactions (*continued*)

Name	Symbol	SI unit	Definition
Stoichiometric number	ν		
Surface concentration	Γ	$\mathrm{mol \cdot m^{-2}}$	$\Gamma_B = n_B/A$
Volume fraction	ϕ		$\phi_B = V_B/\Sigma V_i$

Symbols for particles and nuclear reactions:

Alpha particle	α	Muon, positive	μ^+
Beta particle	β^-, β^+	Neutron	n, n^0
Deuteron	d, ^2H	Photon	γ
Electron	e, e^-	Proton	p, p^+
Helion	h	Triton	t, ^3H
Muon, negative	μ^-		

The meaning of the symbolic expression indicating a nuclear reaction:

$$\begin{matrix}\text{initial}\\ \text{nuclide}\end{matrix}\left(\begin{matrix}\text{incoming particles}\\ \text{or quanta}\end{matrix}, \begin{matrix}\text{outgoing particles}\\ \text{or quanta}\end{matrix}\right)\begin{matrix}\text{final}\\ \text{nuclide}\end{matrix}$$

Examples: $^{14}N(\alpha, p)^{17}O$, $^{23}Na(\gamma, 3n)^{20}Na$

States of aggregation:

am	amorphous solid	cd	condensed phase (solid or liquid)
aq	aqueous solution	cr	crystalline
as, ∞	aqueous solution at infinite dilution	fl	fluid phase (gas or liquid)
		lc	liquid crystal
g	gas	vit	vitreous substance
l	liquid	mon	monomeric form
s	solid	pol	polymeric form
sln	solution	ads	species adsorbed on a substance

C. Chromatography

Name	Symbol	Definition
Adjusted retention time	t'_R	$t'_R = t_R - t_M$
Adjusted retention volume	V'_R	$V'_R = V_R - V_M$
Average linear gas velocity	μ	$\mu = L/t_M$
Band variance	σ^2	
Bed volume	V_g	
Capacity, volume	Q_v	
Capacity, weight	Q_w	
Column length	L	
Column temperature	θ	
Column volume	V_{col}	$V_{col} = \pi D d_c^2/4$
Concentration at peak maximum	C_{max}	
Concentration of solute in mobile phase	C_M	
Concentration of solute in stationary phase	C_S	
Density of liquid phase	ρ_L	
Diffusion coefficient, liquid film	D_f	
Diffusion coefficient, mobile phase	D_M	

TABLE 2.2 Physical and Chemical Symbols and Definitions (*Continued*)

C. Chromatography (*continued*)

Name	Symbol	Definition
Diffusion coefficient, stationary phase	D_S	
Distribution ratio	D_c	$= [A^+]_S/[A^+]_M$
		$= \dfrac{\text{amount of A per cm}^3\text{ stationary phase}}{\text{amount of A per cm}^3\text{ of mobile phase}}$
	D_g	$= \dfrac{\text{amount A per gram dry stationary phase}}{\text{amount A per cm}^3\text{ of mobile phase}}$
	D_v	$= \dfrac{\text{amount A, stationary phase per cm}^3\text{ bed volume}}{\text{amount A per cm}^3\text{ of mobile phase}}$
	D_S	$= \dfrac{\text{amount of A per m}^2\text{ of surface}}{\text{amount of A per cm}^3\text{ of mobile phase}}$
Elution volume, exclusion chromatography	V_e	
Flow rate, column	F_c	$F_c = (\pi d_c^2/4)(\epsilon_{\text{tot}})(L/t_M)$
Gas/liquid volume ratio	β	
Inner column volume	V_i	
Interstitial (outer) volume	V_o	
Kovats retention indices	RI	
Matrix volume	V_g	
Net retention volume	V_N	$V_N = jV'_R$
Obstruction factor	γ	
Packing uniformity factor	λ	
Particle diameter	d_p	$d_p = L/Nh$
Partition coefficient	K	$K = C_S/C_M = (V_R - V_M)/V_S$
Partition ratio	k'	$k' = C_SV_S/C_MV_M = K(V_S/V_M)$
Peak asymmetry factor	AF	Ratio of peak half-widths at 10% peak height
Peak resolution	Rs	$\text{Rs} = (t_{R,2} - t_{R,1})/0.5(W_2 + W_1)$
Plate height	H	$H = L/N_{\text{eff}}$
Plate number	N_{eff}	$N_{\text{eff}} = L/H = 16(t'_R/W_b)^2 = 5.54(t'_R/W_{1/2})^2$
Porosity, column	ϵ	
Pressure, column inlet	p_i	
Presure, column outlet	P_o	
Pressure drop	ΔP	
Pressure-gradient correction	j	$j = \dfrac{3[(p_i/p_o)^2 - 1]}{2[(p_i/p_o)^3 - 1]}$
Recovery factor	R_n	$R_n = 1 - (rD_c + 1)^{-n}$; $r = V_{\text{org}}/V_{\text{aq}}$
Reduced column length	λ	$\lambda = L/d_p$
Reduced plate height	h	$h = H/d_p$
Reduced velocity	v	$v = \mu d_p/D_M = Kd_p/t_MD_M$
Relative retention ratio	α	$\alpha = (k'_2/k'_1)$
Retardation factor[c]	R_f	$R_f = d_{\text{solute}}/d_{\text{mobile phase}}$
Retention time	t_R	$t_R = t_M(1 + k') = L/\mu$
Retention volume	V_R	$V_R = t_RF_c$
Selectivity coefficient[d]	$k_{\text{A,B}}$	$k_{\text{A,B}} = [A^+]_r[B^+]/[B^+]_r[A^+]$
Separation factor	$\alpha_{\text{A/B}}$	$\alpha_{\text{A/B}} = (D_c)_{\text{A}}/(D_c)_{\text{B}}$

[c] The distance d corresponds to the movement of solute and mobile phase from the starting (sample spotting) line.

[d] Subscript "*r*" represents an ion-exchange resin phase. Two immiscible liquid phases might be represented similarly using subscripts "1" and "2."

TABLE 2.2 Physical and Chemical Symbols and Definitions (*Continued*)

C. Chromatography (*continued*)

Name	Symbol	Definition
Specific retention volume	V_g°	$V_g^\circ = 273\ R/(p^\circ\ Mw_L)$
Thickness (effective) of stationary phase	d_f	
Total bed volume	V_{tot}	
Transit time of nonretained solute	t_M, t_0	
Vapor pressure	p	
Volume liquid phase in column	V_L	
Volume mobile phase in column	V_M	
Weight of liquid phase	w_L	
Zone width at baseline	W_b	$W_b = 4\sigma$
Zone width at ½ peak height	$W_{1/2}$	

D. Colloid and surface chemistry

Name	Symbol	SI unit	Definition
Adsorbed amount of B	$n_{\mathrm{B}}^{\mathrm{s}}$	mol	
Area per molecule	a, σ	m^2	$a_{\mathrm{B}} = A/N_{\mathrm{N}}^{\sigma}$
Area per molecule in a filled monolayer	a_{m}	m^2	$a_{\mathrm{m,B}} = A/N_{\mathrm{m,B}}$
Average molar masses:			
Mass-average	M_m	$\mathrm{kg \cdot mol^{-1}}$	$M_m = \Sigma n_i M_i^2/\Sigma n_i M_i$
Number-average	M_n	$\mathrm{kg \cdot mol^{-1}}$	$M_n = \Sigma n_i M_i/\Sigma n_i$
Z-average	M_Z	$\mathrm{kg \cdot mol^{-1}}$	$M_Z = \Sigma n_i M_i^3/\Sigma n_i M_i^2$
Contact angle	θ	rad	
Film tension	Σ_{f}	$\mathrm{N \cdot m^{-1}}\ \phi$	$\Sigma_{\mathrm{f}} = 2\gamma_{\mathrm{f}}$
Film thickness	t, h, δ	m	
Reciprocal thickness of the double layer	κ	m^{-1}	$\kappa = [2F^2 I_c/\epsilon RT]^{1/2}$
Retarded van der Waals constant	β, B	J	
Sedimentation coefficient[e]	s	s	$s = v/a$
Specific surface area	a, s, a_s	$\mathrm{m^2/kg}$	$a = A/m$
Surface coverage	θ		$\theta = N_B^{\sigma}/N_{\mathrm{B}}$
Surface excess of B	n_{B}^{σ}	mol	
Surface pressure	π^{s}, π	$\mathrm{N \cdot m^{-1}}$	$\pi^{\mathrm{s}} = \gamma^0 - \gamma$
Surface tension, interfacial tension	γ, σ	$\mathrm{J \cdot m^{-2}}$	$\gamma = (\partial G/\partial A_{\mathrm{s}})_{\mathrm{T,p}}$
Thickness of (surface or interfacial) layer	τ, δ, t	m	
Total surface excess concentration	Γ	$\mathrm{mol \cdot m^{-2}}$	$\Gamma = \Sigma\Gamma_i$
van der Waals constant	λ	J	
van der Waals-Hamaker constant	A_{H}	J	

[e] v is the velocity of sedimentation and a is the acceleration of free fall or centrifugation.

TABLE 2.2 Physical and Chemical Symbols and Definitions (*Continued*)

E. Electricity and magnetism

Name	Symbol	SI unit	Definition
Admittance	Y	S	$Y = 1/Z$
Capacitance	C	F, $\mathrm{C \cdot V^{-1}}$	$C = Q/U$
Charge density	ρ	$\mathrm{C \cdot m^{-3}}$	$\rho = Q/V$
Conductance	G	S	$G = 1/R$
Conductivity	κ	$\mathrm{S \cdot m^{-1}}$	$\kappa = 1/\rho$
Dielectric polarization (dipole moment per volume)	$\boldsymbol{P}$	$\mathrm{C \cdot m^{-2}}$	$\boldsymbol{P} = \boldsymbol{D} - \boldsymbol{\epsilon}_0\boldsymbol{E}$
Electrical resistance	R	Ω	$R = U/I = \Delta V/I$
Electric current	I	A	$I = dQ/dt$
Electric current density	j, J	$\mathrm{A \cdot m^{-2}}$	$I = \int j\, dA$
Electric dipole moment	$\boldsymbol{p}, \boldsymbol{\mu}$	$\mathrm{C \cdot m}$	$\boldsymbol{p} = \boldsymbol{Qr}$
Electric displacement	$\boldsymbol{D}$	$\mathrm{C \cdot m^{-2}}$	$\boldsymbol{D} = \boldsymbol{\epsilon E}$
Electric field strength	$\boldsymbol{E}$	$\mathrm{V \cdot m^{-1}}$	$\boldsymbol{E} = \boldsymbol{F}/Q = -\mathrm{grad}\ V$
Electric flux	$\boldsymbol{\Psi}$	C	$\Psi = \int \boldsymbol{D}\, d\boldsymbol{A}$
Electric potential	V, ϕ	V, $\mathrm{J \cdot C^{-1}}$	$V = dW/dQ$
Electric potential difference	$U, \Delta V$	V	$U = V_2 - V_1$
Electric susceptibility	χ_e		$\chi_\mathrm{e} = \epsilon_\mathrm{r} - 1$
Electromotive force	E	V	$E = \int (F/Q)\, ds$
Impedance	Z	Ω	$Z = R + iX$
Loss angle[f]	δ	rad	$\delta = (\pi/2) + \phi_I - \phi_U$
Magnetic dipole moment	$\boldsymbol{m}, \boldsymbol{\mu}$	$\mathrm{A \cdot m^2}$	$E_\mathrm{p} = -\boldsymbol{mB}$
Magnetic field strength	H	$\mathrm{A \cdot m^{-1}}$	$\boldsymbol{B} = \boldsymbol{\mu H}$
Magnetic flux	Φ	Wb	$\Phi = \int \boldsymbol{B}\, d\boldsymbol{A}$
Magnetization (magnetic dipole moment per volume)	$\boldsymbol{M}$	$\mathrm{A \cdot m^{-1}}$	$\boldsymbol{M} = (\boldsymbol{B}/\mu_0) - \boldsymbol{H}$
Magnetic susceptibility	χ, κ		$\chi = \mu_\mathrm{r} - 1$
Magnetic vector potential	$\boldsymbol{A}$	$\mathrm{Wb \cdot m^{-1}}$	$\boldsymbol{B} = \nabla\boldsymbol{A}$
Molar magnetic susceptibility	χ_m	$\mathrm{n^3/mol}$	$\chi_\mathrm{m} = V_\mathrm{m}\chi$
Mutual inductance	M, L_{12}	H	$E_1 = L_{12}(dI_2/dt)$
Permeability	μ	$\mathrm{H \cdot m^{-1}}$	$\boldsymbol{B} = \mu\boldsymbol{H}$
Permeability of vacuum	μ_0	$\mathrm{H \cdot m^{-1}}$	
Permittivity	ϵ	$\mathrm{F \cdot m^{-1}}$	$\boldsymbol{D} = \boldsymbol{\epsilon E}$
Permittivity of vacuum	ϵ_0	$\mathrm{F \cdot m^{-1}}$	$\epsilon_0 = \mu_0^{-1}\, c_0^{-2}$
Poynting vector	$\boldsymbol{S}$	$\mathrm{W \cdot m^{-2}}$	$\boldsymbol{S} = \boldsymbol{E} \cdot \boldsymbol{H}$
Quantity of electricity, electric charge	Q	C	
Reactance	X	Ω	$X = (U/I) \sin \delta$
Relative permeability	μ_r		$\mu_\mathrm{r} = \mu/\mu_0$
Relative permittivity[g]	ϵ_r		$\epsilon_\mathrm{r} = \epsilon/\epsilon_0$
Resistivity	ρ	$\Omega \cdot \mathrm{m}$	$\rho = E/j$
Self-inductance	L	H	$E = -L(dI/dt)$
Susceptance	B	S	$Y = G + iB$

[f] ϕ_I and ϕ_U are the phases of current and potential difference.

[g] This quantity was formerly called the dielectric constant.

TABLE 2.2 Physical and Chemical Symbols and Definitions (*Continued*)

F. Electrochemistry

Name	Symbol	SI unit	Definition
Charge density (surface)	σ	$\mathrm{C \cdot n^{-2}}$	$\sigma = Q/A$
Charge number of an ion	z		$z_{\mathrm{B}} = Q_{\mathrm{B}}/e$
Charge number of electrochemical cell reaction	n, (z)		
Conductivity (specific conductance)	κ	$\mathrm{S \cdot m^{-1}}$	$\kappa = j/E$
Conductivity cell constant	K_{cell}	$\mathrm{m^{-1}}$	$K_{\mathrm{cell}} = \kappa R$
Current density (electric)	j	$\mathrm{A \cdot m^{-2}}$	$j = I/A$
Diffusion rate constant, mass transfer coefficient	k_{d}	$\mathrm{m \cdot s^{-1}}$	$k_{\mathrm{d,B}} = \lvert\nu_{\mathrm{B}}\rvert I_{1,\mathrm{B}}/nFcA$
Electric current	I	A	$I = dQ/dt$
Electric mobility	μ	$\mathrm{m^2 \cdot V^{-1} \cdot s^{-1}}$	$\mu_{\mathrm{B}} = v_{\mathrm{B}}/E$
Electric potential difference (of a galvanic cell)	ΔV, E, U	V	$\Delta V = V_{\mathrm{R}} - V_{\mathrm{L}}$
Electrochemical potential	$\tilde{\mu}$	$\mathrm{J \cdot mol^{-1}}$	$\tilde{\mu}_{\mathrm{B}}^{\alpha} = (\partial G/\partial n_{\mathrm{B}}^{\alpha})$
Electrode reaction rate constant	k	(varies)	$k_{\mathrm{ox}} = I_{\mathrm{a}}/\left(nFA\prod_i \mathrm{c}_i^{n_i}\right)$
Electrokinetic potential (zeta potential)	ζ	V	
Elementary charge (proton charge)	e	C	
emf, electromotive force	E	V	$E = \lim_{I\to 0} \Delta V$
emf of the cell	E	V	$E = E° - (RT/nF) \times \Sigma\nu_i \ln a_i$
Faraday constant	F	$\mathrm{C \cdot mol^{-1}}$	$F = eL$
Galvani potential difference	$\Delta\phi$	V	$\Delta_{\alpha}^{\beta}\phi = \phi^{\beta} - \phi^{\alpha}$
Inner electrode potential	ϕ	V	$\nabla\phi = -E$
Ionic conductivity	λ	$\mathrm{S \cdot m^2 \cdot mol^{-1}}$	$\lambda_{\mathrm{B}} = \lvert z_{\mathrm{B}}\rvert Fu_{\mathrm{B}}$
Ionic strength	I_c, I	$\mathrm{mol \cdot m^{-3}}$	$I_c = ½\Sigma c_i z_c^2$
Mean ionic activity	$a_{\pm}$		$a_{\pm} = m_{\pm\gamma\pm}/m^0$
Mean ionic activity coefficient	$\gamma_{\pm}$		$\gamma_{\pm}^{(\nu_+ + \nu_-)} = (\gamma_{\pm}^{\nu_+})(\gamma_{-}^{\nu_-})$
Mean ionic mobility	$m_{\pm}$	$\mathrm{mol \cdot kg^{-1}}$	$m_{\pm}^{(\nu_+ + \nu_-)} = (m_{+}^{\nu_+})(m_{-}^{\nu_-})$
Molar conductivity (of an electrolyte)	Λ	$\mathrm{S \cdot m^{-2}\ mol^{-1}}$	$\Lambda_{\mathrm{B}} = \kappa c_{\mathrm{B}}$
pH	pH		$\mathrm{pH} \simeq -\log\left[\dfrac{c(\mathrm{H^+})}{\mathrm{mol \cdot dm^{-3}}}\right]$
Outer electrode potential	ψ	V	$\psi = Q/4\pi\epsilon_0 r$
Overpotential	η	V	$\eta = E_I - E_{I=0} - IR_u$
Reciprocal radius of ionic atmosphere	κ	$\mathrm{m^{-1}}$	$\kappa = (2F^2I/\epsilon RT)^{1/2}$
Standard emf, standard potential of electrochemical cell reaction	E^0	V	$E^0 = -\Delta_{\mathrm{r}}G^0/nF = (RT/nF) \ln K$
Surface electric potential	χ	V	$\chi = \phi - \psi$
Thickness diffusion layer	δ	m	$\delta_{\mathrm{B}} = D_{\mathrm{B}}/k_{\mathrm{d,B}}$
Transfer coefficient	α		$\alpha_{\mathrm{c}} = \dfrac{-\lvert\nu\rvert RT}{nF}\dfrac{\partial \ln\lvert I_c\rvert}{\partial E}$
Transport number	t		$t_{\mathrm{B}} = j_{\mathrm{B}}/\Sigma j_{\mathrm{i}}$
Volta potential difference	$\Delta\psi$	V	$\Delta_{\alpha}^{\beta} = \psi^{\beta} - \beta^{\alpha}$

TABLE 2.2 Physical and Chemical Symbols and Definitions (*Continued*)

G. Electromagnetic radiation (*continued*)

Name	Symbol	SI unit	Definition
Absorbance	α		$\alpha = \Phi_{\mathrm{abs}}/\Phi_0$
Absorbance (decaidic)	A		$A = -\log(1 - \alpha_{\mathrm{i}})$
Absorbance (napierian)	B		$B = -\ln(1 - \alpha_{\mathrm{i}})$
Absorption coefficient:			
Linear (decaidic)	a, K	m^{-1}	$a = A/l$
Linear (napierian)	α	m^{-1}	$\alpha = B/l$
Molar (decaidic)	ϵ	$\mathrm{m}^2 \cdot \mathrm{mol}^{-1}$	$\epsilon = a/d = A/cl$
Molar (napierian)	κ	$\mathrm{m}^2 \cdot \mathrm{mol}^{-1}$	$\kappa = \alpha/c = B/cl$
Absorption index	k		$k = \alpha/4\pi\tilde{\nu}$
Angle of optical rotation	α	rad	
Circular frequency	ω	s^{-1}, $\mathrm{rad} \cdot \mathrm{s}^{-1}$	$\omega = 2\pi\nu$
Complex refractive index	$\hat{n}$		$\hat{n} = \eta + ik$
Concentration, amount of substance	c	$\mathrm{mol} \cdot \mathrm{m}^3$	
Concentration, mass	γ	$\mathrm{kg} \cdot \mathrm{m}^3$	
Einstein transition probabilities:			
Spontaneous emission	A_{nm}	s^{-1}	$dN_n/dt = -A_{nm}N_n$
Stimulated absorption	B_{mn}	$\mathrm{s} \cdot \mathrm{kg}^{-1}$	$dN_n/dt = \rho_{\tilde{\nu}}(\tilde{\nu}_{nm})B_{mn}N_m$
Stimulated emission	B_{nm}	$\mathrm{s} \cdot \mathrm{kg}^{-1}$	$dN_n/dt = \rho\tilde{\nu}(\tilde{\nu}_{nm})B_{mn}N_m$
Emittance	ϵ		$\epsilon = M/M_{\mathrm{bb}}$
By blackbody	M_{bb}		
First radiation constant	c_1	$\mathrm{W} \cdot \mathrm{m}^2$	$c_1 = 2\pi hc_0^2$
Frequency	ν	Hz	$\nu = c/\lambda$
Irradiance (radiant flux received)	$E, (I)$	$\mathrm{W} \cdot \mathrm{m}^{-2}$	$E = d\Phi/dA$
Molar refraction	R, R_{m}	$\mathrm{m}^3 \cdot \mathrm{mol}^{-1}$	$R = \dfrac{(n^2 - 1)}{(n^2 + 2)} V_m$
Path length (absorbing)	l	m	
Optical rotatory power	$[\alpha]_\lambda^\theta$	rad	$[\alpha]_\lambda^\theta = \alpha/\gamma l$
Planck constant	h	$\mathrm{J} \cdot \mathrm{s}$	
Planck constant/2π	$\hbar$	$\mathrm{J} \cdot \mathrm{s}$	$\hbar = h/2\pi$
Radiant energy	Q, W	J	
Radiant energy density	ρ, w	$\mathrm{J} \cdot \mathrm{m}^{-3}$	$\rho = Q/V$
Radiant exitance, emitted radiant flux	M	$\mathrm{W} \cdot \mathrm{m}^{-2}$	$M = d\Phi/dA_{\mathrm{source}}$
Radiant intensity	I	$\mathrm{W} \cdot \mathrm{sr}^{-1}$	$I = d\Phi/d\Omega$
Radiant power, radiant energy per time	Φ, P	W	$\Phi = dQ/dt$
Refractive index	n		$n = c_0/c$
Reflectance	ρ		$\rho = \Phi_{\mathrm{refl}}/\Phi_0$
Second radiation constant	c_2	$\mathrm{K} \cdot \mathrm{m}$	$c_2 = hc_0/k$
Spectral radiant energy density:			
In terms of frequency	ρ_ν, w_ν	$\mathrm{J} \cdot \mathrm{m}^{-3} \cdot \mathrm{Hz}^{-1}$	$\rho_\nu = d\rho/d\nu$
In terms of wavelength	ρ_λ, w_λ	$\mathrm{J} \cdot \mathrm{m}^{-4}$	$\rho_\lambda = d\rho/d\lambda$
In terms of wavenumber	$\rho_{\tilde{\nu}}, w_{\tilde{\nu}}$	$\mathrm{J} \cdot \mathrm{m}^{-2}$	$\rho_{\tilde{\nu}} = d\rho/d\tilde{\nu}$

TABLE 2.2 Physical and Chemical Symbols and Definitions (*Continued*)

G. Electromagnetic radiation (*continued*)

Name	Symbol	SI unit	Definition
Speed of light:			
In a medium	c	$\mathrm{m \cdot s^{-1}}$	$c = c_0/n$
In vacuum	c_0	$\mathrm{m \cdot s^{-1}}$	
Stefan-Boltzmann constant	σ	$\mathrm{W \cdot m^{-2} \cdot K^{-4}}$	$M_{\mathrm{bb}} = \sigma T^4$
Transmittance	τ, T		$\tau = \Phi_{\mathrm{tr}}/\Phi_0$
Wavelength	λ	m	
Wavenumber:			
In a medium	σ	$\mathrm{m^{-1}}$	$\sigma = 1/\lambda$
In vacuum	$\tilde{\nu}$	$\mathrm{m^{-1}}$	$\tilde{\nu} = \nu/c_0 = 1/n\lambda$

H. Kinetics

Name	Symbol	SI unit	Definition
Activation energy	E_{a}, E	$\mathrm{J \cdot mol^{-1}}$	$E_{\mathrm{a}} = RT^2\, d \ln k/dT$
Boltzmann constant	k, k_{B}	$\mathrm{J \cdot K^{-1}}$	
Collision cross section	σ	$\mathrm{m^2}$	$\sigma_{\mathrm{AB}} = \pi d_{\mathrm{AB}}^2$
Collision diameter	d	m	$d_{\mathrm{AB}} = r_{\mathrm{A}} + r_{\mathrm{B}}$
Collision frequency	Z_{A}	$\mathrm{s^{-1}}$	
Collision frequency factor	z_{AB}, z_{AA}	$\mathrm{m^3 \cdot mol^{-1} \cdot s^{-1}}$	$z_{\mathrm{AB}} = Z_{\mathrm{AB}}/Lc_A c_B$
Collision number	Z_{AB}, Z_{AA}	$\mathrm{m^{-3} \cdot s^{-1}}$	
Half-life	$t_{1/2}$	s	$c(t_{1/2}) = c_0/2$
Overall order of reaction	n		$n = \Sigma n_{\mathrm{B}}$
Partial order of reaction	n_{B}		$v = k\Pi c_{\mathrm{B}}^{n_{\mathrm{B}}}$
Pre-exponential factor	A	$\mathrm{(mol^{-1} \cdot m^3)^{\mathit{n}-1} \cdot s^{-1}}$	$k = A \exp(-E_{\mathrm{a}}/RT)$
Quantum yield, photochemical yield	ϕ		
Rate of change of quantity X	$\dot{X}$	(varies)	$\dot{X} = dX/dt$
Rate of concentration change (chemical reaction)	r_{B}, v_{B}	$\mathrm{mol \cdot m^{-3} \cdot s^{-1}}$	$r_{\mathrm{B}} = dc_{\mathrm{B}}/dt$
Rate constant, rate coefficient	k	$\mathrm{(mol^{-1} \cdot m^3)^{\mathit{n}-1} \cdot s^{-1}}$	$v = k\Pi c_{\mathrm{B}}^{n_{\mathrm{B}}}$
Rate of conversion change due to chemical reaction	$\dot{\zeta}$	$\mathrm{mol \cdot s^{-1}}$	$\dot{\zeta} = d\zeta/dt$
Rate of reaction (based on concentration)	v	$\mathrm{mol \cdot m^{-3} \cdot s^{-1}}$	$v = \zeta/V = \nu_{\mathrm{B}}^{-1}\, dc_{\mathrm{B}}/dt$
Relaxation time	τ	s	$\tau = 1/(k_1 + k_{-1})$
Standard enthalpy of activation	$\Delta H^{\ddagger}$	$\mathrm{J \cdot mol^{-1}}$	
Standard entropy of activation	$\Delta S^{\ddagger}$	$\mathrm{J \cdot mol^{-1} \cdot K^{-1}}$	
Standard Gibbs energy of activation	$\Delta G^{\ddagger}$	$\mathrm{J \cdot mol^{-1}}$	
Volume of activation	$\Delta^{\ddagger} V$	$\mathrm{m^3 \cdot mol^{-1}}$	$\Delta^{\ddagger} V = -RT\,(\partial \ln k/\partial p)_T$

TABLE 2.2 Physical and Chemical Symbols and Definitions (*Continued*)

I. Mechanics

Name	Symbol	SI unit	Definition
Acoustic factors:			
Absorption	α_a		$\alpha_a = 1 - \rho$
Dissipation	δ		$\delta = \alpha_a - \tau$
Reflection	ρ		$\rho = P_r/P_0$
Transmission	τ		$\tau = P_{tr}/P_0$
Angular momentum	$\boldsymbol{L}$	$\mathrm{J \cdot s}$	$L = \boldsymbol{r} \times \boldsymbol{p}$
Bulk modulus, compression modulus	K	Pa	$K = -V_0(dp/dV)$
Density, mass density	ρ	$\mathrm{kg \cdot m^{-3}}$	$\rho = m/V$
Energy	E	J	
Fluidity, kinematic viscosity	ϕ	$\mathrm{m \cdot kg^{-1} \cdot s}$	$\phi = 1/\eta$
Force	$\boldsymbol{F}$	N	$\boldsymbol{F} = d\boldsymbol{p}/dt = m\boldsymbol{a}$
Friction coefficient	μ, (f)		$F_{frict} = \mu F_{norm}$
Gravitational constant	G	$\mathrm{N \cdot m^2 \cdot kg^{-2}}$	$F = Gm_1m_2/r^2$
Hamilton function	H	J	$H(q, p) = T(q, p) + V(q)$
Kinematic viscosity	ν	$\mathrm{m^2 \cdot s^{-1}}$	$\nu = \eta/\rho$
Kinetic energy	E_k	J	$E_k = ½mv^2$
Lagrange function	L	J	$L(q, \dot{q}) = T(q, \dot{q}) - V(q)$
Linear strain, relative elongation	ϵ, e		$\epsilon = \Delta l/l$
Mass	m	kg	
Modulus of elasticity, Young's modulus	E	Pa	$E = \sigma/\epsilon$
Moment of inertia	I, J	$\mathrm{kg \cdot m^2}$	$I = \Sigma m_i r_i^2$
Momentum	$\boldsymbol{p}$	$\mathrm{kg \cdot m \cdot s^{-1}}$	$\boldsymbol{p} = m\boldsymbol{v}$
Normal stress	σ	Pa	$\sigma = F/A$
Potential energy	E_p	J	$E_p = \int -\boldsymbol{F} \cdot ds$
Power	P	W	$P = dW/dt$
Pressure	p, P	Pa, $\mathrm{N \cdot m^{-2}}$	$p = F/A$
Reduced mass	μ	kg	$\mu = m_1m_2/(m_1 + m_2)$
Relative density	d		$d = \rho/\pi^0$
Shear modulus	G	Pa	$G = \tau/\gamma$
Shear strain	γ		$\gamma = \Delta x/d$
Shear stress	τ	Pa	$\tau = F/A$
Sound energy flux	P, P_a	W	$P = dE/dt$
Specific volume	υ	$\mathrm{m^3 \cdot kg^{-1}}$	$\upsilon = V/\mu = 1/\rho$
Surface density	ρ_A, ρ_S	$\mathrm{kg \cdot m^{-2}}$	$\rho_A = m/A$
Surface tension	γ, σ	$\mathrm{N \cdot m^{-1}}$, $\mathrm{J \cdot m^{-2}}$	$\gamma = dW/dA$
Torque, moment of a force	$\boldsymbol{T}$, $(\boldsymbol{M})$	$\mathrm{N \cdot m}$	$\boldsymbol{T} = \boldsymbol{r} \times \boldsymbol{F}$
Viscosity (dynamic)	η, μ	$\mathrm{Pa \cdot s}$	$\tau_{x,z} = \lambda(dv_x/dz)$
Volume (or bulk) strain	θ		$\theta = \Delta V/V_0$
Weight	G, (W, P)	N	$G = m \cdot g$
Work	W, w	J	$W = \int \boldsymbol{F} \cdot ds$

TABLE 2.2 Physical and Chemical Symbols and Definitions (*Continued*)

J. Solid state

Name	Symbol	SI unit	Definition
Acceptor ionization energy	E_a	J	
Bragg angle	θ	rad	$n\lambda = 2d \sin \theta$
Bloch function	$\mathbf{u_k}(\boldsymbol{r})$	$m^{-3/2}$	$\psi(\boldsymbol{r}) = u_k(r) \exp(ik \cdot r)$
Burgers vector	$\boldsymbol{b}$	m	
Charge density of electrons	ρ	$C \cdot m^{-3}$	$\rho(\boldsymbol{r}) = -e\psi^*(r)\psi(r)$
Circular wave vector:			
For particles ($\boldsymbol{k}$)	$\boldsymbol{k}, \boldsymbol{q}$	m^{-1}	$\boldsymbol{k} = 2\pi/\lambda$
For phonons ($\boldsymbol{q}$)			
Conductivity tensor	σ_{ik}	$S \cdot m^{-1}$	$\sigma = \rho^{-1}$
Curie temperature	T_C	K	
Debye circular frequency	ω_D	s^{-1}	
Debye circular wavenumber	q_D	m^{-1}	
Debye-Waller factor	B, D		
Density of states	N_E	$J^{-1} \cdot m^{-3}$	$N_E = dN(E)/dE$
Density of vibrational modes (spectral)	N_ω, g	$s \cdot m^{-3}$	$N_\omega = dN(\omega)/d\omega$
Diffusion coefficient	D	$m^2 \cdot s^{-1}$	$dN/dt = -DA\ dn/dx$
Diffusion length	L	m	$L = (D\tau)^{1/2}$
Displacement vector of an ion	$\boldsymbol{u}$	m	$\boldsymbol{u} = \boldsymbol{R} - \boldsymbol{R_0}$
Donor ionization energy	E_d	J	
Effective mass	m^*	kg	
Equilibrium position vector of an ion	$\boldsymbol{R}_0$	m	
Fermi energy	E_F	J	
Gap energy	E_g		
Grüneisen parameter	γ, Γ		$\gamma = \alpha V/\kappa C_V$
Hall coefficient	A_H, R_H	$m^3 \cdot C^{-1}$	$\boldsymbol{E} = \boldsymbol{\rho} \cdot \boldsymbol{j} + \boldsymbol{R_H}(\boldsymbol{B} \times \boldsymbol{j})$
Lattice plane spacing	d	m	
Lattice vector	$\boldsymbol{R}, \boldsymbol{R}_0$	m	
Lorenz coefficient	L	$V^2 \cdot K^{-2}$	$L = \lambda/\sigma T$
Madelung constant	α		$E_{coul} = \dfrac{\alpha N_A z_+ z_- e^2}{4\pi\epsilon_0 R_0}$
Mobility	μ	$m^2 \cdot V^{-1} \cdot s^{-1}$	$\mu = v_{drift}/E$
Mobility ratio	b		$b = \mu_n/\mu_p$
Neel temperature	T_N	K	
Number density, number concentration	n	m^{-3}	
Order parameters:			
Long range	s		
Short range	σ		
Order of reflection	n		
Particle position vector:			
Electron	$\boldsymbol{r}$	m	
Ion position	$\boldsymbol{R}_j$	m	
Peltier coefficient	Π	V	
Reciprocal lattice vector (circular)	$\boldsymbol{G}$	m^{-1}	$\boldsymbol{G} \cdot \boldsymbol{R} = 2\pi m$

TABLE 2.2 Physical and Chemical Symbols and Definitions (*Continued*)

J. Solid state (*continued*)

Name	Symbol	SI unit	Definition
Relaxation time	τ	s	$\tau = 1/\upsilon_F$
Residual resistivity	ρ_R	m	
Resistivity tensor	$\boldsymbol{\rho}$	$\Omega \cdot m$	$\boldsymbol{E} = \boldsymbol{\rho} \cdot \boldsymbol{j}$
Temperature	θ	K	
Thermal conductivity tensor	$\boldsymbol{\lambda}$	$W \cdot m^{-1} \cdot K^{-1}$	$\boldsymbol{J}_q = -\boldsymbol{\lambda} \cdot \text{grad } T$
Thermoelectric force	E	V	
Thomson coefficient	μ	$V \cdot K^{-1}$	
Translation vectors for the reciprocal lattice (circular)	$\boldsymbol{b}_1; \boldsymbol{b}_2; \boldsymbol{b}_3$ $\boldsymbol{a}^*; \boldsymbol{b}^*; \boldsymbol{c}^*$	$\mathbf{m}^{-1}$	$\boldsymbol{a}_i \cdot \boldsymbol{b}_k = 2\pi\delta_{ik}$
Translation vectors for crystal lattice	$\boldsymbol{a}_1; \boldsymbol{a}_2; \boldsymbol{a}_3$ $\boldsymbol{a}; \boldsymbol{b}; \boldsymbol{c}$	m	$\boldsymbol{R} = n_1\boldsymbol{a}_1 + n_2\boldsymbol{a}_2 + n_3\boldsymbol{a}_3$
Work function	$\boldsymbol{\Phi}$	J	$\boldsymbol{\Phi} = \boldsymbol{E}_\infty - \boldsymbol{E}_F$

K. Space and time

Name	Symbol	SI unit	Definition
Acceleration	$\boldsymbol{a}$, (g)	$m \cdot s^{-2}$	$\boldsymbol{a} = d\upsilon/dt$
Angular velocity	ω	$rad \cdot s^{-1}$, s^{-1}	$\omega = d\phi/dt$
Area	A, A_s, S	m^2	
Breadth	b	m	
Cartesian space coordinates	x, y, z	m	
Circular frequency, angular frequency	ω	$rad \cdot s^{-1}$, s^{-1}	$\omega = 2\pi\nu$
Diameter	d	m	
Distance	d	m	
Frequency	ν, f	Hz	$\nu = 1/T$
Generalized coordinate	q, q_i	(varies)	
Height	h	m	
Length	l	m	
Length of arc	s	m	
Path length	s	m	
Period	T	s	$T = t/N$
Plane angle	$\alpha, \beta, \gamma, \theta, \phi$	rad, l	$\alpha = s/r$
Position vector	$\boldsymbol{r}$	m	$\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$
Radius	r	m	
Relaxation time, time constant	τ, T	s	$\tau = \lvert dt/d \ln x \rvert$
Solid angle	ω, Ω	sr, l	$\Omega = A/r^2$
Speed	υ, u, w, c	$m \cdot s^{-1}$	$\upsilon = \lvert\upsilon\rvert$
Spherical polar coordinates	r, θ, ϕ	m, l, l	
Thickness	d, δ	m	
Time	t	s	
Velocity	$\upsilon, \boldsymbol{u}, \boldsymbol{w}, \boldsymbol{c}$	$m \cdot s$	$\upsilon = d\mathbf{r}/dt$
Volume	$V, (\upsilon)$	m^3	

TABLE 2.2 Physical and Chemical Symbols and Definitions (*Continued*)

L. Spectroscopy

Name	Symbol	SI unit	Definition
Asymmetry parameter	κ		$\kappa = \frac{2B - A - C}{A - C}$
Centrifugal distortion constants:			
A reduction	$\Delta_J\ \Delta_{JK}\ \Delta_K\ \delta_J\ \delta_K$	m^{-1}	
S reduction	$D_J\ D_{JK}\ D_K\ d_1\ d_2$		
Degeneracy, statistical weight	g, d, β		
Electric dipole moment of a molecule	$\boldsymbol{p}, \boldsymbol{\mu}$	C · m	$E_p = -\boldsymbol{p} \cdot \boldsymbol{E}$
Electron spin resonance (ESR), electron paramagnetic resonance (EPR):			
Hyperfine coupling constant:			
In liquids	a, A	Hz	$\hat{\boldsymbol{H}}_{hfs}/h = a\hat{\boldsymbol{S}} \cdot \boldsymbol{I}$
In solids	T	Hz	$\hat{\boldsymbol{H}}_{hfs}/h = \hat{\boldsymbol{S}} \cdot \boldsymbol{T} \cdot \boldsymbol{I}$
g factor	g		$h\nu = g\mu_B B$
Electronic term	T_e	m^{-1}	$T_e = E_e/hc$
Harmonic vibration wavenumber	$\omega_e; \omega_r$	m^{-1}	
Inertial defect	Δ	$kg \cdot m^2$	$\Delta = I_C - I_A - I_B$
Interatomic distances:			
Equilibrium distance	r_e	m	
Ground state distance	r_0	m	
Substitution structure distance	r_s	m	
Zero-point average distance	r_z	m	
Longitudinal relaxation time	T_1	s	
Nuclear magnetic resonance (NMR), chemical shift, δ scale	δ		$\delta = 10^6(\nu - \nu_0)/\nu_0$
Coupling constant, direct (dipolar)	D_{AB}	Hz	
Magnetogyric ratio	γ	$C \cdot kg^{-1}$	$\gamma = 2\pi\mu/Ih$
Shielding constant	σ_A		$B_A = (1 - \sigma_A)B$
Spin-spin coupling constant	J_{AB}	Hz	$H/h = J_{AB}\boldsymbol{I}_A \cdot \boldsymbol{I}_B$
Principal moments of inertia	$I_A; I_B; I_C$	$kg \cdot m^2$	$I_A \leq I_B \leq I_C$
Rotational constants:			
In frequency	$A; B; C$	Hz	$A = h/8\pi^2 I_A$
In wavenumber	$\tilde{A}; \tilde{B}; \tilde{C}$	m^{-1}	$\tilde{A} = h/8\pi^2 I_A$
Rotational term	F	m^{-1}	$F = E_{rot}/hc$
Spin orbit coupling constant	A	m^{-1}	$T_{s.o.} = A \langle \tilde{\boldsymbol{L}} \cdot \tilde{\boldsymbol{S}} \rangle$
Total term	T	m^{-1}	$T = E_{tot}/hc$
Transition dipole moment of a molecule	$\boldsymbol{M}, \boldsymbol{R}$	C · m	$M = \int \psi' \boldsymbol{p} \psi'' \, d\tau$

TABLE 2.2 Physical and Chemical Symbols and Definitions (*Continued*)

L. Spectroscopy (*continued*)

Name	Symbol	SI unit	Definition
Transition frequency	ν	Hz	$\nu = (E' - E'')/h$
Transition wavenumber	$\tilde{\nu}$, (ν)	m^{-1}	$\tilde{\nu} = T' - T''$
Transverse relaxation time	T_2	s	
Vibrational anharmonicity constant	$\omega_e\chi_e$; χ_{rs}; $g_{tt'}$	m^{-1}	
Vibrational coordinates:			
Internal coordinates	R_i, r_i, θ_j, etc.		
Normal coordinates, dimensionless	q_r		
Mass adjusted	Q_r		
Vibrational force constants:			
Diatomic	f, (k)	$J \cdot m^{-2}$	$f = \partial^2 V/\partial r^2$
Polyatomic			
Dimensionless normal coordinates	$\phi_{rst\ldots}$, $k_{rst\ldots}$	m^{-1}	
Internal coordinates	f_{ij}	(varies)	$f_{ij} = \partial^2 V/\partial r_i \partial r_j$
Symmetry coordinates	F_{ij}	(varies)	$F_{ij} = \partial^2 V/\partial S_i \partial S_j$
Vibrational quantum numbers	v_r; l_t		
Vibrational term	G	m^{-1}	$G = E_{vib}/hc$

Angular momentum types	Operator symbol	Quantum number symbol		
		Total	Z axis	z axis
Electron orbital	$\hat{\boldsymbol{L}}$	L	M_L	Λ
One electron only	$\hat{\boldsymbol{l}}$	l	m_l	λ
Electron orbital + spin	$\hat{\boldsymbol{L}} + \hat{\boldsymbol{S}}$			$\Omega = \Lambda + \Sigma$
Electron spin	$\hat{\boldsymbol{S}}$	S	M_S	σ
One electron only	$\hat{\boldsymbol{s}}$	s	m_s	Σ
Internal vibrational:				
Spherical top	$\hat{\boldsymbol{l}}$	$l(l\zeta)$		K_l
Other	$\hat{\boldsymbol{j}}$, $\hat{\boldsymbol{\pi}}$			$l(l\zeta)$
Nuclear orbital (rotational)	$\hat{\boldsymbol{R}}$	R		K_R, k_R
Nuclear spin	$\hat{\boldsymbol{I}}$	I	M_I	
Sum of $J + I$	$\hat{\boldsymbol{F}}$	F	M_F	
Sum of $N + S$	$\hat{\boldsymbol{J}}$	J	M_J	K, k
Sum of $R + L(+j)$	$\hat{\boldsymbol{N}}$	N		K, k

M. Thermodynamics

Name	Symbol	SI unit	Definition
Absolute activity	λ		$\lambda_B = \exp(\mu_B/RT)$
Activity (referenced to Henry's law):			
Concentration basis	a_c		$a_{c,B} = \exp\left[\frac{\mu_B - \mu_B^*}{RT}\right]$

TABLE 2.2 Physical and Chemical Symbols and Definitions (*Continued*)

M. Thermodynamics (*continued*)

Name	Symbol	SI unit	Definition
Molality basis	a_m		$a_{m,B} = \exp\left[\frac{\mu_B - \mu_B^*}{RT}\right]$
Mole fraction basis	a_x		$a_{x,B} = \exp\left[\frac{\mu_B - \mu_B^*}{RT}\right]$
Activity (referenced to Raoult's law)	a		$a_B = \exp\left[\frac{\mu_B - \mu_B^*}{RT}\right]$
Activity coefficient (referenced to Henry's law):			
Concentration basis	γ_c		$a_{c,B} = \gamma_{c,B} c_B / c^0$
Molality basis	γ_m		$a_{m,B} = \gamma_{m,B} m_B / m^0$
Mole fraction basis	γ_x		$a_{x,B} = \gamma_{x,B} x_B$
Activity coefficient (referenced to Raoult's law)	f		$f_B = a_B / x_B$
Affinity of reaction	A	J · mol^{-1}	$A = -(\partial G/\partial \xi)_{p,T}$
Celsius temperature	θ, t	°C	$\theta/°C = T/K - 273.15$
Chemical potential	μ	J · mol^{-1}	$\mu_B = (\partial G/\partial n_B)_{T,p,n}$
Compressibility:			
Isentropic	κ_S	Pa^{-1}	$\kappa_S = -(1/V)(\partial V/\partial p)_S$
Isothermal	κ_T	Pa^{-1}	$\kappa_T = -(1/V)(\partial V/\partial p)_T$
Compressibility factor	Z		$Z = pV_m/RT$
Cubic expansion coefficient	α, α_V, γ	K^{-1}	$\alpha = (1/V)(\partial V/\partial T)_p$
Enthalpy	H	J	$H = U + pV$
Entropy	S	J · K^{-1}	$dS \geq dq/T$
Equilibrium constant	K^0, K		$K° = \exp(-\Delta_r G°/RT)$
Equilibrium constant:			
Concentration basis	K_c	(mol · m^{-3})$^{\Sigma\nu}$	$K_c = \Pi\, c^\nu$
Molality basis	K_m	(mol · m^{-1})$^{\Sigma\nu}$	$K_m = \Pi\, m^\nu$
Pressure basis	K_p	Pa$^{\Sigma\nu}$	$K_p = \Pi\, p^\nu$
Fugacity	f	Pa	$f_B = \lambda_B \lim\limits_{p \to 0} (p_B/\lambda_B)_T$
Fugacity coefficient	ϕ		$\phi_B = f_B/p_B$
Gibbs energy	G	J	$G = H - TS$
Heat	q, Q	J	
Heat capacity:			
At constant pressure	C_p	J · K^{-1}	$C_p = (\partial H/\partial T)_p$
At constant volume	C_V	J · K^{-1}	$C_V = (\partial U/\partial T)_V$
Helmholtz energy	A	J	$A = U - TS$
Internal energy	U		$\Delta U = q + w$
Ionic strength:			
Concentration basis	I_c, I	mol · kg^{-3}	$I_c = ½\Sigma m_B z_B^2$
Molality basis	I_m, I	mol · kg^{-1}	$I_m = ½\Sigma m_B z_B^2$
Joule-Thomson coefficient	μ, μ_{JT}	K · Pa^{-1}	$\mu = (\partial T/\partial p)_H$
Linear expansion coefficient	α_1	K^{-1}	$\alpha_1 = (l/l)(\partial l/T)$
Massieu function	J	J · K^{-1}	$J = -A/T$
Molar quantity *X*	X_m	(varies)	$X_m = X/n$
Osmotic coefficient:			
Molality basis	ϕ_m		$\phi_m = (\mu_A^* - \mu_A)/(RTM_A\, \Sigma m_B)$
Mole fraction basis	ϕ_x		$\phi_x = (\mu_A - \mu_A^*)/(RT \ln x_A)$

TABLE 2.2 Physical and Chemical Symbols and Definitions (*Continued*)

M. Thermodynamics (*continued*)

Name	Symbol	SI unit	Definition
Osmotic pressure (ideal dilute solution)	Π	Pa	$\Pi = c_B RT$
Partial molar quantity X	X_B	(varies)	$X = (\partial X/\partial n_B)_{T,p,n}$
Planck function	Y	$J \cdot K^{-1}$	$Y = -G/T$
Pressure coefficient	β	$Pa \cdot K^{-1}$	$\beta = (\partial P/\partial T)_V$
Ratio of heat capacities	γ		$\gamma = C_p/C_V$
Relative pressure coefficient	α_p	K^{-1}	$\alpha_p = (1/p)(\partial p/\partial T)_V$
Second virial coefficient	B	$m^3 \cdot mol^{-1}$	$pV_m = RT(1 + B/V_m + \cdots)$
Specific quantity X	x	(varies)	$x = X/m$
Standard chemical potential	μ^0	$J \cdot mol^{-1}$	
Standard partial molar enthalpy	H^0	$J \cdot mol^{-1}$	$H^0 = \mu^0 + TS$
Standard partial molar entropy	S^0	$J \cdot mol^{-1} \cdot K^{-1}$	$S^0 = -(\partial \mu^0/\partial T)$
Standard reaction enthalpy	$\Delta_r H^0$	$J \cdot mol^{-1}$	$\Delta_r H^0 = \Sigma \nu H^0$
Standard reaction entropy	$\Delta_r S^0$	$J \cdot mol^{-1} \cdot K^{-1}$	$\Delta_r S^0 = \Sigma \nu S^0$
Standard reaction Gibbs energy	$\Delta_r G^0$	$J \cdot mol^{-1}$	$\Delta_r G^0 = \Sigma \nu \mu^0$
Surface tension	γ, σ	$J \cdot m^{-2}$, $N \cdot m^{-1}$	$\gamma = (\partial G/\partial A_s)_{T,p}$
Thermodynamic temperature	T	K	
Work	w, W	J	

Symbols used as subscripts to denote a chemical reaction or process:

ads	adsorption	mix	mixing of fluids
at	atomization	r	reaction in general
c	combustion reaction	sol	solution of solute in solvent
dil	dilution of a solution	sub	sublimation (solid to gas)
f	formation reaction	trs	transition (two phases)
fus	melting, fusion (solid to liquid)		

Recommended superscripts:

‡	activated complex, transition state	∞	infinite solution
E	excess quantity	*	pure substance
id	ideal	°	standard

N. Transport properties

Name	Symbol	SI unit	Definition
Coefficient of heat transfer	$h, (k, K)$	$W \cdot m^{-2} \cdot K^{-1}$	$h = J_q/\Delta T$
Diffusion coefficient	D	$m^2 \cdot s^{-1}$	$D = J_n/(dc/dl)$
Flux (of a quantity X)	J_X, J	(varies)	$J_X = A^{-1}\, dX/dt$
Heat flow rate	ϕ	W	$\phi = dq/dt$
Heat flux	J_q	$W \cdot m^{-2}$	$J_q = \phi/A$
Mass flow rate	q_m, m	$kg \cdot s^{-1}$	$q_m = dm/dt$
Mass transfer coefficient	k_d	$m \cdot s^{-1}$	
Thermal conductance	G	$W \cdot K^{-1}$	$G = \phi/\Delta T$
Thermal conductivity	λ, k	$W \cdot m^{-1} \cdot K^{-1}$	$\lambda = J_q/(dT/dl)$

TABLE 2.2 Physical and Chemical Symbols and Definitions (*Continued*)

N. Transport properties (*continued*)

Name	Symbol	SI unit	Definition
Thermal diffusivity	a	$\mathrm{m^2 \cdot s^{-1}}$	$a = \lambda/\rho c_\mathrm{p}$
Thermal resistance	R	$\mathrm{K \cdot W^{-1}}$	$R = 1/G$
Volume flow rate	q_v, V	$\mathrm{m^3 \cdot s^{-1}}$	$q_\mathrm{v} = dV/dt$
Dimensionless quantities:			
Alfvén number	Al		$Al = \upsilon(\rho\mu)^{1/2}/B$
Cowling number	Co		$Co = B^2/\mu\rho\upsilon^2$
Euler number	Eu		$Eu = \Delta p/\rho\upsilon^2$
Fourier number	Fo		$Fo = at/l^2$
Fourier number for mass transfer in binary mixtures	Fo^*		$Fo^* = Dt/l^2$
Froude number	Fr		$Fr = \upsilon/(lg)^{1/2}$
Grashof number	Gr		$Gr = l^3\, go\, \Delta T\rho^2/\eta^2$
Grashof number for mass transfer in binary mixtures	Gr^*		$Gr^* = l^3 g\, (\partial\rho/\partial x)_{\mathrm{T,p}}\, (\Delta x \pi/\eta)$
Hartmann number	Ha		$Ha = Bl(\kappa/\eta)^{1/2}$
Knudsen number	Kn		$Kn = \lambda/l$
Lewis number	Le		$Le = a/D$
Mach number	Ma		$Ma = \upsilon/c$
Magnetic Reynolds number	Rm, Re_m		$Rm = \upsilon\mu\kappa l$
Nusselt number	Nu		$Nu = hl/k$
Nusselt number for mass transfer in binary mixtures	Nu^*		$Nu^* = k_\mathrm{d} l/D$
Péclet number	Pe		$Pe = \upsilon l/a$
Péclet number for mass transfer in binary mixtures	Pe^*		$Pe^* = \upsilon l/D$
Prandtl number	Pr		$Pr = \eta/\rho a$
Rayleigh number	Ra		$Ra = l^3\, g\alpha\, \Delta T\rho/\eta a$
Reynolds number	Re		$Re = \rho\upsilon l/\eta$
Schmidt number	Sc		$Sc = \eta/\rho D$
Sherwood number	Sh		$Sh = k_\mathrm{d} l/D$
Stanton number	St		$St = h/\rho\upsilon c_\mathrm{p}$
Stanton number for mass transfer in binary mixtures	St^*		$St^* = k_\mathrm{d}/\upsilon$
Strouhal number	Sr		$Sr = lf/\upsilon$
Weber number	We		$We = \rho\upsilon^2\, l/\gamma$

Symbols used in the definitions of dimensionless quantities:

Acceleration of free fall	g	Pressure	p
Area	A	Speed	υ
Cubic expansion coefficient	α	Speed of sound	c
Density	ρ	Surface tension	γ
Frequency	f	Temperature	T
Length	l	Time	t
Mass	m	Viscosity	η
Mean free path	λ	Volume	V
Mole fraction	x		

TABLE 2.3 Mathematical Symbols and Abbreviations

Symbol or abbreviation	Meaning
$+$	Plus
$-$	Minus
$\pm$	Plus or minus
$\mp$	Minus or plus
$\equiv$	Identically equal to
$\times$, center dot	Multiplied by (ab, $a \times b$, $a \cdot b$)
$\div$	Divided by (a/b, ab^{-1})
$\neq$	Not equal to
$\approx$	Approximately equal to
$\simeq$	Asymptotically equal to
$>$	Greater than
$<$	Less than
$\gg$	Much greater than
$\ll$	Much less than
$\geq$	Greater than or equal to
$\leq$	Less than or equal to
$\propto$, $\sim$	Proportional to
$\longrightarrow$	Tends to, approaches
∞	Infinity
$\lvert a \rvert$	Magnitude of a
a^n	nth power of a
$\sqrt[n]{a}$, $a^{1/n}$	nth root of a
$\sqrt{a}$, $a^{1/2}$	Square root of a
$\langle a \rangle$, $\bar{a}$	Mean value of a
$\prod_{i=1}^{n} a_i$, Πa_i	Product of a_i
$\log a$ or $\log_{10} a$	Common (or Briggsian) logarithm to the base 10 of a
$\log_a b$	Logarithm to the base a of b
$\ln b$, $\log_e b$	Natural (Napierian) logarithm (to the base e) of b
e	Base (2.718) of natural system of logarithms
π	Pi (3.1416)
i	Imaginary quantity, square root of minus one
$n!$	n factorial ($n! = 1 \cdot 2 \cdot 3 \cdots n$)
$\angle$	Angle
$\perp$	Perpendicular to
$\parallel$	Parallel to
a°	a degrees (angle)
a'	a minutes (angle); a prime
a''	a seconds (angle); a double prime
$\sin a$	sine of a
$\cos a$	cosine of a
$\tan a$	tangent of a
$\cot a$	cotangent of a
$\sec a$	secant of a
$\cos a$	cosecant of a
$\arcsin a$, $\sin^{-1} a$	Inverse sine of a (angle whose sine is a)
$\arccos a$, $\cos^{-1} a$	Inverse sine of a (angle whose cosine is a)
$\arctan a$, $\tan^{-1} a$	Inverse tangent of a (angle whose tangent is a)
$\sinh a$	Hyperbolic sine of a
$\cosh a$	Hyperbolic cosine of a
$\tanh a$	Hyperbolic tangent of a

TABLE 2.3 Mathematical Symbols and Abbreviations (*Continued*)

Symbol or abbreviation	Meaning
cotanh a	Hyperbolic cotangent of a
$P(x, y)$	Rectangular coordinate of point P
$P(r, \theta)$	Polar coordinate of point P
$f(x)$, $F(x)$	Function of x
Δx	Increment of x
dy	Total differential of y
$\frac{dy}{dx}$ or $f'(x)$	Derivative of $y = f(x)$ with respect to x
$\frac{d^2y}{dx^2}$ or $f''(x)$	Second derivative of $y = f(x)$ with respect to x
$\frac{\partial z}{\partial x}$	Partial derivative of z with respect to x
$\frac{\partial^2 z}{\partial x\, \partial y}$	Second partial derivative of z with respect to x and y
$\int$	Integral of
$\int_a^b$	Integral between the limits a and b
$\lim_{x \to a} f(x)$	limit of $f(x)$ as x tends to a
$\sum_{i=1}^{n}$	Summation of a_i between the limits 1 and n

TABLE 2.4 SI Prefixes

Submultiple	Prefix	Symbol	Multiple	Prefix	Symbol
10^{-1}	deci	d	10	deka	da
10^{-2}	centi	c	10^{2}	hecto	h
10^{-3}	milli	m	10^{3}	kilo	k
10^{-6}	micro	μ	10^{6}	mega	M
10^{-9}	nano	n	10^{9}	giga	G
10^{-12}	pico	p	10^{12}	tera	T
10^{-15}	femto	f	10^{15}	peta	P
10^{-18}	atto	a	10^{18}	exa	E
10^{-21}	zepto	z	10^{21}	zetta	Z
10^{-24}	yocto	y	10^{24}	yotta	Y

TABLE 2.4 SI Prefixes (*Continued*)

Numerical (multiplying) prefixes

Number	Prefix	Number	Prefix	Number	Prefix
0.5	hemi	19	nonadeca	39	nonatriaconta
1	mono	20	icosa	40	tetraconta
l.5	sesqui	21	henicosa	41	hentetraconta
2	di (bis)*	22	docosa	42	dotetraconta
3	tri (tris)*	23	tricosa	43	tritetraconta
4	tetra (tetrakis)*	24	tetracosa	44	tetratetraconta
5	penta	25	pentacosa	45	pentatetraconta
6	hexa	26	hexacosa	46	hexatetraconta
7	hepta	27	heptacosa	47	heptatetraconta
8	octa	28	octacosa	48	octatetraconta
9	nona	29	nonacosa	49	nonatetraconta
10	deca	30	triaconta	50	pentaconta
11	undeca	31	hentriaconta	60	hexaconta
12	dodeca	32	dotriaconta	70	heptaconta
13	trideca	33	tritriaconta	80	octaconta
14	tetradeca	34	tetratriaconta	90	nonaconta
15	pentadeca	35	pentatriaconta	100	hecta
16	hexadeca	36	hexatriaconta	110	decahecta
17	heptadeca	37	heptatriaconta	120	icosahecta
18	octadeca	38	octatriaconta	130	triacontahecta

* In the case of complex entities such as organic ligands (particularly if they are substituted) the multiplying prefixes bis-, tris-, tetrakis-, pentakis-, . . . are used, i.e., -kis is added starting from tetra-. The modified entity is often placed within parentheses to avoid ambiguity.

TABLE 2.5 Greek Alphabet

Capital	Lower case	Name	Capital	Lower case	Name
Α	α	Alpha	Ν	ν	Nu
Β	β	Beta	Ξ	ξ	Xi
Γ	γ	Gamma	Ο	o	Omicron
Δ	δ	Delta	Π	π	Pi
Ε	ϵ	Epsilon	Ρ	ρ	Rho
Ζ	ζ	Zeta	Σ	σ	Sigma
Η	η	Eta	Τ	τ	Tau
Θ	θ	Theta	Υ	υ	Upsilon
Ι	ι	Iota	Φ	ϕ	Phi
Κ	κ	Kappa	Χ	χ	Chi
Λ	λ	Lambda	Ψ	ψ	Psi
Μ	μ	Mu	Ω	ω	Omega

TABLE 2.6 Abbreviations and Standard Letter Symbols

Abampere	abamp
Absolute	abs
Absolute activity	λ
Absorbance (decaidic)	A
Absorbance (napierian)	B
Absorptance	α
Absorption coefficient, linear decaidic	a, K
Absorption coefficient, linear napierian	α
Absorption coefficient, molar decaidic	μ, ϵ
Absorption coefficient, molar napierian	κ
Absorption index	k
Acceleration	a
Acceleration due to gravity	g, g_n
Acetyl	Ac
Acoustic absorption factor	α_a
Acoustic dissipation factor	δ
Acoustic reflection factor	ρ
Acoustic transmission factor	τ
Activation energy	E_a
Activity (referenced to Raoult's law)	a
Activity (referenced to Henry's law):	
Concentration basis	a_c
Molality basis	a_m
Mole fraction basis	a_x
Activity (radioactive)	A
Activity coefficient (referenced to Raoult's law)	f
Activity coefficient (referenced to Henry's law):	
Concentration basis	γ_c
Molality basis	γ_m
Mole fraction basis	γ_x
Adjusted retention time	t'_R
Adjusted retention volume	V'_R
Admittance	Y
Affinity of reaction	A
Alcohol	alc
Alfvén number	Al
Alkaline	alk
Alpha particle	α
Alternating current	ac
Amorphous	am
Amount concentration	c
Amount of substance	n
Ampere	A
Amplification factor	μ
Angle of optical rotation	α
Angstrom	Å, A
Angular dispersion	$d\theta/d\lambda$
Angular momentum	π
Angular momentum terms	j, J, l, L, N
Angular velocity	ω
Anhydrous	anhyd
Approximate (circa)	ca.
Aqueous solution	aq
Aqueous solution at infinite dilution	aq, ∞
Are, unit of area	a
Area	A, S
Area per molecule	a, σ
Astronomical unit	AU
Asymmetry parameter	κ
Atmosphere, unit of pressure	atm
Atomic mass	m_a
Atomic mass constant	m_u
Atomic mass unit	amu
Atomic number	Z
Atomic percent	at.%
Atomic weight	at. wt.
Average	av
Average linear gas velocity	μ
Avogadro constant	L, N_A
Axial angular momentum	λ, Λ, Ω
Axial spin angular momentum	σ, Σ
Bandwidth (10%) of a spectral filter	$\Delta\lambda_{0.1}$
Band variance	σ^2
Bar, unit of pressure	bar
Barn, unit of area	b
Barrel	bbl
Base of natural logarithms	e
Becquerel	Bq
Bed volume	V_g
Beta particle	β
Bloch function	$\boldsymbol{u}_\mathbf{k}(\boldsymbol{r})$
Body-centered cubic	bcc
Bohr	b
Bohr magneton	μ_B
Bohr radius	a_0
Boiling point	bp
Boltzmann constant	k, k_B
Bragg angle	θ
Breadth	b
British thermal unit	Btu
Bulk modulus	K
Bulk strain	θ
Burgers vector	$\boldsymbol{b}$
Butyl	Bu
Calorie, unit of energy	cal
Calorie, international steam table	cal_IT
Candela	cd
Capacitance	C

TABLE 2.6 Abbreviations and Standard Letter Symbols (*Continued*)

Capacity, volume	Q_V
Capacity, weight	Q_W
Cartesian space coordinates	x, y, z
Celsius temperature	t, θ
Centimeter-gram-second system	cgs
Centrifugal distortion constants:	
A reduction	Δ, δ
S reduction	D, d
Charge density of electrons	ρ
Charge number of electrochemical reaction	n
Charge number of an ion	z
Chemically pure	CP
Chemical potential	μ
Chemical shift	δ
Circa (approximate)	ca.
Circular frequency	ω
Circular wave vector:	
For particles	$\boldsymbol{k}$
For phonons	$\boldsymbol{q}$
Circumference divided by the diameter	π
Citrate	Cit
Coefficient of heat transfer	h
Collision cross section	σ
Collision diameter	d
Collision frequency	Z
Collision frequency factor	z
Collision number	Z
Column volume	V_col
Compare (confer)	cf.
Complex refractive index	$\hat{n}$
Component of angular momentum	k, K, m, M
Compressibility:	
Isentropic	κ_S
Isothermal	κ_T
Compression factor	Z
Compression modulus	K
Compton wavelength of electron	λ_c
Compton wavelength of neutron	$\lambda_\mathrm{c,n}$
Compton wavelength of proton	$\lambda_\mathrm{c,p}$
Concentration (amount of substance)	c
Concentration (mass)	γ
Concentration at peak maximum	C_max
Concentration of solute in mobile phase	C_M
Concentration of solute in stationary phase	C_S
Condensed phase (solid or liquid)	cd
Conductance	G
Conductivity	γ, κ
Conductivity cell constant	K_cell
Conductivity tensor	σ_{ik}
Contact angle	θ
Coordinate, position vector	r
Coulomb	C
Counts per minute	cpm, c/m
Coupling constant, direct dipolar	D_AB
Critical density	d_c
Critical temperature	t_c
Cross section	σ
Crystalline	cr, cryst
Cubic	cub
Cubic expansion coefficient	$\alpha, \alpha_\mathrm{v}, \gamma$
Curie	Ci
Cycles per second	Hz
Curie temperature	T_c
Dalton (atomic mass unit)	Da
Day	d
Debye, unit of electric dipole	D
Debye circular frequency	ω_D
Debye circular wave-number	q_D
Debye-Waller factor	D, B
Decay constant (radioactive)	λ
Decibel	dB
Decompose	dec
Degeneracy, statistical weight	d, g, β
Degree of dissociation	α
Degrees Baume	°Be
Degrees Celsius	°C
Degrees Fahrenheit	°F
Density (mass)	ρ, γ
Density, critical	d_c
Density, relative	d
Density of liquid phase	ρ_L
Density of states	N_E, ρ
Density of vibrational modes (spectral)	N_ω
Detect, determine (d)	det(d)
Determination	detn
Deuteron	d
Diamagnetic shielding factor	$1 + \sigma$
Diameter	d
Dielectric polarization	$\boldsymbol{P}$
Differential thermal analysis	DTA
Diffusion coefficient	D
Diffusion coefficient, liquid film	D_f

TABLE 2.6 Abbreviations and Standard Letter Symbols (*Continued*)

Diffusion coefficient, mobile phase	D_{M}
Diffusion coefficient, stationary phase	D_{S}
Diffusion current	i_{d}
Diffusion length	L
Diffusion rate constant, mass transfer coefficient	k_{d}
Dilute	dil
Dirac delta function	δ
Direct current	dc
Direct dipolar coupling constant	D_{AB}
Disintegration energy	Q
Disintegrations per minute	dpm
Displacement vector of an ion	$\boldsymbol{u}$
Dissociation energy	D, E_{d}
From ground state	D_0
From the potential minimum	D_{e}
Distribution ratio	D
Donor ionization energy	E_{d}
Dropping mercury electrode	dme
Dyne, unit of force	dyn
Einstein transition probabilities	A, B
Spontaneous emission	A_{nm}
Stimulated absorption	B_{mn}
Stimulated emission	B_{nm}
Electric charge	Q
Electric current	I
Electric current density	j, J
Electric dipole moment of a molecule	$\boldsymbol{p}$, $\boldsymbol{\mu}$
Electric displacement	$\boldsymbol{D}$
Electric field gradient	q
Electric field strength	$\boldsymbol{E}$
Electric flux	$\boldsymbol{\Psi}$
Electric mobility	u, μ
Electric polarizability of a molecule	α
Electric potential	V, ϕ
Electric potential difference	U, ΔV
Electric susceptibility	χ_{e}
Electrical conductivity	σ
Electrical conductance	G
Electrical resistance	R
Electrochemical transfer coefficient	α
Electrokinetic potential	ζ
Electromagnetic unit	emu
Electromotive force	E, emf
Electron	e^-, e
Electron affinity	E_{ea}
Electron magnetic moment	μ_{e}
Electron paramagnetic resonance	EPR
Electron radius	r_{e}
Electron rest mass	m_{e}
Electron spin resonance	ESR
Electronvolt	eV
Electrostatic unit	esu
Elementary charge	e
Elution volume, exclusion chromatography	V_{e}
Emittance	ϵ
By blackbody	M_{bb}
Energy	E
Energy density	w, ρ
Energy per electron hole pair of ion pair in detector	ϵ
Enthalpy	H
Entropy	S
Entropy unit	e.u.
Equilibrium constant	K, K^0
Concentration basis	K_{c}
Molality basis	K_{m}
Pressure basis	K_{p}
Equilibrium position vector of an ion	$\boldsymbol{R}_0$
Equivalent weight	equiv wt
Erg, unit of energy	erg
Especially	esp.
et alii (and others)	et al.
et cetera (and so forth)	etc.
Ethyl	Et
Ethylenediamine	en
Ethylenediamine-*N*,*N*,*N*′,*N*′-tetraacetic acid	EDTA
Euler number	*Eu*
Exempli gratia (for example)	e.g.
Expansion coefficient	α
Exponential	exp
Extent of reaction	ξ
Fano factor	F
Farad	F
Faraday constant	F
Fermi, unit of length	f
Fermi energy	E_{F}
Film tension	Σ_{f}
Film thickness	h, t
Fine structure constant	α
Finite change	Δ
First radiation constant	c_1
Flow rate	q
Flow rate, column chromatography	F_{c}
Fluid phase (gas or liquid)	fl
Fluidity	ϕ

TABLE 2.6 Abbreviations and Standard Letter Symbols (*Continued*)

Fluorescent efficiency	$\boldsymbol{\Phi}_{\mathrm{F}}$
Fluorescent power	P_{F}
Flux	F, J
Focal length	f
Foot	ft
For example (exempli gratia)	e.g.
Force	F
Force constant (vibrational levels)	k
Formal concentration	F
Fourier number	Fo
Franklin, unit of electric charge	Fr
Freezing point	fp
Frequency	f, ν
Friction coefficient	f, μ
Froude number	Fr
Fugacity	f
Fugacity coefficient	ϕ
Gallon	gal
Galvani potential difference	$\Delta\phi$
Gamma, unit of mass	γ
Gamma radiation	γ
Gap energy (solid state)	E_{g}
Gas (physical state)	g
Gas constant	R
Gauss	G
g factor	g
Gibbs energy	G
Grade	grad
Grain, unit of mass	gr
Gram	g
Grand partition function	Ξ
Grashof number	Gr
Gravimetric	grav
Gravitational constant	G
Gray	Gy
Grüneisen parameter	γ, Γ
Half-life	$t_{1/2}$
Half-wave potential	$E_{1/2}$
Hall coefficient	$A_{\mathrm{H}}, R_{\mathrm{H}}$
Hamilton function	H
Harmonic vibration wavenumber	ω
Hartmann number	Ha
Hartree energy	E_{h}
Heat	q, Q
Heat capacity	C
At constant pressure	C_{p}
At constant volume	C_{v}
Heat flow rate	ϕ
Heat flux	J
Hectare, unit of area	ha
Height	h
Helion	h
Helmholtz energy	A
Henry	H
Hertz	Hz
Hexagonal	hex
Horsepower	hp
Hour	h
Hygroscopic	hygr
Hyperfine coupling constant	a, A
Hyperfine coupling tensor	$\boldsymbol{T}$
ibidem (in the same place)	ibid.
id est (that is)	i.e.
Ignition	ign
Impedance	Z
Inch	in
Indices of a family of crystallographic planes	hkl
Indirect spin-spin coupling constant	J_{AB}
Inductance	L
Inertial defect	Δ
Infinitesimal change	δ
Infrared	ir
Inner column volume	V_{i}
Inner electric potential	ϕ
Inner electrode potential	ϕ
Inorganic	inorg
Inside diameter	i.d.
Insoluble	insol
Interatomic distances:	
Equilibrium distance	r_{e}
Ground-state distance	r_0
Substitution structure distance	r_{s}
Zero-point average distance	r_{z}
Internal energy	U
Interstitial (outer) volume	V_{o}
In the place cited (loco citato)	loc. cit.
In the same place	ibid.
In the work cited	op. cit.
Ionic conductivity	λ, Λ
Ionic strength	I
Concentration basis	I_{c}
Molality basis	I_{m}
Ionization energy	E_{i}
Irradiance	E
Joule	J
Joule-Thomson coefficient	μ, μ_{JT}
Kelvin	K
Kilocalorie	kcal
Kilogram	kg
Kilogram-force	kgf
Kilowatt-hour	kWh
Kinematic viscosity	ν, ϕ

TABLE 2.6 Abbreviations and Standard Letter Symbols (*Continued*)

Kinetic energy	K, T, E_{k}
Knudsen number	Kn
Kovats retention indices	RI
Lagrange function	L
Lambda, unit of volume	λ
Landé g-factor	g, g_{e}
Larmor circular frequency	ω_{L}
Larmor frequency	ν_{L}
Lattice plane spacing	d
Lattice vector	$\boldsymbol{R}$, $\boldsymbol{R}_{\mathbf{0}}$
Lattice vectors	$\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$
Length	l, L
Length of arc	s
Lewis number	Le
Light year	l.y.
Limit (mathematics)	lim
Linear expansion coefficient	α_1
Linear reciprocal dispersion	D^{-1}, $d\lambda/dx$
Linear strain	e, ϵ
Liquid	l, lq
Liquid crystal	lc
Liter	L, 1
loco citato (in the place cited)	loc. cit.
Logarithm, common	log
Logarithm, base e	ln
Longitudinal relaxation time	T_1
Lorenz coefficient	L
Loss angle	δ
Lumen	lm
Luminous intensity	I
Lux	lx
Mach number	Ma
Madelung constant	α
Magnetic dipole moment of a molecule	$\boldsymbol{m}$, $\boldsymbol{\mu}$
Magnetic field strength	$\boldsymbol{H}$
Magnetic flux	$\boldsymbol{\Phi}$
Magnetic flux density	B
Magnetic moment of protons in water	$\mu_{\mathrm{p}}/\mu_{\mathrm{B}}$
Magnetic quantum number	M_{j}
Magnetic Reynolds number	Rm
Magnetic susceptibility	κ, χ
Magnetic vector potential	$\boldsymbol{A}$
Magnetizability	ξ
Magnetization	$\boldsymbol{M}$
Magnetogyric ratio	γ
Mass	m
Mass absorption coefficient	μ/ρ, μ_{m}
Mass concentration	γ, ρ
Mass density	ρ
Mass flow rate	q_{m}
Mass fraction	w
Massieu function	J
Mass number	A
Mass of atom	m, m_{a}
Mass transfer coefficient	k_{d}
Matrix volume	V_{g}
Maximum	max
Maxwell, unit of magnetic flux	Mx
Mean ionic activity	$a_{\pm}$
Mean ionic activity coefficient	$\gamma_{\pm}$
Mean ionic mobility	$W_{\pm}$
Melting point	mp
Metallic	met
Metastable	m
Metastable peaks	m^*
Meter	m
Methyl	Me
Micrometer	μm
Micron	μ
Mile	mi
Miller indices	h, l, k
Milliequivalent	meq
Millimeters of mercury, unit of pressure	mmHg
Millimole	mM
Minimum	min
Minute	m, min
Mixture	mixt
Mobility	μ
Mobility ratio	b
Modulus of elasticity	E
Molal	m
Molality	b
Molar	M, **M**
Molar (decadic) absorption coefficient	ϵ
Molar ionic conductivity	λ, Λ
Molar magnetic susceptibility	χ_{m}
Molar mass	M
Molar quantity X	X_{m}
Molar refraction	R, R_{m}
Molar volume	V_{m}
Mole	mol
Mole fraction, condensed phase	x
Gaseous mixtures	y
Mole percent	mol %
Molecular weight	mol wt
Moment of inertia	I, J
Momentum	p
Monoclinic	mn
Monomeric form	mon
Muon, negative	μ^-
Muon, positive	μ^+

TABLE 2.6 Abbreviations and Standard Letter Symbols (*Continued*)

Mutual inductance	M, L
Napierian absorbance	B
Napierian base	e
Napierian molar absorption coefficient	κ
Neel temperature	T_{N}
Net retention volume	V_{N}
Neutrino	ν_e
Neutron	n
Neutron magnetic moment	μ_{N}
Neutron number	N
Neutron rest mass	m_{n}
Newton	N
Normal concentration	N
Normal stress	σ
Nuclear magnetic resonance	NMR
Nuclear magneton	μ_{N}
Nuclear spin angular momentum	I
Nucleon number	A
Number concentration	C
Number density	n
Number of entities	N
Numerical aperture	NA
Nusselt number	*Nu*
Obstruction factor	γ
Oersted, unit of magnetic field	Oe
Ohm	Ω
opere citato (in the work cited)	op. cit.
Optical speed	*f*/number
Orbital angular momentum:	
Quantum number	$L = 0, 1, 2, 3, \ldots$
Series symbol	$S, P, D, F, \ldots$
Orbital angular momentum (molecules):	
Quantum number	$\Lambda = 0, 1, 2, \ldots$
Symbol	$\Sigma, \Pi, \Delta, \ldots$
Orbital angular momenta of individual electrons	$l = 0, 1, 2, 3, \ldots$ $s, p, d, f, \ldots$
Order of Bragg reflection	n
Order of reaction	n
Order of reflection	n
Order parameters (solid state), long range	s
Short range	σ
Organic	org
Orthorhombic	o-rh
Osmotic coefficient	ϕ
Molality basis	ϕ_{m}
Mole fraction basis	ϕ_x
Osmotic pressure (ideal dilute solution)	Π
Ounce	oz
Outer diameter	o.d.
Outer electric potential	ψ
Overall order of reaction	n
Overpotential	η
Oxalate	Ox
Oxidant	ox
Packing uniformity factor	λ
Page(s)	p. (pp.)
Parsec, unit of length	pc
Partial molar quantity	X
Partial order of reaction	n_{B}
Particle diameter	d_{p}
Particle position vector:	
Electron	$\boldsymbol{r}$
Ion position	$\boldsymbol{R}_{\mathrm{j}}$
Partition coefficient	K
Partition function	q, Q, z, Z, Ω
Partition ratio	k'
Parts per billion, volume	ng/mL
Parts per billion, weight	ng/g
Parts per million, volume	μg/mL
Parts per million, weight	μ/g
Pascal	Pa
Path length (absorbing)	l
Peak asymmetry factor	*AF*
Peak resolution	*Rs*
Péclet number	*Pe*
Peltier coefficient	Π
Percent	%
Period of time	T
Permeability	μ
Permeability of vacuum	μ_0
Permittivity	ϵ
Permittivity of vacuum	ϵ_0
pH, expressed in activity	p*a*H
Expressed in molarity	pH
Phenyl	Ph, ϕ
Phosphorescent efficiency	Φ_{P}
Phosphorescent power	P_{P}
Photochemical yield	ϕ
Photoluminescence power	P
Photon	γ
Pion	π
Planck constant	h
Planck constant/2π	$\hbar$
Planck function	Y
Plane angle	$\alpha, \beta, \gamma, \theta, \phi$
Plate height	H
Plate number, effective	N_{eff}
Poise	P
Polymeric form	pol
Porosity, column	ϵ
Positron	β^+
Potential energy	V, Φ, E_{P}
Pound	lb

TABLE 2.6 Abbreviations and Standard Letter Symbols (*Continued*)

Pounds per square inch	psi
Powder	pwd
Power	p
Poynting vector	$\boldsymbol{S}$
Prandtl number	Pr
Pressure (partial)	p
Pressure (total)	p, P
Pressure coefficient	β
Pressure, column inlet	p_i
Pressure, column outlet	p_o
Pressure, critical	p_c
Pressure drop	ΔP
Pressure-gradient correction	j
Principal moments of inertia	I_A; I_B; I_C
Principal quantum number	n
Probability	P
Probability density	P
Product sign	Π
Propyl	Pr
Proton	p
Proton magnetic resonance	pmr
Proton magnetogyric ratio	γ_p
Proton number	Z
Proton rest mass	m_p
Pyridine	py
Quadrupole interaction energy tensor	$\boldsymbol{\chi}$
Quadrupole moment of a molecule	Q, Θ
Quantity of electricity, electric charge	Q
Quantum of energy	$h\nu$
Quantum yield	ϕ
Rad, unit of radiation dose	rad
Radian	rad
Radiant energy	Q, W
Radiant energy density	ρ, w
Radiant energy flux	dQ/dt
Radiant exitance	M
Radiant flux received	E
Radiant intensity	I
Radiant intensity at time t after termination of excitation	$I(t)$
Radiant power	Φ
Radiant power incident on sample	P_0
Radiofrequency	rf
Radius	r
Rate of concentration change	r
Rate constant	k
Rate of reaction	v
Ratio of heat capacities	γ
Reactance	X
Reciprocal lattice	$\boldsymbol{a^*}$, $\boldsymbol{b^*}$, $\boldsymbol{c^*}$
Reciprocal lattice vector (circular)	$\boldsymbol{G}$
Vectors for	$\boldsymbol{a^*}$; $\boldsymbol{b^*}$, $\boldsymbol{c^*}$
Reciprocal radius of ionic atmosphere	κ
Reciprocal temperature parameter, $1/kT$	β
Reciprocal thickness of double layer	κ
Reduced column length	λ
Reduced mass	μ
Reduced plate height	h
Reduced velocity	v
Reductant	red
Reference	ref
Reflectance	ρ
Reflection plane	σ
Refractive index	n
Relative permeability	μ_r
Relative permittivity (dielectric constant)	ϵ_r
Relative pressure coefficient	α_p
Relative retention ratio	α
Relaxation time	τ
Rem, unit of dose equivalent	rem
Residual resistivity (solid state)	ρ_R
Resistivity tensor	$\boldsymbol{\rho}$
Retardation factor	R_f
Retarded van der Waals constant	B, β
Retention time	t_R
Retention volume	V_R
Revolutions per minute	rpm
Reynolds number	Re
Rhombic	rh
Rhombohedral	rh-hed
Roentgen	R
Root-mean-square	rms
Rotational constants:	
In frequency	A, B, C
In wavenumber	$\tilde{A}$; $\tilde{B}$; $\tilde{C}$
Rotational term (spectroscopy)	F
Rotation-reflection	S_n
Rydberg, unit of energy	Ry
Rydberg constant	R, R_∞
Saturated	satd
Saturated calomel electrode	SCE
Schmidt number	Sc
Second	s
Second radiation constant	c_2
Second virial coefficient	B
Sedimentation coefficient	s
Selectivity coefficient	k

TABLE 2.6 Abbreviations and Standard Letter Symbols (*Continued*)

Self-inductance	L
Separation factor	α
Shear modulus	G
Shear strain	γ
Shear stress	τ
Sherwood number	Sh
Shielding constant (NMR)	σ
Short-range order parameter	σ
Siemens	S
Sievert	Sv
Signal-to-noise ratio	S/N
Slightly	sl
Solid	c, s
Solid angle	ω, Ω
Solid angle over which luminescence is measured (F, fluorescence; P, phosphorescence; DF, delayed fluorescence)	$\Omega_{\mathrm{F(P,DF)}}$
Solid angle over which radiation is absorbed in cell	Ω_{A}
Solubility	s
Soluble	sol
Solution	soln, sln
Solvent	solv
Sound energy flux	P, P_{a}
Spacing between crystal diffracting planes	d
Species adsorbed on a substance	ads
Specific gravity	sp gr
Specific retention volume	V_{g}^{0}
Specific surface area	s
Specific volume	v, υ
Spectral bandwidth of emission monochromator	$\Delta\lambda_{\mathrm{em}}$
Spectral bandwidth of excitation monochromator	$\Delta\lambda_{\mathrm{ex}}$
Spectral bandwidth of monochromator	$\Delta\lambda_{\mathrm{m}}$
Spectral radiant energy	Q_{λ}, $dQ/d\lambda$
Spectral radiant energy density:	
In terms of frequency	ρ_{ν}, w_{ν}
In terms of wavelength	ρ_{ν}, w_{ν}
In terms of wavenumber	$\rho_{\tilde{\nu}}$, $w_{\tilde{\nu}}$
Spectral radiant energy flux	$d\phi/d\lambda$
Spectroscopic splitting factor	g
Speed	u, w
Speed of light:	
In a medium	c
In vacuum	c_0
Spherical polar coordinates	r, θ, ϕ
Spin angular momentum	s, S
Spin-lattice relaxation time	T_1
Spin orbit coupling constant	A
Spin-spin coupling constant	J_{AB}
Spin-spin (or transverse) relaxation time	T_2
Spin wavefunctions	α, β
Square	sq
Standard	std
Standard enthalpy of activation	$H^{\ddagger}$
Standard enthalpy of formation	ΔHf^{0}
Standard entropy	S^{0}
Standard entropy of activation	$\Delta S^{\ddagger}$
Standard Gibbs energy of activation	$\Delta G^{\ddagger}$
Standard Gibbs energy of formation	ΔGf^{0}
Standard heat capacity	C_{p}
Standard hydrogen electrode	SHE
Standard partial molar enthalpy	H^{0}
Standard partial molar entropy	S^{0}
Standard potential of electrochemical cell reaction	E^{0}
Standard reaction enthalpy	$\Delta_{\mathrm{r}}H^{0}$
Standard reaction entropy	$\Delta_{\mathrm{r}}S^{0}$
Standard reaction Gibbs energy	$\Delta_{\mathrm{r}}G^{0}$
Standard temperature and pressure	STP
Stanton number	St
Statistical weight	W, β, ω
Statistical weight of atomic states	g
Stefan-Boltzmann constant	σ
Steradian	sr
Stoichiometric number	v
Stokes	St
Summation sign	Σ
Surface charge density	σ
Surface concentration	Γ
Surface coverage	θ
Surface density	ρ_{A}, ρ_{S}
Surface electric potential	χ
Surface pressure	π
Surface tension	γ, σ
Susceptance	B
Svedberg, unit of time	Sv
Symmetrical	sym
Symmetry coordinate	S
Symmetry number	s, σ
Tartrate	Tart
Temperature	θ, Θ

TABLE 2.6 Abbreviations and Standard Letter Symbols (*Continued*)

Temperature, thermodynamic	T
Temperature at boiling point	T_b
Term value spectroscopy	T
Tesla	T
Tetragonal	tetr
Thermal conductance	G
Thermal conductivity	λ, k
Thermal diffusivity	a
Thermal resistance	R
Thermoelectric force	E
Thickness of diffusion layer	δ
Thickness of layer	t
Thickness (effective) of stationary phase	$\mathrm{d_f}$
Thickness of surface layer	τ
Thickness of various layers	δ
Thomson coefficient	μ, τ
Thomson cross section	σ_e
Time	t
Time interval, characteristic	T, τ
Tonne	t, ton
Torr (mm of mercury)	Torr
Torque	$\boldsymbol{T}$
Total bed volume	V_{tot}
Total term (spectroscopy)	T
Transconductance	g_m
Transfer coefficient	α
Transit time of nonretained solute	t_M, t_0
Transition	tr
Transition dipole moment of a molecule	$\boldsymbol{M}$, $\boldsymbol{R}$
Transition frequency	ν
Transition wavenumber	$\tilde{\nu}$
Translation (circular)	b_1; b_2; b_3
Translation vectors for crystal lattice	$\boldsymbol{a}_1$; $\boldsymbol{a}_2$; $\boldsymbol{a}_3$ $\boldsymbol{a}$; $\boldsymbol{b}$; $\boldsymbol{c}$
Transmission factor	τ
Transmittance	T, τ
Transport number	t
Transverse relaxation time	T_2
Triclinic	tric
Trigonal	trig
Triton (tritium nucleus)	t
Ultrahigh frequency	uhf
Ultraviolet	uv
Unified atomic mass unit	u
United States Pharmacopoeia	USP
Vacuum	vac
van der Waals constant	λ
Vapor pressure	p, vp
Velocity	u, w
Versus	vs
Vibrational anharmonicity constant	χ
Vibrational coordinates:	
Internal coordinates	R_i, r_I, θ_j, etc.
Normal coordinates, dimensionless	q_r
Mass adjusted	Q_r
Vibrational force constants:	
Diatomic	f
Polyatomic, dimensionless normal coordinates	$\phi_{rst\ldots}$; $k_{rst\ldots}$
Internal coordinates	f_{ij}
Symmetry coordinates	F_{ij}
Vibrational quantum number	v
Vibrational term	G
Viscosity	η, μ
Vitreous substance	vit
Volt	V
Volt-ampere-reactive	var
Volta potential difference	$\Delta\psi$
Volume	V, v
Volume flow rate	q_v
Volume fraction	ϕ
Volume in space phase	Ω
Volume liquid phase in column	V_L
Volume mobile phase in column	V_M
Volume of activation	$\Delta^{\ddagger}V$
Volume percent	vol %
Volume per volume	v/v
Volume strain	θ
Watt	W
Wavefunction	ϕ, ψ, Ψ
Wavelength	λ
Wavenumber (in a medium)	σ
Wavenumber in vacuum	$\tilde{\nu}$
Weber	Wb
Weber number	*We*
Weight	W
Weight of liquid phase	w_L
Weight percent	wt %
Weight per volume	w/v
Wien displacement constant	b
Work	w, W
Work function	Φ
x unit	X
Yard	yd
Young's modulus	E
Zeeman splitting constant	μ_B/hc
Zone width at baseline	W_b
Zone width at one-half peak height	$W_{1/2}$

TABLE 2.7 Conversion Factors

The data were compiled by L. P. Buseth for the 13th edition; some entries have been added or modified in view of recent data and SI units.

Relations which are exact are indicated by an asterisk (*). Factors in parentheses are also exact. Other factors are within ±5 in the last significant figure.

To convert	Into	Multiply by
Abampere	ampere*	10
Abcoulomb	coulomb*	10
	statcoulomb	2.998×10^{10}
Abfarad	farad*	10^{9}
Abhenry	henry*	10^{-9}
Abmho	siemens*	10^{9}
Abvolt	volt	10^{-8}
Acre	hectare or square hectometer	0.404 685 64
	square chain (Gunter's)*	10
	square kilometer*	0.004 046 873
	square meter*	4046.873
	square mile*	(1/640)
	square rod*	160
	square yard*	4840
Acre (U.S. survey)	square meter	4046.873
Acre-foot	cubic foot*	4.3560×10^{4}
	cubic meter	1233.482
	gallon (U.S.)	3.259×10^{5}
Acre-inch	cubic foot*	3630
	cubic meter	102.7902
Ampere per square centimeter	ampere per square inch*	6.4516
Ampere-hour	coulomb*	3600
	faraday	0.037 31
Ampere-turn	gilbert	1.256 637
Ampere-turn per centimeter	ampere-turn per inch	2.540
Ångström	meter*	10^{-10}
	nanometer*	0.1
Apostilb	candela per square meter	0.318 309 9; $(1/\pi)$
	lambert*	10^{-4}
Are	acre	0.024 710 54
	square meter*	100
Assay ton	gram	29.1667
Astronomical unit	meter	$1.496\,00 \times 10^{-11}$
	light-year	$1.581\,284 \times 10^{-5}$
Atmosphere	bar*	1.013 25.0
	foot of water (at 4°C)	33.898 54
	inch of mercury (at 0°C)	29.921 26
	kilogram per square centimeter	1.033 227
	millimeter of mercury*	760
	millimeter of water (4°C)	$1.033\,227 \times 10^{4}$
	newton per square meter*	$1.013\,250 \times 10^{5}$
	pascal*	101 325.0
	pound per square inch	14.695 95
	ton per square inch	0.007 348
	torr*	760
Atomic mass unit	gram	1.6605×10^{-24}
Avogadro number	molecules per mole	$6.022\,137 \times 10^{23}$

TABLE 2.7 Conversion Factors (*Continued*)

To convert	Into	Multiply by
Bar	atmosphere	0.986 923
	dyne per square centimeter*	10^6
	kilogram per square centimeter	1.019 716
	millimeter of mercury	750.062
	millimeter of water (4°C)	$1.019\ 716 \times 10^4$
	newton per square meter	10^5
	pascal*	10^5
	pound per square inch	14.503 77
Barn	square meter*	10^{-28}
Barrel (British)	gallon (British)*	36
	liter	163.659
Barrel (petroleum)	gallon (British)	34.9723
	gallon (U.S.)*	42
	liter	158.987
Barrel (U.S. dry)	bushel (U.S.)	3.281 22
	cubic foot	4.083 33
	liter	115.6271
	quart (U.S. dry)	104.9990
Barrel (U.S. liquid)	gallon (U.S.)	31.5 (variable)
	liter	119.2405
Barye	dyne per square centimeter*	1
Becquerel	curie*	2.7×10^{-11}
Biot	ampere*	10
Board foot	cubic foot	(1/12)
	cubic meter	$2.359\ 737 \times 10^{-3}$
Bohr	meter	$5.291\ 77 \times 10^{-11}$
Bohr magneton	joule per tesla	$9.274\ 02 \times 10^{-24}$
Bolt (U.S. cloth)	foot*	120
	meter	36.576
Boltzmann constant	joule per degree	1.3806×10^{-23}
British thermal unit (Btu)	calorie	251.996
	cubic foot-atmosphere	0.367 717
	erg	1.0550×10^{10}
	foot-pound	778.169
	horsepower-hour (British)	$3.930\ 15 \times 10^{-4}$
	horsepower-hour (metric)	$3.984\ 66 \times 10^{-4}$
	joule (International table)	1055.056
	joule (thermochemical)	1054.350
	kilogram-calorie	0.2520
	kilogram-meter	107.5
	kilowatt-hour	$2.930\ 71 \times 10^{-4}$
	liter-atmosphere	10.4126
Btu per foot^3	kilocalorie per cubic meter	8.899 15
Btu (International table)/ft^3	joule per meter^3	$3.725\ 895 \times 10^4$
Btu (thermochemical)/ft^3	joule per meter^3	$3.723\ 402 \times 10^4$
Btu (International table)/hour	watt	0.293 071 1
Btu (thermochemical)/hour	watt	0.292 875 1
Btu (International table)/pound	joule per kilogram*	2.326×10^3
Btu (thermochemical)/pound	joule per kilogram	$2.324\ 444 \times 10^3$
Btu (thermochemical)/(ft^2 · h)	watt per meter^2	3.154 591
Btu (thermochemical)/minute	watt	17.572 50
Btu (thermochemical)/pound	joule per kilogram	$2.324\ 444 \times 10^3$
Btu per square foot	joule per square meter	$1.135\ 65 \times 10^4$
Bucket (British, dry)	gallon (British)*	4

TABLE 2.7 Conversion Factors (*Continued*)

To convert	Into	Multiply by
Bushel (British)	bushel (U.S.)	1.032 057
	cubic foot	1.284 35
	gallon (British)*	8
	gallon (U.S.)	9.607 60
	liter	36.3687
Bushel (U.S.)	barrel (U.S., dry)	0.304 765
	bushel (British)	0.968 939
	cubic foot	1.244 456
	cubic meter	0.035 239 07
	gallon (British)	7.751 51
	gallon (U.S.)	9.309 18
	liter	35.239 07
	peck (U.S.)*	4
	pint (U.S., dry)*	64
Cable length (international)	foot	607.611 55
	meter*	185.2
	mile (nautical)*	0.1
Cable length (U.S. or British)	foot*	720
	meter	219.456
	mile (nautical)	0.118 407
	mile (statute)	0.136 364
Caliber	inch*	0.01
	millimeter*	0.254
Calorie	Btu	0.003 968 320
	foot-pound	3.088 03
	foot-poundal	99.3543
	horsepower-hour (British)	$1.559\ 61 \times 10^{-6}$
	joule*	4.184
	kilowatt-hour	1.163×10^{-6}
	liter-atmosphere	0.041 320 5
Calorie (15°C)	joule	4.1858
Calorie (international)	joule	4.1868
Calorie per minute	foot-pound per second	0.051 467 1
	horsepower (British)	$9.357\ 65 \times 10^{-5}$
	watt*	0.069 78
Candela	Hefner unit	1.11
	lumen per steradian*	1
Candela per square centimeter	candela per square foot*	929.0304
	candela per square meter*	10^4
	lambert	3.141 593; (π)
Carat (metric)	gram*	0.2
Celsius temperature	Fahrenheit temperature	(9/5)°C + 32
	kelvin	°C − 273.15
Centigrade heat unit or chu	Btu*	1.8
	calorie	453.592
	joule	1899.10
Centimeter	foot	0.032 808 4
	inch	0.393 700 8
	mil	393.700 8
Centimeter of mercury (0°C)	pascal	1333.22
Centimeter of water (4°C)	pascal	98.063 8
Centimeter per second	foot per minute	1.986 50
	kilometer per hour*	0.036

TABLE 2.7 Conversion Factors (*Continued*)

To convert	Into	Multiply by
Centimeter per second (*continued*)	knot	0.019 438 4
	mile per hour	0.022 369 4
Centimeter per second squared	foot per second squared	0.032 808 4
	meter per second squared*	0.01
Centimeter-dyne	erg*	1
	joule*	10^{-7}
	meter-kilogram	1.020×10^{-8}
	pound-foot	7.376×10^{-8}
Centimeter-gram	erg*	980.665
	joule*	$9.806\ 65 \times 10^{-5}$
Centipoise	kilogram per (meter-second)*	0.001
	pascal-second*	0.001
	pound per (foot-second)	0.006 72
Chain (Ramsden's)	foot*	100
	meter*	30.48
Chain (Gunter's)	foot*	66
	meter*	20.1168
Circular inch	circular mil*	10^6
	square centimeter	5.067 075
	square inch	$(\pi/4)$
Circular millimeter	square millimeter	$(\pi/4)$
Circumference	degree*	360
	gon (grade)	400
	radian	(2π)
Cord	cord foot*	8
	cubic foot*	128
Coulomb	ampere-second*	1
Coulomb per square centimeter	coulomb per square inch*	6.4516
Cubic centimeter	cubic foot	$3.531\ 47 \times 10^{-5}$
	cubic inch	0.061 023 744
	dram (U.S., fluid)	0.270 512 2
	gallon (British)	$2.199\ 69 \times 10^{-4}$
	gallon (U.S.)	$2.641\ 72 \times 10^{-4}$
	liter*	0.001
	minim (U.S.)	16.230 73
	ounce (British, fluid)	0.035 195 1
	ounce (U.S., fluid)	0.033 814 02
	pint (British)	0.001 759 75
	pint (U.S., dry)	0.001 816 17
	pint (U.S., liquid)	0.002 113 376
Cubic centimeter-atmosphere	joule*	0.101 325
	watt-hour	$2.814\ 58 \times 10^{-5}$
Cubic centimeter per gram	cubic foot per pound	0.016 018 5
Cubic centimeter per second	cubic foot per minute	0.002 118 88
	liter per hour*	3.6
Cubic decimeter (dm^3)	liter*	1
Cubic foot	acre-foot	$2.295\ 68 \times 10^{-5}$
	board foot*	12
	cord*	(1/128)
	cord foot*	(1/16)
	cubic inch*	1728
	cubic meter*	0.028 316 846 592
	cubic yard	(1/27)

TABLE 2.7 Conversion Factors (*Continued*)

To convert	Into	Multiply by
Cubic foot (*continued*)	gallon (British)	6.228 835
	gallon (U.S.)	7.480 519
	liter	28.316 847
Cubic foot per hour	liter per minute	0.471 947
Cubic foot per pound	cubic meter per kilogram	0.062 428 0
Cubic foot-atmosphere	Btu	2.719 48
	calorie	685.298
	joule	2869.205
	kilogram-meter	292.577
	liter-atmosphere	28.3168
	watt-hour	0.797 001
Cubic inch	cubic foot	(1/1728)
	milliliter*	16.387 064
Cubic inch per minute	cubic centimeter per second	0.273 118
Cubic kilometer	cubic mile	0.239 913
Cubic meter per kilogram	cubic foot per pound	16.0185
Cubic yard	bushel (British)	21.0223
	bushel (U.S.)	21.6962
	cubic foot*	27
	cubic meter	0.764 554 86
	liter	764.555
Cubic yard per minute	cubic foot per second*	0.45
	gallon (British) per second	2.802 98
	gallon (U.S.) per second	3.366 23
	liter per second	12.742 58
Cubit	inch*	18
Cup (U.S.)	milliliter; centimeter^3	236.6
Cup (metric)	cubic centimeter*	200
Curie	becquerel*	3.7×10^{10}
Cycle per second	hertz*	1
Dalton	kilogram	$1.660\ 54 \times 10^{-27}$
	unified atomic mass*	1
Day (mean solar)	hour*	24
	minute*	1440
	second*	86 400
Debye	coulomb-meter	$3.335\ 64 \times 10^{-30}$
Decibel	neper	0.115 129 255
Degree (plane angle)	circumference	(1/366)
	gon (grade)	1.111 11
	minute (angle)*	60
	quadrant	(1/90)
	radian	(π/180)
	revolution	(1/360)
	second (angle)*	3600
Degree (angle) per foot	radian per meter	0.057 261 5
Degree (angle) per second	radian per second	0.017 453 3
Degree Celsius	degree Fahrenheit*	1.8
	degree Rankine*	1.8
	kelvin*	1
Degree Fahrenheit	degree Celsius	(5/9)
Degree Rankine	kelvin	(5/9)
Denier	tex	(1/9)
Dipole length (e cm)	coulomb-meter	$1.602\ 18 \times 10^{-21}$

TABLE 2.7 Conversion Factors (*Continued*)

To convert	Into	Multiply by
Drachm (British)	dram (apothecaries or troy)*	1
Drachm (British, fluid)	cubic centimeter	3.551 633
	dram (U.S., fluid)	0.960 760
	minim (British)	60
	ounce (British, fluid)	(1/8)
Dram (apothecaries or troy)	dram (weight)	2.194 285 7
	grain*	60
	gram*	3.887 934 6
	ounce (troy)*	(1/8)
	pennyweight*	2.5
	pound (troy)*	(1/96)
	scruple*	3
Dram (weight)	grain*	27.343 75
	gram	1.771 845 2
	ounce (weight)	(1/16)
	pound (weight)	(1/256)
Dram (U.S., fluid)	cubic centimeter	3.696 691 2
	gallon (U.S.)	(1/1024)
	gill (U.S.)	(1/32)
	milliliter	3.696 691 2
	minim (U.S.)*	60
	ounce (U.S., fluid)	(1/8)
	pint (U.S., fluid)	(1/128)
Dyne	kilogram (force)	1.019 716 × 10^{-6}
	newton*	10^{-5}
	pound (force)	2.248 09 × 10^{-6}
Dyne per centimeter	newton per meter*	0.001
Dyne per square centimeter	bar*	10^{-6}
	kilogram per square centimeter	1.019 716 × 10^{-6}
	millimeter of mercury (0°C)	7.500 617 × 10^{-4}
	millimeter of water (4°C)	0.010 197 16
	newton per square meter*	0.1
	pascal*	0.1
	pound per square inch (psi)	1.450 38 × 10^{-5}
Dyne-centimeter	erg*	1
	foot-pound (force)	7.375 62 × 10^{-8}
	foot-poundal	2.373 04 × 10^{-6}
	joule*	10^{-7}
	kilogram-meter (force)	1.019 716 × 10^{-8}
	newton-meter*	10^{-7}
Dyne-second/centimeter^2	poise*	1
	pascal-second*	0.1
Electron charge	coulomb	1.602 18 × 10^{-19}
Electron charge-centimeter (*e* cm)	coulomb-meter	1.602 18 × 10^{-21}
Electron charge-centimeter^2	coulomb-meter squared	1.602 18 × 10^{-23}
Electron mass	atomic mass unit	0.000 548 6
	gram	9.1096 × 10^{-28}
Electronvolt	erg	1.602 18 × 10^{-12}
	joule	1.602 18 × 10^{-19}
	kilojoule per mole	96.4853
Ell	inch*	45
Em, pica	inch	0.167
	millimeter	4.217 52

TABLE 2.7 Conversion Factors (*Continued*)

To convert	Into	Multiply by
EMU[1] of capacitance	farad*	10^9
EMU of current	ampere*	10
EMU of electric potential	volt*	10^{-8}
EMU of inductance	henry*	10^{-9}
EMU of quantity (charge)	coulomb	10
EMU of resistance	ohm	10^{-9}
EMU of work	joule	10^{-7}
ESU[2] of capacitance	farad	$1.112\ 650 \times 10^{-12}$
ESU of current	ampere	$3.335\ 641 \times 10^{-10}$
ESU of electric potential	volt	299.792 5
ESU of inductance	henry	$8.987\ 552 \times 10^{11}$
ESU of quantity (charge)	coulomb	$3.335\ 556 \times 10^{-11}$
ESU of resistance	ohm	$8.987\ 552 \times 10^{11}$
ESU of work	joule	10^{-7}
Erg	dyne-centimeter*	1
	joule*	10^{-7}
	watt-hour	$2.777\ 78 \times 10^{-11}$
Erg per second	Btu	5.69×10^{-6}
	watt*	10^{-7}
Erg per ($cm^2 \times$ second)	watt per square meter*	0.001
Erg per gauss	ampere-centimeter squared*	10
	joule per tesla*	0.001
Fahrenheit scale	centigrade scale	(5/9)
Fahrenheit temperature (°F)	Celsius temperature (°C)	$(°F - 32)(5/9)$
Faraday (based on carbon-12)	coulomb	96 487.0
Faraday (chemical)	coulomb	96 495.7
Faraday (physical)	coulomb	96 521.9
Fathom	foot*	6
	meter	1.828 8
Fermi	meter*	10^{-15}
Foot	centimeter*	30.48
	inch*	12
	mile (nautical)	$1.645\ 788 \times 10^{-4}$
	mile (statute)	$1.893\ 939 \times 10^{-4}$
	yard	(1/3)
Foot of water (4°C)	atmosphere	0.029 499 8
	bar	0.029 499 8
	gram per square centimeter	30.48
	inch of mercury (0°C)	0.882 671
	pascal	2989.067
Foot per minute	centimeter per second*	0.508
	knot	0.009 874 73
	mile per hour	0.011 363 6
Foot-candle	lumen per square foot*	1
	lumen per square meter	10.7639
	lux	10.76391
Foot-lambert	candela per square centimeter	$3.426\ 26 \times 10^{-4}$
	candela per square foot	$(1/\pi)$
	lambert	0.001 076 39
	meter-lambert	10.7639

[1] EMU, the electromagnetic system of electrical units based on dynamics.
[2] ESU, the electrostatic system of electrical units based on static data.

TABLE 2.7 Conversion Factors (*Continued*)

To convert	Into	Multiply by
Foot-pound	Btu	0.001 285 07
	calorie	0.323 832
	foot-poundal	32.1740
	horsepower (British)	$5.050\ 51 \times 10^{-7}$
	joule	1.355 818
	kilogram-meter	0.138 255
	liter-atmosphere	0.013 380 9
	newton-meter	1.355 818
	watt-hour	$3.766\ 161 \times 10^{-4}$
Foot-pound per minute	horsepower (British)	$3.030\ 30 \times 10^{-5}$
	horsepower (metric)	$3.072\ 33 \times 10^{-5}$
	watt	0.022 597 0
Foot-poundal	Btu	$3.994\ 11 \times 10^{-5}$
	calorie	0.010 064 99
	foot-pound	0.031 081 0
	joule	0.042 140 11
	kilogram-meter	0.004 297 10
	liter-atmosphere	$4.158\ 91 \times 10^{-4}$
	watt-hour	$1.170\ 56 \times 10^{-5}$
Franklin	coulomb	$3.335\ 64 \times 10^{-10}$
Franklin per cm^3	coulomb per cubic meter	$3.335\ 64 \times 10^{-4}$
Franklin per cm^2	coulomb per square meter	$3.335\ 64 \times 10^{-6}$
Furlong	chain (Gunter's)*	10
	foot*	600
	meter*	201.168
	mile	(1/8)
Gallon (British, imperial)	bushel (British)	(1/8)
	cubic decimeter, liter*	4.546 90
	cubic foot	0.160 544
	gallon (U.S., fluid)	1.200 95
	gill (British)*	32
	liter	4.546 09
	ounce (British)*	160
	quart (British)*	4
Gallon (U.S.)	barrel (petroleum)	(1/42)
	cubic decimeter, liter	3.785 41
	cubic foot	0.133 680 56
	gallon (British)	0.832 674
	liter	3.785 41
	ounce (U.S., fluid)*	128
	quart (U.S., fluid)*	4
Gallon (U.S.) per minute	cubic foot per hour	8.020 83
	cubic meter per hour	0.227 125
	liter per minute	3.785 412
Gamma	microgram*	1
Gas constant	calorie per mole-degree	1.987
	joule per mole-degree	8.3143
	liter-atmosphere per mole-degree	0.082 057
Gauss	tesla*	10^{-4}
	weber per square meter*	10^{-4}
Gilbert	ampere-turn	0.795 775

TABLE 2.7 Conversion Factors (*Continued*)

To convert	Into	Multiply by
Gill (British)	cubic centimeter, mL	142.065
	cubic inch	8.669 36
	gallon (British)	(1/32)
	gill (U.S.)	1.200 95
	ounce (British, fluid)*	5
	pint (British)	(1/4)
Gill (U.S.)	cubic centimeter, mL	118.2941
	gallon (U.S.)	(1/32)
	liter	0.118 294 1
	ounce (U.S., fluid)*	4
	quart (U.S.)	(1/8)
Gon (grade)	circumference	(1/400)
	minute (angle)*	54
	radian	($2\pi/400$)
Grade	radian	($2\pi/400$)
Grain	carat (metric)*	0.323 994 55
	milligram*	64.798 91
	ounce (weight)	0.002 285 714 3
	ounce (troy)	(1/480)
	pennyweight	(1/24)
	pound	(1/7000)
	scruple	(1/20)
Gram	carat (metric)*	5
	dram	0.564 383 39
	grain	15.432 358
	ounce (weight)	0.035 273 962
	ounce (troy)	0.032 150 747
	pennyweight	0.643 014 93
	pound	0.002 204 622 6
	ton (metric)*	10^{-6}
Gram per (centimeter-second)	poise*	1
Gram per cubic centimeter	kilogram per liter*	1
	pound per cubic foot	62.4280
	pound per gallon (U.S.)	8.345 40
Gram per square meter	ounce per square foot	0.327 706
Gram per ton (long)	gram per ton (metric)	0.984 207
	gram per ton (short)	0.892 857
Gram (force)	dyne*	980.665
	newton*	0.009 806 65
Gram per square centimeter	pascal*	98.0665
Gram-centimeter	joule*	$9.806\,65 \times 10^{-5}$
Gram-square centimeter	pound-square foot	$2.373\,04 \times 10^{-6}$
Gray	joule per kilogram*	1
Hartree	electron volt	27.211 40
	hertz	$6.579\,683\,90 \times 10^{15}$
	joule	$4.359\,75 \times 10^{-18}$
Hectare	acre	2.471 054
	are*	100
	meter squared	10^{4}
Hefner unit	candela	0.9
Hemisphere	sphere*	0.5
	spherical right angle*	4
	steradian	(2π)

TABLE 2.7 Conversion Factors (*Continued*)

To convert	Into	Multiply by
Hertz	cycle per second*	1
Hogshead	gallon (U.S.)*	63
Horsepower (British)	Btu per hour	2544.43
	foot pound per hour*	1.98×10^6
	horsepower (metric)	1.013 87
	joule per second	745.700
	kilocalorie per hour	641.186
	kilogram-meter per second	76.0402
	watt	745.70
Horsepower (electric)	watt*	746
Horsepower-hour (British)	Btu	2544.43
	foot-pound*	1.98×10^6
	joule	$2.684\ 52 \times 10^6$
	kilocalorie	641.186
	kilogram-meter	$2.737\ 45 \times 10^5$
	watt-hour	745.7
Hour (mean solar)	day	(1/24)
	minute*	60
	second*	3600
	week	(1/168)
Hundredweight (long)	kilogram*	50.802 345 44
	pound*	112
	ton (long)	(1/20)
	ton (metric)	0.050 802 345
	ton (short)*	0.056
Hundredweight (short)	hundredweight (long)	0.892 857
Inch	centimeter*	2.54
	foot	(1/12)
	mil*	1000
Inch of mercury (0°C)	atmosphere	0.033 421 05
	inch of water (4°C)	13.5951
	millibar	33.863 88
	millimeter of water (4°C)	345.316
	pascal	3386.388
	pound per square inch, psi	0.491 1541
Inch of water (4°C)	inch of mercury (0°C)	0.073 5559
	millibar	2.490 89
	millimeter of mercury (0°C)	1.868 32
	pascal	249.089
	pound per square inch, psi	0.036 1273
Inch per minute	foot per hour*	5
	meter per hour*	1.524
	millimeter per second	0.423 333
Joule	Btu	$9.478\ 170 \times 10^{-4}$
	calorie*	0.2390
	centigrade heat unit, chu	5.265 65
	centimeter-dyne*	10^7
	cubic foot-atmosphere	0.000 348 529
	cubic foot-(pound per in^2)	0.005 121 959
	erg*	10^7
	foot-pound	0.737 562
	foot-poundal	23.7304
	horsepower-hour (British)	$3.725\ 06 \times 10^{-7}$
	liter-atmosphere	0.009 869 233

TABLE 2.7 Conversion Factors (*Continued*)

To convert	Into	Multiply by
Joule (*continued*)	newton-meter*	1
	watt-second*	1
Joule per centimeter	kilogram (force)	10.197 16
	newton*	100
	pound (force)	22.4809
Joule per gram	Btu per pound	0.429 923
	kilocalorie per kilogram	0.238 846
	watt-hour per pound	0.125 998
Joule per second	watt*	1
Kilogram (force)	dyne*	$9.806\ 65 \times 10^5$
	newton*	9.806 65
	pound (force)	2.204 62
	poundal	70.9316
Kilometer	astronomical unit	$6.684\ 59 \times 10^{-9}$
	mile (nautical)	0.539 956 80
	mile (statute)	0.621 371 192
Kilowatt	Btu per minute	56.8690
	foot-pound per second	737.562
	horsepower (British)	1.341 02
	horsepower (metric)	1.359 62
	joule per second*	1000
	kilocalorie per hour	859.845
Kilowatt-hour	Btu	3412.14
	horsepower-hour (British)	1.341 02
	joule*	3.6×10^6
	kilocalorie	859.845
Knot	foot per minute	101.2686
	kilometer per hour*	1.852
	mile (nautical) per hour*	1
	mile (statute) per hour	1.150 78
Lambda	decimeter cubed*	10^{-6}
	microliter*	1
Lambert	candela per square meter	$(1/\pi) \times 10^4$; 3183.099
	candela per square inch	2.053 61
	foot-lambert	929.030
Langley	joule per square meter*	4.184×10^4
League (nautical)	mile (nautical)*	3
League (statute)	mile (statute)*	3
Light-year	astronomical unit	$6.323\ 97 \times 10^4$
	meter	$9.460\ 73 \times 10^{15}$
Link	chain*	0.01
Liter	cubic decimeter (dm^3)*	1
	cubic foot	0.035 314 67
	gallon (British)	0.219 969
	gallon (U.S.)	0.264 172 1
	quart (British)	0.879 877
	quart (U.S.)	1.056 688
Liter per minute	cubic foot per hour	2.118 88
	gallon (British) per hour	13.198
	gallon (U.S.) per hour	15.8503
Liter-atmosphere	Btu	0.096 037 6
	calorie	24.2011
	cubic foot-atmosphere	0.035 314 7
	cubic foot-pound per in^2	0.518 983

TABLE 2.7 Conversion Factors (*Continued*)

To convert	Into	Multiply by
Liter-atmosphere (*continued*)	horsepower (British)	$3.774\ 42 \times 10^{-5}$
	horsepower (metric)	$3.826\ 77 \times 10^{-5}$
	joule*	101.325
	kilogram-meter	10.332 27
	watt-hour	0.028 145 8
Lumen per square centimeter	lux*	10^4
	phot*	1
Lumen per square meter	lumen per square foot	0.092 903 0
Lux	lumen per square meter*	1
Maxwell	weber*	10^{-8}
Meter	ångström*	10^{10}
	fathom	0.546 807
	foot	3.280 839 895
	inch	39.370 078 740
	mile (nautical)	$5.399\ 568 \times 10^{-4}$
	mile (statute)	$6.213\ 712 \times 10^{-4}$
Meter per second	foot per minute	196.850
	kilometer per hour*	3.6
	knot	1.943 844
	mile per hour	2.236 936
Meter-candle	lux*	1
Meter-lambert	candela per square meter	$(1/\pi)$
	foot-lambert	0.092 903 0
	lambert*	10^{-4}
Mho (ohm−1)	siemen*	1
Micron	meter	10^{-6}
Mil	inch*	0.001
	micrometer*	25.4
Mile (nautical)	foot	6076.115 49
	kilometer*	1.852
	mile (statute)	1.150 78
Mile (statute)	chain (Gunter's)*	80
	chain (Ramsden's)*	52.8
	foot*	5280
	furlong*	8
	kilometer*	1.609 344
	light-year	$1.701\ 11 \times 10^{-11}$
	link (Gunter's)*	8000
	link (Ramsden's)*	5280
	mile (nautical)	0.868 976
	rod*	320
Mile per gallon (British)	kilometer per liter	0.354 006
Mile per gallon (U.S.)	kilometer per liter	0.425 144
Mile per hour	foot per minute	88
	kilometer per hour*	1.609 344
	knot	0.868 976
Milliliter	cubic centimeter*	1
Millimeter of mercury (0°C)	atmosphere	(1/760)
	dyne per square centimeter	1333.224
	millimeter of water (4°C)	13.5951
	pascal	133.322
	pound per square inch (psi)	0.019 336 8
	torr*	1

TABLE 2.7 Conversion Factors (*Continued*)

To convert	Into	Multiply by
Millimeter of water (4°C)	atmosphere	0.009 678 41
	millibar*	0.098 066 5
	millimeter of mercury (0°C)	0.073 555 9
	pascal*	9.806 65
	pound per square inch	0.001 422 33
Minim (British)	milliliter	0.059 193 9
	minim (U.S.)	0.960 760
Minim (U.S.)	milliliter	0.061 611 5
Minute (plane angle)	circumference	$4.629\ 63 \times 10^{-5}$
	degree (angle)	(1/60)
	gon	(1/54)
	radian	(π/10,800)
Minute	hour	(1/60)
	second	60
Month (mean of 4-year period)	day	30.4375
	hour	730.5
	week	4.348 21
Nail (British)	inch*	2.25
Nanometer	ångström*	10
Neper	decibel	8.685 890
Nuclear magneton	joule per tesla	$5.050\ 79 \times 10^{-27}$
Neutron mass	atomic mass unit	1.008 66
	gram	1.6749×10^{-24}
Newton	dyne*	10^5
	kilogram (force)	0.101 971 6
	pound (force)	0.224 809
	poundal	7.233 01
Newton per square meter	*See* pascal	
Newton-meter	foot-pound	0.737 562
	joule*	1
	kilogram-meter	0.101 971 6
	watt-second*	1
Nit	candela per square meter*	1
Noggin (British)	gill (British)*	1
Nox	lux*	0.001
Oersted	ampere per meter (in practice)	(1000/4π); 79.577 47
Ohm (mean international)	ohm	1.000 49
Ohm (U.S. international)	ohm	1.000 495
Ohm per foot	ohm per meter	3.280 84
Ounce (avoirdupois)	dram*	16
	grain*	437.5
	gram*	28.349 5
	ounce (troy)	0.911 458 33
	pound	(1/16)
Ounce (troy)	grain*	480
	gram*	31.1035
	ounce (avoirdupois)	1.097 142 9
	pennyweight*	20
	pound (avoirdupois)	0.068 571 429
	scruple*	24
Ounce (British, fluid)	cubic centimeter	28.413 06
	gallon (British)	(1/160)
	milliliter	28.413 06

TABLE 2.7 Conversion Factors (*Continued*)

To convert	Into	Multiply by
Ounce (British, fluid) (*continued*)	minim (British)	480
	ounce (U.S., fluid)	0.960 760
	pint (British)	(1/20)
	quart (British)	(1/40)
Ounce (U.S., fluid)	cubic centimeter	29.573 530
	gallon (U.S.)	(1/128)
	milliliter	29.573 530
	pint (U.S., fluid)	(1/16)
	quart (U.S., fluid)	(1/32)
Ounce (avoirdupois) per cubic foot	kilogram per cubic meter	1.001 154
Ounce (avoirdupois)/gallon (U.S.)	gram per liter	7.489 15
Ounce (avoirdupois) per ton (long)	gram per ton (metric)	27.9018
	milligram per kilogram	27.9018
Ounce (avoirdupois) per ton (short)	gram per ton (metric)*	31.25
	milligram per kilogram*	31.25
Parsec	light-year	3.261 636
Part per million	milligram per kilogram*	1
	milliliter per cubic meter*	1
Pascal	atmosphere	$9.869\,233 \times 10^{-6}$
	bar*	10^{-5}
	dyne per square centimeter*	10
	inch of mercury	$2.953\,00 \times 10^{-4}$
	millimeter of mercury	$7.500\,62 \times 10^{-3}$
	millimeter of water	0.101 972
	newton per square meter*	1
	pound per square inch	$1.450\,377 \times 10^{-4}$
	poundal per square foot	0.671 969
Pascal-second	poise*	10
Peck (British)	gallon (British)*	2
Peck (U.S.)	bushel (U.S.)*	0.25
Pennyweight	grain*	24
	gram*	1.555 173 84
	ounce (troy)	(1/20)
	pound	0.003 428 571 4
Phot	lux*	10^4
Pica (printer's)	inch	0.167
	point*	12
Pint (British)	gallon (British)	(1/8)
	liter	0.568 261
	pint (U.S., fluid)	1.200 95
	quart (British)	0.5
Pint (U.S., dry)	bushel (U.S.)	(1/64)
	liter	0.550 610 5
	peck (U.S.)	(1/16)
	pint (British)	0.968 939
	quart (U.S., dry)	0.5
Pint (U.S., fluid)	gallon (U.S.)	(1/8)
	liter	0.473 176 5

TABLE 2.7 Conversion Factors (*Continued*)

To convert	Into	Multiply by
Pint (U.S., fluid)	pint (British)	0.832 674
(*continued*)	quart (U.S., fluid)*	0.5
Planck's constant	joule-second	$6.626\ 08 \times 10^{-34}$
Point (printer's, Didot)	millimeter	0.376 065 03
Point (printer's, U.S.)	millimeter*	0.351 459 8
Poise	dyne-second per square centimeter*	1
	pascal-second*	0.1
Polarizability volume ($4\pi\epsilon_0$ cm^3)	coulomb squared-(meter squared per joule)	$1.112\ 65 \times 10^{-16}$
Pole (British)	foot*	16.5
Pottle (British)	gallon (British)*	0.5
Pound	gram*	453.592 37
	ounce (weight)*	16
	ton (long)	$4.464\ 285\ 7 \times 10^{-4}$
	ton (short)	(1/2000)
Pound (troy)	grain	5760
	gram*	373.241 721 6
	ounce (troy)*	12
	pennyweight	240
	pound (weight)	0.822 857 14
	scruple*	288
Pound per cubic foot	kilogram per cubic meter	16.018 46
Pound per cubic inch	gram per cubic centimeter	27.679 905
	pound per cubic foot*	1728
Pound per foot	kilogram per meter	1.488 16
Pound per (foot-second)	pascal-second	1.488 16
Pound per gallon (U.S.)	gram per liter	119.8264
Pound per hour	kilogram per day	10.886 22
Pound per inch	kilogram per meter	17.857 97
Pound per minute	kilogram per hour	27.215 54
Pound per square foot	kilogram per square meter	4.882 43
Pound (force)	kilogram (force)	0.453 592
	newton	4.448 222
	poundal	32.1740
Pound per square inch	atmosphere	0.068 046 0
	bar	0.068 948 0
	inch of mercury (0°C)	2.036 02
	millimeter of mercury (0°C)	51.7149
	millimeter of water (4°C)	703.070
	pascal	6894.757
	pound per square foot	144
Pound-second per square inch	pascal-second	6894.76
Poundal	gram (force)	14.0981
	newton	0.138 255
	pound (force)	0.031 081 0
Poundal per square foot	pascal	1.488 164
Poundal-foot	newton-meter	0.042 140 1
Poundal-second per square foot	pascal-second	1.488 164
Proof (U.S.)	percent alcohol by volume*	0.5
Proton mass	atomic mass unit	1.007 28
	gram	1.6726×10^{-24}
Puncheon (British)	gallon (British)	70

TABLE 2.7 Conversion Factors (*Continued*)

To convert	Into	Multiply by
Quad	Btu	10^{15}
	joule	1.055×10^{18}
Quadrant	circumference*	0.25
	degree (angle)*	90
	gon (grade)*	100
	minute (angle)*	5400
	radian	$(\pi/2)$
Quadrupole area (e cm^2)	coulomb meter squared	$1.602\ 18 \times 10^{-23}$
Quart (British)	gallon (British)*	0.25
	liter	1.136 523
	ounce (British, fluid)*	40
	pint (British)*	2
	quart (U.S., fluid)	1.200 95
Quart (U.S., dry)	bushel (U.S.)	(1/32)
	cubic foot	0.038 889 25
	liter	1.101 221
	peck (U.S.)	(1/8)
	pint (U.S., dry)*	2
Quart (U.S., fluid)	gallon (U.S.)*	0.25
	liter	0.946 529
	ounce (U.S., fluid)*	32
	pint (U.S., fluid)	2
	quart (British)	0.832 674
Quartern (British, fluid)	gill (British)*	0.5
Quintal (metric)	kilogram*	100
Rad (absorbed dose)	gray*	0.01
	joule per kilogram*	0.01
Radian	circumference	$(1/2\pi)$
	degree (angle)	57.295 780
	minute (angle)	3437.75
	quadrant	$(2/\pi)$
	revolution	$(1/2\pi)$
Radian per centimeter	degree per millimeter	5.729 58
	degree per inch	145.531
Radian per second	revolution per minute	9.549 30
Radian per second squared	revolution per minute squared	572.958
Rankin (degree)	kelvin	(5/9)
Ream	quire*	20
	sheet	480 or 500
Register ton	cubic foot*	100
	cubic meter	2.831 685
Rem (dose equivalent)	sievert*	0.01
Revolution	degree (angle)	360
	gon*	400
	quadrant*	4
	radian	(2π)
Revolution per minute	degree (angle) per second*	6
	radian per second	0.104 720
Revolution per minute squared	radian per second squared	0.001 745 33
Revolution per second squared	radian per second squared	6.283 185
	revolution per minute squared	3600
Reyn	pascal-second	6894.76
	pound-second per square inch	1

TABLE 2.7 Conversion Factors (*Continued*)

To convert	Into	Multiply by
Rhe	per pascal-second*	10
Right angle	degree*	90
	radian	$(\pi/2)$
Rod (British, volume)	cubic foot*	1000
Rod (surveyer's measure)	chain (Gunter's)*	0.25
	foot*	16.5
	link (Gunter's)*	25
	meter*	5.0292
Roentgen	coulomb per kilogram	2.58×10^{-4}
Rood (British)	acre*	0.25
	square meter	1011.714 1
Rydberg	joule	$2.179\ 87 \times 10^{-18}$
Scruple	dram (troy)	(1/3)
	grain*	20
	gram*	1.295 978 2
	ounce (weight)	0.045 714 286
	ounce (troy)	(1/24)
	pennyweight	(10/12)
	pound	(1/350)
Second (plane angle)	degree	$2.777\ 78 \times 10^{-4}$
	minute	(1/60)
	radian	$(\pi/6.48 \times 10^{5})$
Section	square mile*	1
Siemens	mho (ohm^{-1})*	1
Slug	geepound*	1
	kilogram	14.593 90
	pound	32.1740
Speed of light	centimeter per second	$2.997\ 924\ 58 \times 10^{10}$
Sphere	steradian	(4π)
Square centimeter	circular mil	$1.973\ 53 \times 10^{5}$
	circular millimeter	127.3240
	square inch	0.155 000 31
Square chain (Gunter's)	acre*	0.1
	square foot*	4356
	square meter	404.686
Square chain (Ramsden's)	square foot*	10^{4}
Square degree (angle)	steradian	$3.046\ 17 \times 10^{-4}$
Square foot	acre	$2.295\ 68 \times 10^{-5}$
	square centimeter	929.0304
	square meter	0.092 903 04
	square rod	0.003 673 09
Square inch	circular mil	$1.273\ 240 \times 10^{6}$
	circular millimeter	821.4432
	square centimeter	6.4516
Square kilometer	acre	247.1054
	hectare*	100
	square mile	0.386 102 16
Square link (Gunter's)	square foot*	0.4356
Square link (Ramsden's)	square foot*	1
Square meter	are*	0.01
	square foot	10.763 91
	square mile	$3.861\ 01 \times 10^{-7}$

TABLE 2.7 Conversion Factors (*Continued*)

To convert	Into	Multiply by
Square meter (*continued*)	square rod	0.039 536 9
	square yard	1.195 990
Square mile	acre*	640
	square kilometer	2.589 988 110
	township	(1/36)
Square rod	acre	(1/160)
	square foot	272.25
	square meter	25.292 853
Square yard	square foot*	9
	square inch*	1296
	square meter*	0.836 127 36
	square rod	0.033 057 85
Statampere	ampere	$3.335\ 641 \times 10^{-10}$
Statcoulomb	coulomb	$3.335\ 641 \times 10^{-10}$
Statfarad	farad	$1.112\ 650 \times 10^{-12}$
Stathenry	henry	$8.987\ 552 \times 10^{11}$
Statmho	siemens	$1.112\ 650 \times 10^{-12}$
Statohm	ohm	$8.987\ 552 \times 10^{11}$
Statvolt	volt	299.7925
Statweber	weber	299.7925
Steradian	sphere	$(1/4\pi)$
	spherical right angle	$(2/\pi)$
	square degree	3282.81
Stere	cubic meter*	1
Stilb	candela/cm^2	1
Stokes (kinematic viscosity)	square meter per second*	10^{-4}
Stone (British)	pound*	14
Svedberg	second*	10^{-13}
Tablespoon (metric)	cubic centimeter*; milliter	14.79
Teaspoon (metric)	cubic centimeter*; milliliter	4.929
Tesla	weber per square meter*	1
Tex	denier*	9
	gram per kilometer*	1
Therm	Btu*	10^5
	joule*	$1.054\ 804 \times 10^8$
Ton (assay)	gram	29.166 67
Ton (long)	hundredweight (long)*	20
	hundredweight (short)*	22.4
	kilogram	1016.046 908 8
	pound*	2240
	ton (metric)	1.016 046 9
	ton (short)	1.12
Ton (metric)	hundredweight (long)	19.684 131
	hundredweight (short)	22.046 226
	kilogram*	1000
	pound	2204.6226
	ton (long)	0.984 206 53
	ton (short)*	1.102 311 3
Ton (short)	kilogram	907.184 74
	pound*	2000
Ton (force, long)	newton	1186.553
Ton (force, metric)	newton	9806.65

TABLE 2.7 Conversion Factors (*Continued*)

To convert	Into	Multiply by
Ton (force, short)	newton	8896.44
Ton (force, long)/ft^2	bar	1.072 518
	pascal	$1.072\ 518 \times 10^5$
Ton (force, metric)/m^2	bar	0.098 066 5
	pascal	9806.65
Ton (force, short)/ft^2	bar	0.957 605
	pascal	$9.576\ 05 \times 10^4$
Tonne (metric)	kilogram*	1000
Torr	atmosphere	(1/760)
	millibar	1.333 224
	millimeter of mercury* (0°C)	1
	pascal	133.322; (101 325/760)
Township (U.S.)	square kilometer	93.2396
	square mile*	36
Unified atomic mass unit	kilogram	$1.660\ 54 \times 10^{-27}$
Unit pole	weber	$1.256\ 637 \times 10^{-7}$
Volt (mean international)	volt	1.000 34
Volt (U.S. international)	volt	1.000 330
Volt-second	weber*	1
Watt	Btu per hour	3.412 14
	calorie per minute	14.3308
	erg per second*	10^7
	foot-pound per minute	44.2537
	horsepower (British)	0.001 341 02
	horsepower (metric)	0.001 359 62
	joule per second*	1
	kilogram-meter per second	0.101 972
Watt per square inch	watt per square meter	1550.003
Watt-hour	Btu	3.412 14
	calorie	859.845
	foot-pound	2655.22
	horsepower-hour (British)	0.001 341 02
	horsepower-hour (metric)	0.001 359 62
	joule*	3600
	liter-atmosphere	35.5292
Watt-second	joule*	1
Weber	maxwell*	10^8
Week	day*	7
	hour*	168
Wey (British, capacity)	bushel (British)	40 (variable)
Wey (British, mass)	pound	252 (variable)
X unit	meter	$1.002\ 02 \times 10^{-13}$
Yard	fathom*	0.5
	meter	0.9144
Year (mean of 4-years)	day	365.25
	week	52.178 87
Year (sidereal)	day (mean solar)	365.256 36

TABLE 2.8 Temperature Conversion Table

The column of figures in bold and which is headed “Reading in °F. or °C. to be converted” refers to the temperature either in degrees Fahrenheit or Celsius which it is desired to convert into the other scale. If converting from Fahrenheit degrees to Celsius degrees, the equivalent temperature will be found in the column headed “°C.”; while if converting from degrees Celsius to degrees Fahrenheit, the equivalent temperature will be found in the column headed “°F.” This arrangement is very similar to that of Sauveur and Boylston, copyrighted 1920, and is published with their permission.

°F.	Reading in °F. or °C. to be converted	°C.
....	−458	−272.22
....	−456	−271.11
....	−454	−270.00
....	−452	−268.89
....	−450	−267.78
....	−448	−266.67
....	−446	−265.56
....	−444	−264.44
....	−442	−263.33
....	−440	−262.22
....	−438	−261.11
....	−436	−260.00
....	−434	−258.89
....	−432	−257.78
....	−430	−256.67
....	−428	−255.56
....	−426	−254.44
....	−424	−253.33
....	−422	−252.22
....	−420	−251.11
....	−418	−250.00
....	−416	−248.89
....	−414	−247.78
....	−412	−246.67
....	−410	−245.56
....	−408	−244.44
....	−406	−243.33
....	−404	−242.22
....	−402	−241.11
....	−400	−240.00
....	−398	−238.89
....	−396	−237.78
....	−394	−236.67
....	−392	−235.56
....	−390	−234.44
....	−388	−233.33
....	−386	−232.22
....	−384	−231.11
....	−382	−230.00
....	−380	−228.89
....	−378	−227.78
....	−376	−226.67
....	−374	−225.56
....	−372	−224.44
....	−370	−223.33
....	−368	−222.22
....	−366	−221.11
....	−364	−220.00
....	−362	−218.89
....	−360	−217.78
....	−358	−216.67
....	−356	−215.56
....	−354	−214.44
....	−352	−213.33
....	−350	−212.22
....	−348	−211.11
....	−346	−210.00
....	−344	−208.89
....	−342	−207.78
....	−340	−206.67
....	−338	−205.56
....	−336	−204.44
....	−334	−203.33
....	−332	−202.22
....	−330	−201.11
....	−328	−200.00
....	−326	−198.89
....	−324	−197.78
....	−322	−196.67
....	−320	−195.56
....	−318	−194.44
....	−316	−193.33
....	−314	−192.22
....	−312	−191.11
....	−310	−190.00
....	−308	−188.89
....	−306	−187.78
....	−304	−186.67
....	−302	−185.56
....	−300	−184.44

TABLE 2.8 Temperature Conversion Table (*Continued*)

°F.	Reading in °F. or °C. to be converted	°C.
....	−298	−183.33
....	−296	−182.22
....	−294	−181.11
....	−292	−180.00
....	−290	−178.89
....	−288	−177.78
....	−286	−176.67
....	−284	−175.56
....	−282	−174.44
....	−280	−173.33
....	−278	−172.22
....	−276	−171.11
....	−274	−170.00
−457.6	−272	−168.89
−454.0	−270	−167.78
−450.4	−268	−166.67
−446.8	−266	−165.56
−443.2	−264	−164.44
−439.6	−262	−163.33
−436.0	−260	−162.22
−432.4	−258	−161.11
−428.8	−256	−160.00
−425.2	−254	−158.89
−421.6	−252	−157.78
−418.0	−250	−156.67
−414.4	−248	−155.56
−410.8	−246	−154.44
−407.2	−244	−153.33
−403.6	−242	−152.22
−400.0	−240	−151.11
−396.4	−238	−150.00
−392.8	−236	−148.89
−389.2	−234	−147.78
−385.6	−232	−146.67
−382.0	−230	−145.56
−378.4	−228	−144.44
−374.8	−226	−143.33
−371.2	−224	−142.22
−367.6	−222	−141.11
−364.0	−220	−140.00
−360.4	−218	−138.89
−356.8	−216	−137.78
−353.2	−214	−136.67
−349.6	−212	−135.56
−346.0	−210	−134.44
−342.4	−208	−133.33
−338.8	−206	−132.22
−335.2	−204	−131.11
−331.6	−202	−130.00
−328.0	−200	−128.89
−324.4	−198	−127.78
−320.8	−196	−126.67
−317.2	−194	−125.56
−313.6	−192	−124.44
−310.0	−190	−123.33
−306.4	−188	−122.22
−302.8	−186	−121.11
−299.2	−184	−120.00
−295.6	−182	−118.89
−292.0	−180	−117.78
−288.4	−178	−116.67
−284.8	−176	−115.56
−281.2	−174	−114.44
−277.6	−172	−113.33
−274.0	−170	−112.22
−270.4	−168	−111.11
−266.8	−166	−110.00
−263.2	−164	−108.89
−259.6	−162	−107.78
−256.0	−160	−106.67
−252.4	−158	−105.56
−248.8	−156	−104.44
−245.2	−154	−103.33
−241.6	−152	−102.22
−238.0	−150	−101.11
−234.4	−148	−100.00
−230.8	−146	−98.89
−227.2	−144	−97.78
−223.6	−142	−96.67
−220.0	−140	−95.56
−216.4	−138	−94.44
−212.8	−136	−93.33
−209.2	−134	−92.22
−205.6	−132	−91.11
−202.0	−130	−90.00
−198.4	−128	−88.89
−194.8	−126	−87.78
−191.2	−124	−86.67
−187.6	−122	−85.56
−184.0	−120	−84.44

TABLE 2.8 Temperature Conversion Table (*Continued*)

°F.	Reading in °F. or °C. to be converted	°C.
−180.4	−118	−83.33
−176.8	−116	−82.22
−173.2	−114	−81.11
−169.6	−112	−80.00
−166.0	−110	−78.89
−162.4	−108	−77.78
−158.8	−106	−76.67
−155.2	−104	−75.56
−151.6	−102	−74.44
−148.0	−100	−73.33
−144.4	−98	−72.22
−140.8	−96	−71.11
−137.2	−94	−70.00
−133.6	−92	−68.89
−130.0	−90	−67.78
−126.4	−88	−66.67
−122.8	−86	−65.56
−119.2	−84	−64.44
−115.6	−82	−63.33
−112.0	−80	−62.22
−108.4	−78	−61.11
−104.8	−76	−60.00
−101.2	−74	−58.89
−97.6	−72	−57.78
−94.0	−70	−56.67
−90.4	−68	−55.56
−86.8	−66	−54.44
−83.2	−64	−53.33
−79.6	−62	−52.22
−76.0	−60	−51.11
−72.4	−58	−50.00
−68.8	−56	−48.89
−65.2	−54	−47.78
−61.6	−52	−46.67
−58.0	−50	−45.56
−54.4	−48	−44.44
−50.8	−46	−43.33
−47.2	−44	−42.22
−43.6	−42	−41.11
−40.0	−40	−40.00
−36.4	−38	−38.89
−32.8	−36	−37.78
−29.2	−34	−36.67
−25.6	−32	−35.56
−22.0	−30	−34.44
−18.4	−28	−33.33
−14.8	−26	−32.22
−11.2	−24	−31.11
−7.6	−22	−30.00
−4.0	−20	−28.89
−0.4	−18	−27.78
+3.2	−16	−26.67
+6.8	−14	−25.56
+10.4	−12	−24.44
+14.0	−10	−23.33
+17.6	−8	−22.22
+19.4	−7	−21.67
+21.2	−6	−21.11
+23.0	−5	−20.56
+24.8	−4	−20.00
+26.6	−3	−19.44
+28.4	−2	−18.89
+30.2	−1	−18.33
+32.0	±0	−17.78
+33.8	+1	−17.22
+35.6	+2	−16.67
+37.4	+3	−16.11
+39.2	+4	−15.56
+41.0	+5	−15.00
+42.8	+6	−14.44
+44.6	+7	−13.89
+46.4	+8	−13.33
+48.2	+9	−12.78
+50.0	+10	−12.22
+51.8	+11	−11.67
+53.6	+12	−11.11
+55.4	+13	−10.56
+57.2	+14	−10.00
+59.0	+15	−9.44
+60.8	+16	−8.89
+62.6	+17	−8.33
+64.4	+18	−7.78
+66.2	+19	−7.22
+68.0	+20	−6.67
+69.8	+21	−6.11
+71.6	+22	−5.56
+73.4	+23	−5.00
+75.2	+24	−4.44
+77.0	+25	−3.89
+78.8	+26	−3.33

TABLE 2.8 Temperature Conversion Table (*Continued*)

°F.	Reading in °F. or °C. to be converted	°C.
+80.6	+27	−2.78
+82.4	+28	−2.22
+84.2	+29	−1.67
+86.0	+30	−1.11
+87.8	+31	−0.56
+89.6	+32	±0.00
+91.4	+33	+0.56
+93.2	+34	+1.11
+95.0	+35	+1.67
+96.8	+36	+2.22
+98.6	+37	+2.78
+100.4	+38	+3.33
+102.2	+39	+3.89
+104.0	+40	+4.44
+105.8	+41	+5.00
+107.6	+42	+5.56
+109.4	+43	+6.11
+111.2	+44	+6.67
+113.0	+45	+7.22
+114.8	+46	+7.78
+116.6	+47	+8.33
+118.4	+48	+8.89
+120.2	+49	+9.44
+122.0	+50	+10.00
+123.8	+51	+10.56
+125.6	+52	+11.11
+127.4	+53	+11.67
+129.2	+54	+12.22
+131.0	+55	+12.78
+132.8	+56	+13.33
+134.6	+57	+13.89
+136.4	+58	+14.44
+138.2	+59	+15.00
+140.0	+60	+15.56
+141.8	+61	+16.11
+143.6	+62	+16.67
+145.4	+63	+17.22
+147.2	+64	+17.78
+149.0	+65	+18.33
+150.8	+66	+18.89
+152.6	+67	+19.44
+154.4	+68	+20.00
+156.2	+69	+20.56
+158.0	+70	+21.11
+159.8	+71	+21.67
+161.6	+72	+22.22
+163.4	+73	+22.78
+165.2	+74	+23.33
+167.0	+75	+23.89
+168.8	+76	+24.44
+170.6	+77	+25.00
+172.4	+78	+25.56
+174.2	+79	+26.11
+176.0	+80	+26.67
+177.8	+81	+27.22
+179.6	+82	+27.78
+181.4	+83	+28.33
+183.2	+84	+28.89
+185.0	+85	+29.44
+186.8	+86	+30.00
+188.6	+87	+30.56
+190.4	+88	+31.11
+192.2	+89	+31.67
+194.0	+90	+32.22
+195.8	+91	+32.78
+197.6	+92	+33.33
+199.4	+93	+33.89
+201.2	+94	+34.44
+203.0	+95	+35.00
+204.8	+96	+35.56
+206.6	+97	+36.11
+208.4	+98	+36.67
+210.2	+99	+37.22
+212.0	+100	+37.78
+213.8	+101	+38.33
+215.6	+102	+38.89
+217.4	+103	+39.44
+219.2	+104	+40.00
+221.0	+105	+40.56
+222.8	+106	+41.11
+224.6	+107	+41.67
+226.4	+108	+42.22
+228.2	+109	+42.78
+230.0	+110	+43.33
+231.8	+111	+43.89
+233.6	+112	+44.44
+235.4	+113	+45.00
+237.2	+114	+45.56
+239.0	+115	+46.11
+240.8	+116	+46.67

TABLE 2.8 Temperature Conversion Table (*Continued*)

°F.	Reading in °F. or °C. to be converted	°C.
+242.6	+117	+47.22
+244.4	+118	+47.78
+246.2	+119	+48.33
+248.0	+120	+48.89
+249.8	+121	+49.44
+251.6	+122	+50.00
+253.4	+123	+50.56
+255.2	+124	+51.11
+257.0	+125	+51.67
+258.8	+126	+52.22
+260.6	+127	+52.78
+262.4	+128	+53.33
+264.2	+129	+53.89
+266.0	+130	+54.44
+267.8	+131	+55.00
+269.6	+132	+55.56
+271.4	+133	+56.11
+273.2	+134	+56.67
+275.0	+135	+57.22
+276.8	+136	+57.78
+278.6	+137	+58.33
+280.4	+138	+58.89
+282.2	+139	+59.44
+284.0	+140	+60.00
+285.8	+141	+60.56
+287.6	+142	+61.11
+289.4	+143	+61.67
+291.2	+144	+62.22
+293.0	+145	+62.78
+294.8	+146	+63.33
+296.6	+147	+63.89
+298.4	+148	+64.44
+300.2	+149	+65.00
+302.0	+150	+65.56
+303.8	+151	+66.11
+305.6	+152	+66.67
+307.4	+153	+67.22
+309.2	+154	+67.78
+311.0	+155	+68.33
+312.8	+156	+68.89
+314.6	+157	+69.44
+316.4	+158	+70.00
+318.2	+159	+70.56
+320.0	+160	+71.11
+321.8	+161	+71.67
+323.6	+162	+72.22
+325.4	+163	+72.78
+327.2	+164	+73.33
+329.0	+165	+73.89
+330.8	+166	+74.44
+332.6	+167	+75.00
+334.4	+168	+75.56
+336.2	+169	+76.11
+338.0	+170	+76.67
+339.8	+171	+77.22
+341.6	+172	+77.78
+343.4	+173	+78.33
+345.2	+174	+78.89
+347.0	+175	+79.44
+348.8	+176	+80.00
+350.6	+177	+80.56
+352.4	+178	+81.11
+354.2	+179	+81.67
+356.0	+180	+82.22
+357.8	+181	+82.78
+359.6	+182	+83.33
+361.4	+183	+83.89
+363.2	+184	+84.44
+365.0	+185	+85.00
+366.8	+186	+85.56
+368.6	+187	+86.11
+370.4	+188	+86.67
+372.2	+189	+87.22
+374.0	+190	+87.78
+375.8	+191	+88.33
+377.6	+192	+88.89
+379.4	+193	+89.44
+381.2	+194	+90.00
+383.0	+195	+90.56
+384.8	+196	+91.11
+386.6	+197	+91.67
+388.4	+198	+92.22
+390.2	+199	+92.78
+392.0	+200	+93.33
+393.8	+201	+93.89
+395.6	+202	+94.44
+397.4	+203	+95.00
+399.2	+204	+95.56
+401.0	+205	+96.11
+402.8	+206	+96.67

TABLE 2.8 Temperature Conversion Table (*Continued*)

°F.	Reading in °F. or °C. to be converted	°C.
+404.6	+207	+97.22
+406.4	+208	+97.78
+408.2	+209	+98.33
+410.0	+210	+98.89
+411.8	+211	+99.44
+413.6	+212	+100.00
+415.4	+213	+100.56
+417.2	+214	+101.11
+419.0	+215	+101.67
+420.8	+216	+102.22
+422.6	+217	+102.78
+424.4	+218	+103.33
+426.2	+219	+103.89
+428.0	+220	+104.44
+431.6	+222	+105.56
+435.2	+224	+106.67
+438.8	+226	+107.78
+442.4	+228	+108.89
+446.0	+230	+110.00
+449.6	+232	+111.11
+453.2	+234	+112.22
+456.8	+236	+113.33
+460.4	+238	+114.44
+464.0	+240	+115.56
+467.6	+242	+116.67
+471.2	+244	+117.78
+474.8	+246	+118.89
+478.4	+248	+120.00
+482.0	+250	+121.11
+485.6	+252	+122.22
+489.2	+254	+123.33
+492.8	+256	+124.44
+496.4	+258	+125.56
+500.0	+260	+126.67
+503.6	+262	+127.78
+507.2	+264	+128.89
+510.8	+266	+130.00
+514.4	+268	+131.11
+518.0	+270	+132.22
+521.6	+272	+133.33
+525.2	+274	+134.44
+528.8	+276	+135.56
+532.4	+278	+136.67
+536.0	+280	+137.78
+539.6	+282	+138.89
+543.2	+284	+140.00
+546.8	+286	+141.11
+550.4	+288	+142.22
+554.0	+290	+143.33
+557.6	+292	+144.44
+561.2	+294	+145.56
+564.8	+296	+146.67
+568.4	+298	+147.78
+572.0	+300	+148.89
+575.6	+302	+150.00
+579.2	+304	+151.11
+582.8	+306	+152.22
+586.4	+308	+153.33
+590.0	+310	+154.44
+593.6	+312	+155.56
+597.2	+314	+156.67
+600.8	+316	+157.78
+604.4	+318	+158.89
+608.0	+320	+160.00
+611.6	+322	+161.11
+615.2	+324	+162.22
+618.8	+326	+163.33
+622.4	+328	+164.44
+626.0	+330	+165.56
+629.6	+332	+166.67
+633.2	+334	+167.78
+636.8	+336	+168.89
+640.4	+338	+170.00
+644.0	+340	+171.11
+647.6	+342	+172.22
+651.2	+344	+173.33
+654.8	+346	+174.44
+658.4	+348	+175.56
+662.0	+350	+176.67
+665.6	+352	+177.78
+669.2	+354	+178.89
+672.8	+356	+180.00
+676.4	+358	+181.11
+680.0	+360	+182.22
+683.6	+362	+183.33
+687.2	+364	+184.44
+690.8	+366	+185.56
+694.4	+368	+186.67
+698.0	+370	+187.78
+701.6	+372	+188.89

TABLE 2.8 Temperature Conversion Table (*Continued*)

°F.	Reading in °F. or °C. to be converted	°C.
+705.2	+374	+190.00
+708.8	+376	+191.11
+712.4	+378	+192.22
+716.0	+380	+193.33
+719.6	+382	+194.44
+723.2	+384	+195.56
+726.8	+386	+196.67
+730.4	+388	+197.78
+734.0	+390	+198.89
+737.6	+392	+200.00
+741.2	+394	+201.11
+744.8	+396	+202.22
+748.4	+398	+203.33
+752.0	+400	+204.44
+755.6	+402	+205.56
+759.2	+404	+206.67
+762.8	+406	+207.78
+766.4	+408	+208.89
+770.0	+410	+210.00
+773.6	+412	+211.11
+777.2	+414	+212.22
+780.8	+416	+213.33
+784.4	+418	+214.44
+788.0	+420	+215.56
+791.6	+422	+216.67
+795.2	+424	+217.78
+798.8	+426	+218.89
+802.4	+428	+220.00
+806.0	+430	+221.11
+809.6	+432	+222.22
+813.2	+434	+223.33
+816.8	+436	+224.44
+820.4	+438	+225.56
+824.0	+440	+226.67
+827.6	+442	+227.78
+831.2	+444	+228.89
+834.8	+446	+230.00
+838.4	+448	+231.11
+842.0	+450	+232.22
+845.6	+452	+233.33
+849.2	+454	+234.44
+852.8	+456	+235.56
+856.4	+458	+236.67
+860.0	+460	+237.78
+863.6	+462	+238.89
+867.2	+464	+240.00
+870.8	+466	+241.11
+874.4	+468	+242.22
+878.0	+470	+243.33
+881.6	+472	+244.44
+885.2	+474	+245.56
+888.8	+476	+246.67
+892.4	+478	+247.78
+896.0	+480	+248.89
+899.6	+482	+250.00
+903.2	+484	+251.11
+906.8	+486	+252.22
+910.4	+488	+253.33
+914.0	+490	+254.44
+917.6	+492	+255.56
+921.2	+494	+256.67
+924.8	+496	+257.78
+928.4	+498	+258.89
+932.0	+500	+260.00
+935.6	+502	+261.11
+939.2	+504	+262.22
+942.8	+506	+263.33
+946.4	+508	+264.44
+950.0	+510	+265.56
+953.6	+512	+266.67
+957.2	+514	+267.78
+960.8	+516	+268.89
+964.4	+518	+270.00
+968.0	+520	+271.11
+971.6	+522	+272.22
+975.2	+524	+273.33
+978.8	+526	+274.44
+982.4	+528	+275.56
+986.0	+530	+276.67
+989.6	+532	+277.78
+993.2	+534	+278.89
+996.8	+536	+280.00
+1000.4	+538	+281.11
+1004.0	+540	+282.22
+1007.6	+542	+283.33
+1011.2	+544	+284.44
+1014.8	+546	+285.56
+1018.4	+548	+286.67
+1022.0	+550	+287.78
+1025.6	+552	+288.89

TABLE 2.8 Temperature Conversion Table (*Continued*)

°F.	Reading in °F. or °C. to be converted	°C.
+1029.2	+554	+290.00
+1032.8	+556	+291.11
+1036.4	+558	+292.22
+1040.0	+560	+293.33
+1043.6	+562	+294.44
+1047.2	+564	+295.56
+1050.8	+566	+296.67
+1054.4	+568	+297.78
+1058.0	+570	+298.89
+1061.6	+572	+300.00
+1065.2	+574	+301.11
+1068.8	+576	+302.22
+1072.4	+578	+303.33
+1076.0	+580	+304.44
+1079.6	+582	+305.56
+1083.2	+584	+306.67
+1086.8	+586	+307.78
+1090.4	+588	+308.89
+1094.0	+590	+310.00
+1097.6	+592	+311.11
+1101.2	+594	+312.22
+1104.8	+596	+313.33
+1108.4	+598	+314.44
+1112.0	+600	+315.56
+1115.6	+602	+316.67
+1119.2	+604	+317.78
+1122.8	+606	+318.89
+1126.4	+608	+320.00
+1130.0	+610	+321.11
+1133.6	+612	+322.22
+1137.2	+614	+323.33
+1140.8	+616	+324.44
+1144.4	+618	+325.56
+1148.0	+620	+326.67
+1151.6	+622	+327.78
+1155.2	+624	+328.89
+1158.8	+626	+330.00
+1162.4	+628	+331.11
+1166.0	+630	+332.22
+1169.6	+632	+333.33
+1173.2	+634	+334.44
+1176.8	+636	+335.56
+1180.4	+638	+336.67
+1184.0	+640	+337.78
+1187.6	+642	+338.89
+1191.2	+644	+340.00
+1194.8	+646	+341.11
+1198.4	+648	+342.22
+1202.0	+650	+343.33
+1205.6	+652	+344.44
+1209.2	+654	+345.56
+1212.8	+656	+346.67
+1216.4	+658	+347.78
+1220.0	+660	+348.89
+1223.6	+662	+350.00
+1227.2	+664	+351.11
+1230.8	+666	+352.22
+1234.4	+668	+353.33
+1238.0	+670	+354.44
+1241.6	+672	+355.56
+1245.2	+674	+356.67
+1248.8	+676	+357.78
+1252.4	+678	+358.89
+1256.0	+680	+360.00
+1259.6	+682	+361.11
+1263.2	+684	+362.22
+1266.8	+686	+363.33
+1270.4	+688	+364.44
+1274.0	+690	+365.56
+1277.6	+692	+366.67
+1281.2	+694	+367.78
+1284.8	+696	+368.89
+1288.4	+698	+370.00
+1292.0	+700	+371.11
+1295.6	+702	+372.22
+1299.2	+704	+373.33
+1302.8	+706	+374.44
+1306.4	+708	+375.56
+1310.0	+710	+376.67
+1313.6	+712	+377.78
+1317.2	+714	+378.89
+1320.8	+716	+380.00
+1324.4	+718	+381.11
+1328.0	+720	+382.22
+1331.6	+722	+383.33
+1335.2	+724	+384.44
+1338.8	+726	+385.56
+1342.4	+728	+386.67
+1346.0	+730	+387.78
+1349.6	+732	+388.89

TABLE 2.8 Temperature Conversion Table (*Continued*)

°F.	Reading in °F. or °C. to be converted	°C.
+1353.2	+734	+390.00
+1356.8	+736	+391.11
+1360.4	+738	+392.22
+1364.0	+740	+393.33
+1367.6	+742	+394.44
+1371.2	+744	+395.56
+1374.8	+746	+396.67
+1378.4	+748	+397.78
+1382.0	+750	+398.89
+1385.6	+752	+400.00
+1389.2	+754	+401.11
+1392.8	+756	+402.22
+1396.4	+758	+403.33
+1400.0	+760	+404.44
+1403.6	+762	+405.56
+1407.2	+764	+406.67
+1410.8	+766	+407.78
+1414.4	+768	+408.89
+1418.0	+770	+410.00
+1421.6	+772	+411.11
+1425.2	+774	+412.22
+1428.8	+776	+413.33
+1432.4	+778	+414.44
+1436.0	+780	+415.56
+1439.6	+782	+416.67
+1443.2	+784	+417.78
+1446.8	+786	+418.89
+1450.4	+788	+420.00
+1454.0	+790	+421.11
+1457.6	+792	+422.22
+1461.2	+794	+423.33
+1464.8	+796	+424.44
+1468.4	+798	+425.56
+1472.0	+800	+426.67
+1475.6	+802	+427.78
+1479.2	+804	+428.89
+1482.8	+806	+430.00
+1486.4	+808	+431.11
+1490.0	+810	+432.22
+1493.6	+812	+433.33
+1497.2	+814	+434.44
+1500.8	+816	+435.56
+1504.4	+818	+436.67
+1508.0	+820	+437.78
+1511.6	+822	+438.89
+1515.2	+824	+440.00
+1518.8	+826	+441.11
+1522.4	+828	+442.22
+1526.0	+830	+443.33
+1529.6	+832	+444.44
+1533.2	+834	+445.56
+1536.8	+836	+446.67
+1540.4	+838	+447.78
+1544.0	+840	+448.89
+1547.6	+842	+450.00
+1551.2	+844	+451.11
+1554.8	+846	+452.22
+1558.4	+848	+453.33
+1562.0	+850	+454.44
+1565.6	+852	+455.56
+1569.2	+854	+456.67
+1572.8	+856	+457.78
+1576.4	+858	+458.89
+1580.0	+860	+460.00
+1583.6	+862	+461.11
+1587.2	+864	+462.22
+1590.8	+866	+463.33
+1594.4	+868	+464.44
+1598.0	+870	+465.56
+1601.6	+872	+466.67
+1605.2	+874	+467.78
+1608.8	+876	+468.89
+1612.4	+878	+470.00
+1616.0	+880	+471.11
+1619.6	+882	+472.22
+1623.2	+884	+473.33
+1626.8	+886	+474.44
+1630.4	+888	+475.56
+1634.0	+890	+476.67
+1637.6	+892	+477.78
+1641.2	+894	+478.89
+1644.8	+896	+480.00
+1648.4	+898	+481.11
+1652.0	+900	+482.22
+1655.6	+902	+483.33
+1659.2	+904	+484.44
+1662.8	+906	+485.56
+1666.4	+908	+486.67
+1670.0	+910	+487.78
+1673.6	+912	+488.89

TABLE 2.8 Temperature Conversion Table (*Continued*)

°F.	Reading in °F. or °C. to be converted	°C.
+1677.2	+914	+490.00
+1680.8	+916	+491.11
+1684.4	+918	+492.22
+1688.0	+920	+493.33
+1691.6	+922	+494.44
+1695.2	+924	+495.56
+1698.8	+926	+496.67
+1702.4	+928	+497.78
+1706.0	+930	+498.89
+1709.6	+932	+500.00
+1713.2	+934	+501.11
+1716.8	+936	+502.22
+1720.4	+938	+503.33
+1724.0	+940	+504.44
+1727.6	+942	+505.56
+1731.2	+944	+506.67
+1734.8	+946	+507.78
+1738.4	+948	+508.89
+1742.0	+950	+510.00
+1745.6	+952	+511.11
+1749.2	+954	+512.22
+1752.8	+956	+513.33
+1756.4	+958	+514.44
+1760.0	+960	+515.56
+1763.6	+962	+516.67
+1767.2	+964	+517.78
+1770.8	+966	+518.89
+1774.4	+968	+520.00
+1778.0	+970	+521.11
+1781.6	+972	+522.22
+1785.2	+974	+523.33
+1788.8	+976	+524.44
+1792.4	+978	+525.56
+1796.0	+980	+526.67
+1799.6	+982	+527.78
+1803.2	+984	+528.89
+1806.8	+986	+530.00
+1810.4	+988	+531.11
+1814.0	+990	+532.22
+1817.6	+992	+533.33
+1821.2	+994	+534.44
+1824.8	+996	+535.56
+1828.4	+998	+536.67
+1832.0	+1000	+537.78
+1850.0	+1010	+543.33
+1868.0	+1020	+548.89
+1886.0	+1030	+554.44
+1904.0	+1040	+560.00
+1922.0	+1050	+565.56
+1940.0	+1060	+571.11
+1958.0	+1070	+576.67
+1976.0	+1080	+582.22
+1994.0	+1090	+587.78
+2012.0	+1100	+593.33
+2030.0	+1110	+598.89
+2048.0	+1120	+604.44
+2066.0	+1130	+610.00
+2084.0	+1140	+615.56
+2102.0	+1150	+621.11
+2120.0	+1160	+626.67
+2138.0	+1170	+632.22
+2156.0	+1180	+637.78
+2174.0	+1190	+643.33
+2192.0	+1200	+648.89
+2210.0	+1210	+654.44
+2228.0	+1220	+660.00
+2246.0	+1230	+665.56
+2264.0	+1240	+671.11
+2282.0	+1250	+676.67
+2300.0	+1260	+682.22
+2318.0	+1270	+687.78
+2336.0	+1280	+693.33
+2354.0	+1290	+698.89
+2372.0	+1300	+704.44
+2390.0	+1310	+710.00
+2408.0	+1320	+715.56
+2426.0	+1330	+721.11
+2444.0	+1340	+726.67
+2462.0	+1350	+732.22
+2480.0	+1360	+737.78
+2498.0	+1370	+743.33
+2516.0	+1380	+748.89
+2534.0	+1390	+754.44
+2552.0	+1400	+760.00
+2570.0	+1410	+765.56
+2588.0	+1420	+771.11
+2606.0	+1430	+776.67
+2624.0	+1440	+782.22
+2642.0	+1450	+787.78
+2660.0	+1460	+793.33

TABLE 2.8 Temperature Conversion Table (*Continued*)

°F.	Reading in °F. or °C. to be converted	°C.
+2678.0	+1470	+798.89
+2696.0	+1480	+804.44
+2714.0	+1490	+810.00
+2732.0	+1500	+815.56
+2750.0	+1510	+821.11
+2768.0	+1520	+826.67
+2786.0	+1530	+832.22
+2804.0	+1540	+837.78
+2822.0	+1550	+843.33
+2840.0	+1560	+848.89
+2858.0	+1570	+854.44
+2876.0	+1580	+860.00
+2894.0	+1590	+865.56
+2912.0	+1600	+871.11
+2930.0	+1610	+876.67
+2948.0	+1620	+882.22
+2966.0	+1630	+887.78
+2984.0	+1640	+893.33
+3002.0	+1650	+898.89
+3020.0	+1660	+904.44
+3038.0	+1670	+910.00
+3056.0	+1680	+915.56
+3074.0	+1690	+921.11
+3092.0	+1700	+926.67
+3110.0	+1710	+932.22
+3128.0	+1720	+937.78
+3146.0	+1730	+943.33
+3164.0	+1740	+948.89
+3182.0	+1750	+954.44
+3200.0	+1760	+960.00
+3218.0	+1770	+965.56
+3236.0	+1780	+971.11
+3254.0	+1790	+976.67
+3272.0	+1800	+982.22
+3290.0	+1810	+987.78
+3308.0	+1820	+993.33
+3326.0	+1830	+998.89
+3344.0	+1840	+1004.4
+3362.0	+1850	+1010.0
+3380.0	+1860	+1015.6
+3398.0	+1870	+1021.1
+3416.0	+1880	+1026.7
+3434.0	+1890	+1032.2
+3452.0	+1900	+1037.8
+3470.0	+1910	+1043.3
+3488.0	+1920	+1048.9
+3506.0	+1930	+1054.4
+3524.0	+1940	+1060.0
+3542.0	+1950	+1065.6
+3560.0	+1960	+1071.1
+3578.0	+1970	+1076.7
+3596.0	+1980	+1082.2
+3614.0	+1990	+1087.8
+3632.0	+2000	+1093.3
+3650.0	+2010	+1098.9
+3668.0	+2020	+1104.4
+3686.0	+2030	+1110.0
+3704.0	+2040	+1115.6
+3722.0	+2050	+1121.1
+3740.0	+2060	+1126.7
+3758.0	+2070	+1132.2
+3776.0	+2080	+1137.8
+3794.0	+2090	+1143.3
+3812.0	+2100	+1148.9
+3830.0	+2110	+1154.4
+3848.0	+2120	+1160.0
+3866.0	+2130	+1165.6
+3884.0	+2140	+1171.1
+3902.0	+2150	+1176.7
+3920.0	+2160	+1182.2
+3938.0	+2170	+1187.8
+3956.0	+2180	+1193.3
+3974.0	+2190	+1198.9
+3992.0	+2200	+1204.4
+4010.0	+2210	+1210.0
+4028.0	+2220	+1215.6
+4046.0	+2230	+1221.1
+4064.0	+2240	+1226.7
+4082.0	+2250	+1232.2
+4100.0	+2260	+1237.8
+4118.0	+2270	+1243.3
+4136.0	+2280	+1248.9
+4154.0	+2290	+1254.4
+4172.0	+2300	+1260.0
+4190.0	+2310	+1265.6
+4208.0	+2320	+1271.1
+4226.0	+2330	+1276.7
+4244.0	+2340	+1282.2
+4262.0	+2350	+1287.8
+4280.0	+2360	+1293.3

TABLE 2.8 Temperature Conversion Table (*Continued*)

°F.	Reading in °F. or °C. to be converted	°C.
+4298.0	+2370	+1298.9
+4316.0	+2380	+1304.4
+4334.0	+2390	+1310.0
+4352.0	+2400	+1315.6
+4370.0	+2410	+1321.1
+4388.0	+2420	+1326.7
+4406.0	+2430	+1332.2
+4424.0	+2440	+1337.8
+4442.0	+2450	+1343.3
+4460.0	+2460	+1348.9
+4478.0	+2470	+1354.4
+4496.0	+2480	+1360.0
+4514.0	+2490	+1365.6
+4532.0	+2500	+1371.1
+4550.0	+2510	+1376.7
+4568.0	+2520	+1382.2
+4586.0	+2530	+1387.8
+4604.0	+2540	+1393.3
+4622.0	+2550	+1398.9
+4640.0	+2560	+1404.4
+4658.0	+2570	+1410.0
+4676.0	+2580	+1415.6
+4694.0	+2590	+1421.1
+4712.0	+2600	+1426.7
+4730.0	+2610	+1432.2
+4748.0	+2620	+1437.8
+4766.0	+2630	+1443.3
+4784.0	+2640	+1448.9
+4802.0	+2650	+1454.4
+4820.0	+2660	+1460.0
+4838.0	+2670	+1465.6
+4856.0	+2680	+1471.1
+4874.0	+2690	+1476.7
+4892.0	+2700	+1482.2
+4910.0	+2710	+1487.8
+4928.0	+2720	+1493.3
+4946.0	+2730	+1498.9
+4964.0	+2740	+1504.4
+4982.0	+2750	+1510.0
+5000.0	+2760	+1515.6
+5018.0	+2770	+1521.1
+5036.0	+2780	+1526.7
+5054.0	+2790	+1532.2
+5072.0	+2800	+1537.8
+5090.0	+2810	+1543.3
+5108.0	+2820	+1548.9
+5126.0	+2830	+1554.4
+5144.0	+2840	+1560.0
+5162.0	+2850	+1565.6
+5180.0	+2860	+1571.1
+5198.0	+2870	+1576.7
+5216.0	+2880	+1582.2
+5234.0	+2890	+1587.8
+5252.0	+2900	+1593.3
+5270.0	+2910	+1598.9
+5288.0	+2920	+1604.4
+5306.0	+2930	+1610.0
+5324.0	+2940	+1615.6
+5342.0	+2950	+1621.1
+5360.0	+2960	+1626.7
+5378.0	+2970	+1632.2
+5396.0	+2980	+1637.8
+5414.0	+2990	+1643.3
+5432.0	+3000	+1648.9
+5450.0	+3010	+1654.4
+5468.0	+3020	+1660.0
+5486.0	+3030	+1665.6
+5504.0	+3040	+1671.1
+5522.0	+3050	+1676.7
+5540.0	+3060	+1682.2
+5558.0	+3070	+1687.8
+5576.0	+3080	+1693.3
+5594.0	+3090	+1698.9
+5612.0	+3100	+1704.4

2.1.1 Conversion of Thermometer Scales

The following abbreviations are used: °F, degrees Fahrenheit; °C, degrees Celsius; °K, degrees Kelvin; °Ré, degrees Reaumur; °R, degrees Rankine; °Z, degrees on any scale; (fp)"Z", the freezing point of water on the Z scale; and (bp)"Z", the boiling point of water on the Z scale. Reference: Dodds, *Chemical and Metallurgical Engineering* **38:**476 (1931).

$$\frac{°\text{F} - 32}{180} = \frac{°\text{C}}{100} = \frac{°\text{Ré}}{80} = \frac{\text{K} - 273}{100} = \frac{°\text{R} - 492}{180} = \frac{°\text{Z} - (\text{fp})“\text{Z}”}{(\text{bp})“\text{Z}” - (\text{fp})“\text{Z}”}$$

Examples

(1) To find the Fahrenheit temperature corresponding to −20°C:

$$\frac{°\text{F} - 32}{180} = \frac{°\text{C}}{100} \qquad \text{or} \qquad \frac{°\text{F} - 32}{180} = \frac{-20}{100}$$

$$°\text{F} - 32 = \frac{(-20)(180)}{100} = -36$$

$$°\text{F} = -4$$

(2) To find the Reaumur temperature corresponding to 20°F:

$$\frac{°\text{F} - 32}{180} = \frac{°\text{Ré}}{80} = \frac{20 - 32}{180} = \frac{°\text{Ré}}{80}$$

i.e.,

$$20°\text{F} = -5.33°\text{Ré}$$

(3) To find the correct temperature on a thermometer reading 80°C and that shows a reading of −0.30°C in a melting ice/water mixture and 99.0°C in steam at 760 mm pressure of mercury:

$$\frac{°\text{C}}{100} = \frac{\text{Z} - (\text{fp})“\text{Z}”}{(\text{bp})“\text{Z}” - (\text{fp})“\text{Z}”} = \frac{80 - (-0.30)}{99.0 - (-0.30)}$$

i.e.,

$$°\text{C} = 80.87 \quad \text{(corrected)}$$

2.1.2 Density and Specific Gravity

2.1.2.1 Hydrometers. Various hydrometers and the relation between the various scales.

Alcoholometer. This hydrometer is used in determining the density of aqueous ethyl alcohol solutions; the reading in degrees is numerically the same as the percentage of alcohol by volume. The scale known as Tralle gives the percentage by volume. Wine and Must hydrometer relations are given below.

Ammoniameter. This hydrometer, employed in finding the density of aqueous ammonia solutions, has a scale graduated in equal divisions from 0° to 40°. To convert the reading to specific gravity multiply by 3 and subtract the resulting number from 1000.

Balling Hydrometer. See under Saccharometers.

Barkometer or Barktrometer. This hydrometer, which is used in determining the density of tanning liquors, has a scale from 0° to 80° Bk; the number to the right of the decimal point of a specific gravity reading is the corresponding Bk degree; thus, a specific gravity of 1.015 is 15° Bk.

Baumé Hydrometers. For liquids heavier than water: This hydrometer was originally based on the density of a 10% sodium chloride solution, which was given the value of 10°, and the density of pure water, which was given the value of 0°; the interval between these two values was divided into 10 equal parts. Other reference points have been taken with the result that so much confusion exists that there are about 36 different scales in use, many of which are incorrect. In general a Baumé hydrometer should have inscribed on it the temperature at which it was calibrated and also the temperature of the water used in relating the density to a specific gravity. The following expression gives the relation between the specific gravity and several of the Baumé scales:

$$\text{Specific gravity} = \frac{m}{m - \text{Baumé}}$$

m = 145 at 60°/60°F (15.56°C) for the American Scale
= 144 for the old scale used in Holland
= 146.3 at 15°C for the Gerlach Scale
= 144.3 at 15°C for the Rational Scale generally used in Germany

For liquids lighter than water: Originally the density of a solution of 1 gram of sodium chloride in 9 grams of water at 12.5°C was given a value of 10°Bé. The scale between these points was divided into ten equal parts and these divisions were repeated throughout the scale giving a relation which could be expressed by the formula: Specific gravity = 145.88/(135.88 + Bé), which is approximately equal to 146/(136 + Bé). Other scales have since come into more general use such as that of the Bureau of Standards in which the specific gravity at 60°/60°F = 140/(130 + Bé) and that of the American Petroleum Institute (A.P.I. Scale) in which the specific gravity at 60°/60°F = 141.5/(131.5 + API°).

See also special table for conversion to density and Twaddell scale.

Beck's Hydrometer. This hydrometer is graduated to show a reading of 0° in pure water and a reading of 30° in a solution with a specific gravity of 0.850, with equal scale divisions above and below these two points.

Brix Hydrometer. See under Saccharometers.

Cartier's Hydrometer. This hydrometer shows a reading of 22° when immersed in a solution having a density of 22° Baumé but the scale divisions are smaller than on the Baumé hydrometer in the ratio of 16 Cartier to 15 Baumé.

Fatty Oil Hydrometer. The graduations on this hydrometer are in specific gravity within the range 0.908 to 0.938. The letters on the scale correspond to the specific gravity of the various common oils as follows: *R*, rape; *O*, olive; *A*, almond; *S*, sesame; *HL*, hoof oil; *HP*, hemp; *C*, cotton seed; *L*, linseed. See also Oleometer below.

Lactometers. These hydrometers are used in determining the density of milk. The various scales in common use are the following:

New York Board of Health has a scale graduated into 120 equal parts, 0° being equal to the specific gravity of water and 100° being equal to a specific gravity of 1.029.

Quevenne lactometer is graduated from 15° to 40° corresponding to specific gravities from 1.015 to 1.040.

Soxhlet lactometer has a scale from 25° to 35° corresponding to specific gravities from 1.025 to 1.035 respectively.

Oleometer. A hydrometer for determining the density of vegetable and sperm oils with a scale from 50° to 0° corresponding to specific gravities from 0.870 to 0.970. See also Fatty Oil Hydrometer above.

Saccharometers. These hydrometers are used in determining the density of sugar solutions. Solutions of the same concentration but of different carbohydrates have very nearly the same specific gravity and in general a concentration of 10 grams of carbohydrate per 100 mL of solution shows a specific gravity of 1.0386. Thus, the wt. of sugar in 1000 mL soln. is (a) for conc. $<$12g/100 mL: (wt. of 1000 mL soln. $-$ 1000) $\div$ 0.386; (b) for conc. $>$12g/100 mL: (wt of 1000 mL soln. $-$ 1000) $\div$ 0.385.

Brix hydrometer is graduated so that the number of degrees is identical with the percentage by weight of cane sugar and is used at the temperature indicated on the hydrometer.

Balling's saccharometer is used in Europe and is practically identical with the Brix hydrometer.

Bates brewers' saccharometer which is used in determining the density of malt worts is graduated so that the divisions express pounds per barrel (32 gallons). The relation between degrees Bates (=b) and degrees Balling (=B) is shown by the following formula: $B = 260b/(360 + b)$.

See also below under Wine and Must Hydrometer.

Salinometer. This hydrometer, which is used in the pickling and meat packing plants, is graduated to show percentage of saturation of a sodium chloride solution. An aqueous solution is completely saturated when it contains 26.4% pure sodium chloride. The range from 0% to 26.4% is divided into 100 parts, each division therefore representing 1% of saturation. In another type of salinometer, the degrees correspond to percentages of sodium chloride expressed in grams of sodium chloride per 100 mL of water.

Sprayometer (Parrot and Stewart). This hydrometer which is used in determining the density of *lime sulfur* solutions has two scales; one scale is graduated from 0° to 38° Baumé and the other scale is from 1.000 to 1.350 specific gravity.

Tralle Hydrometer. See Alcoholometer above.

Twaddell Hydrometer. This hydrometer, which is used only for liquids heavier than water, has a scale such that when the reading is multiplied by 5 and added to 1000 the resulting number is the specific gravity with reference to water as 1000. To convert specific gravity at 60°/60°F to Twaddell degrees, take the decimal portion of the specific gravity value and multiply it by 200; thus a specific gravity of $1.032 = 0.032 \times 200 = 6.4°$ Tw. See also special table for conversion to density and Baumé scale.

Wine and Must Hydrometer. This instrument has three scales. One scale shows readings of 0° to 15° Brix for sugar (see Brix Hydrometer above); another scale from 0° to 15° Tralle is used for sweet wines to indicate the percentage of alcohol by volume; and a third scale from 0° to 20° Tralle is used for tart wines to indicate the percentage of alcohol by volume.

2.1.2.2 Conversion of Specific Gravity at 25°/25°C to Density at any Temperature from 0° to 40°C.* Liquids change volume with change in temperature, but the amount of this change, β (coefficient of cubical expansion), varies widely with different liquids, and to some extent for the same liquid at different temperatures.

The table below, which is calculated from the relationship:

$$F_{\beta_t} = \frac{\text{density of water at 25°C } (=0.99705)}{1 - \beta(25 - t)} \tag{2.1}$$

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* Cf. Dreisbach, *Ind. Eng. Chem., Anal. Ed.* **12:**160 (1940).

may be used to find d^t, the density (weight of 1 mL) of a liquid at any temperature (t) between 0° and 40°C if the specific gravity at 25°/25°C (S) and the coefficient of cubical expansion (β) are known. Substitutions are made in the equations:

$$d^t = SF_{\beta_t} \tag{2.2}$$

$$S = \frac{d^t}{F_{\beta_t}} \tag{2.3}$$

Factors ($F\beta_t$)

Density t°C = sp. gr. 25°/25° × F_{β_t}

°C. / *$\beta \times 10^3$	0	5	10	15	20	25	30	35	40
1.3	1.0306	1.0237	1.0169	1.0102	1.0036	0.99705	0.99065	0.9843	0.9780
1.2	1.0279	1.0216	1.0154	1.0092	1.0031	0.99705	0.9911	0.9853	0.9794
1.1	1.0253	1.0195	1.0138	1.0082	1.0026	0.99705	0.9916	0.9963	0.9809
1.0	1.0227	1.0174	1.0123	1.0072	1.0021	0.99705	0.9921	0.9872	0.98234
0.9	1.0200	1.0153	1.0107	1.0060	1.0016	0.99705	0.99262	0.9882	0.9838
0.8	1.0174	1.0133	1.0092	1.0051	1.0011	0.99705	0.9931	0.98918	0.9851
0.7	1.0148	1.0113	1.0077	1.0041	1.0006	0.99705	0.9935	0.99015	0.98672
0.6	1.0122	1.0092	1.0061	1.0031	1.0001	0.99705	0.9941	0.9911	0.9882
0.5	1.0097	1.0072	1.0046	1.0021	0.99958	0.99705	0.9944	0.9921	0.9897
0.	1.0071	1.0051	1.0031	1.0011	0.99908	0.99705	0.9951	0.9931	0.9911

* β = coefficient of cubical expansion.

Examples. All examples are based upon an assumed coefficient of cubical expansion, β, of 1.3×10^{-3}.

Example 1. To find the density of a liquid at 20°C, d^{20}, which has a specific gravity (S) of $1.2500\frac{25}{25}$:

From the table above F_{β_t} at 20°C = 1.0036.

$$d^{20} = d^t = SF_{\beta_t} = 1.2500 \times 1.0036 = 1.2545$$

Example 2. To find the density at 20°C (d^{20}) of a liquid which has a specific gravity of $1.2500\frac{17}{4}$:

Since the density of water at 4°C is equal to 1, specific gravity at 17°/4° = d^{17} = 1.2500.

Substitution in Equation 3 with F_{β_t} at 17°C, by interpolation from the table, equal to 1.00756, gives

$$\text{Sp. gr. } 25°/25° = S = 1.2500 \div 1.00756$$

Substitution of this value for S in Equation 2 with F_{β_t} at 20°C, from the table, equal to 1.0036, gives

$$d^{20} = d^t = (1.2500 \div 1.00756) \times 1.0036 = 1.2451$$

Example 3. To find the specific gravity at 20°/4°C of a liquid which has a specific gravity of $1.2500\frac{25}{4}$:

Since the density of water at 4°C, is equal to 1, specific gravity $25°/4° = d^{25} = 1.2500$; and, specific gravity $20°/4° = d^{20}$.

Substitution in Equation 3, with $d^t = 1.2500$; and, with F_{β_t} at 25°C, from the table, equal to 0.99705, gives

$$\text{Sp. gr. } 25°/25° = S = 1.2500 \div 0.99705$$

Substitution of this value for S in Equation 2, with F_{β_t} at 20°C, from the table, equal to 1.0036, gives

$$\text{Sp. gr. } 20°/4° = d^{20} = (1.2500 \div 0.99705) \times 1.0036 = 1.2582$$

Example 4. To find the density at 25°C of a liquid which has a specific gravity of $1.2500\frac{15}{15}$:

Since the density of water at 15°C = 0.99910,

$$d^{15} = \text{sp. gr. } 15°/15° \times 0.99910 = 1.2500 \times 0.99910$$

Substitution in Equation 3, with F_{β_t} at 15°C, from the table, equal to 1.0102, gives

$$\text{Sp. gr. } 25°/25° = S = (1.2500 \times 0.99910) \div 1.0102$$

Substitution of this value for S in Equation 2, with F_{β_t} at 25°, from the table, equal to 0.99705, gives

$$d^{26} = d^t = (1.2500 \times 0.99910 \div 1.0102) \times 0.99705 = 1.2326$$

2.1.3 Barometry and Barometric Corrections

In principle, the mercurial barometer balances a column of pure mercury against the weight of the atmosphere. The height of the column above the level of the mercury in the reservoir can be measured and serves as a direct index of atmospheric pressure. The space above the mercury in a barometer tube should be a Torricellian vacuum, perfect except for the practically negligible vapor pressure of mercury. The perfection of the vacuum is indicated by the sharpness of the click noted when the barometer tube is inclined. A barometer should be in a vertical position, suspended rather than fastened to a wall, and in a good light but not exposed to direct sunlight or too near a source of heat. The standard conditions for barometric measurements are 0°C and gravity as at 45° latitude and sea level. There are numerous sources of error, but corrections for most of these are readily applied. Some of the corrections are very small, and their application may be questionable in view of the probably larger errors. The degree of consistency to be expected in careful measurements is about 0.13 mm with a 6.4-mm tube, increasing to 0.04 mm with a tube 12.7 mm in diameter.

In reading a barometer of the Fortin type (the usual laboratory instrument for precision measurements), the procedure should be as follows: (1) Observe and record the temperature as indicated by the thermometer attached to the barometer. The temperature correction is very important and may be affected by heat from the observer's body. (2) Set the mercury in the reservoir at zero level, so that the point of the pin above the mercury just touches the surface, making a barely noticeable dimple therein. Tap the tube at the top and verify the zero setting. (3) Bring the vernier down until the view at the light background is cut off at the highest point of the meniscus. Record the reading.

The corrections to be made on the reading are as follows: (1) Temperature, to correct for the difference in thermal expansion of the mercury and the brass (or glass) to which the scale is attached.

This correction converts the reading into the value of 0°C. The brass scale table is applicable to the Fortin barometer. See Tables 2.10 latitude-gravity correction, and 2.11 altitude-gravity correction, to compensate for differences in gravity, which would affect the height of the mercury column by variation in mass. If local gravity is unknown, an approximate correction may be made from the tables. Local values of gravity are often subject to irregularities which lead to errors even when the corrections here provided are made. It is, therefore, advisable to determine the local value of gravity, from which the correction can be effected in the following manner:

$$Bt = Br + \left(\frac{g_1 - g_0}{g_0}\right) \times Br$$

in which *Bt* and *Br* are the true and the observed heights of the barometer, respectively. g_0 is standard gravity (980 665 cm · s^{-2}), and g_1 is the local gravity. It may be noted that for most localities, g_1 is smaller than g_0, which makes the correction negative. These corrections compensate the reading to gravity at 45° latitude and sea level. (3) Correction for capillary depression of the level of the meniscus. This varies with the tube diameter and actual height of the meniscus in a particular case. Some barometers are calibrated to allow for an average value of the latter and approximating the correction. See table. (4) Correction for vapor pressure of mercury. This correction is usually negligible, being only 0.001 mm at 20°C and 0.006 mm at 40°C. This correction is added. See table of vapor pressure of mercury.

The corrections above do not apply to aneroid barometers. These instruments should be calibrated at regular intervals by checking them against a corrected mercurial barometer.

For records on weather maps, meteorologists customarily correct barometer readings to sea level, and some barometers may be calibrated accordingly. Such instruments are not suitable for laboratory use where true pressure under standard conditions is required. Scale corrections should be specified in the maker's instructions with the instrument, and are also indicated by the lack of correspondence between a gauge mark usually placed exactly 76.2 cm from the zero point and the 76.2-cm scale graduation.

TABLE 2.9 Barometer Temperature Correction—Metric Units

The values in the table below are to be subtracted from the observed readings to correct for the difference in the expansion of the mercury and the glass scale at different temperatures.

A. Glass scale

Temp. °C.	Observed barometer height in millimeters						
	700	730	740	750	760	770	800
	mm.	mm.	mm.	mm.	mm.	mm.	mm.
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	0.12	0.13	0.13	0.13	0.13	0.13	0.14
2	0.24	0.25	0.26	0.26	0.26	0.27	0.27
3	0.36	0.38	0.38	0.39	0.40	0.40	0.42
4	0.49	0.51	0.51	0.52	0.53	0.53	0.55
5	0.61	0.63	0.64	0.65	0.66	0.67	0.69
6	0.73	0.76	0.77	0.78	0.79	0.80	0.83
7	0.85	0.89	0.90	0.91	0.92	0.93	0.97
8	0.97	1.01	1.03	1.04	1.05	1.07	1.11
9	1.09	1.14	1.15	1.17	1.18	1.20	1.25
10	1.21	1.26	1.28	1.30	1.32	1.33	1.39
11	1.33	1.39	1.41	1.43	1.45	1.47	1.52
12	1.45	1.52	1.54	1.56	1.58	1.60	1.66
13	1.58	1.64	1.67	1.69	1.71	1.73	1.80
14	1.70	1.77	1.79	1.82	1.84	1.87	1.94
15	1.82	1.90	1.92	1.95	1.97	2.00	2.08
16	1.94	2.02	2.05	2.08	2.10	2.13	2.21
17	2.06	2.15	2.18	2.21	2.23	2.26	2.35
18	2.18	2.27	2.30	2.33	2.37	2.40	2.49
19	2.30	2.40	2.43	2.46	2.50	2.53	2.63
20	2.42	2.52	2.56	2.59	2.63	2.66	2.77
21	2.54	2.65	2.69	2.72	2.76	2.79	2.90
22	2.66	2.78	2.81	2.85	2.89	2.93	3.04
23	2.78	2.90	2.94	2.98	3.02	3.06	3.18
24	2.90	3.03	3.07	3.11	3.15	3.19	3.32
25	3.02	3.15	3.20	3.24	3.28	3.32	3.45
26	3.14	3.28	3.32	3.37	3.41	3.46	3.59
27	3.26	3.40	3.45	3.50	3.54	3.59	3.73
28	3.38	3.53	3.58	3.63	3.67	3.72	3.87
29	3.50	3.65	3.70	3.75	3.80	3.85	4.00
30	3.62	3.78	3.83	3.88	3.93	3.99	4.14
31	3.74	3.90	3.96	4.01	4.06	4.12	4.28
32	3.86	4.03	4.08	4.14	4.20	4.25	4.42
33	3.98	4.15	4.21	4.27	4.33	4.38	4.55
34	4.10	4.28	4.34	4.40	4.46	4.51	4.69
35	4.22	4.40	4.47	4.53	4.59	4.65	4.83

TABLE 2.9 Barometer Temperature Correction—Metric Units (*Continued*)

The values in the table below are to be subtracted from the observed readings to correct for the difference in the expansion of the mercury and the glass scale at different temperatures.

B. Brass scale

Temp. °C.	Observed barometer height in millimeters						
	640	650	660	670	680	690	700
	mm.	mm.	mm.	mm.	mm.	mm.	mm.
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	0.10	0.11	0.11	0.11	0.11	0.11	0.11
2	0.21	0.21	0.22	0.22	0.22	0.23	0.23
3	0.31	0.32	0.32	0.33	0.33	0.34	0.34
4	0.42	0.42	0.43	0.44	0.44	0.45	0.46
5	0.52	0.53	0.54	0.55	0.55	0.56	0.57
6	0.63	0.64	0.65	0.66	0.66	0.67	0.68
7	0.73	0.74	0.75	0.76	0.78	0.79	0.80
8	0.84	0.85	0.86	0.87	0.89	0.90	0.91
9	0.94	0.95	0.97	0.98	1.00	1.01	1.03
10	1.04	1.06	1.07	1.09	1.11	1.12	1.14
11	1.15	1.16	1.18	1.20	1.22	1.24	1.25
12	1.25	1.27	1.29	1.31	1.33	1.35	1.37
13	1.35	1.38	1.40	1.42	1.44	1.46	1.48
14	1.46	1.48	1.50	1.53	1.55	1.57	1.59
15	1.56	1.59	1.61	1.64	1.66	1.68	1.71
16	1.67	1.69	1.72	1.74	1.77	1.80	1.82
17	1.77	1.80	1.82	1.85	1.88	1.91	1.94
18	1.87	1.90	1.93	1.96	1.99	2.02	2.05
19	1.98	2.01	2.04	2.07	2.10	2.13	2.16
20	2.08	2.11	2.15	2.18	2.21	2.24	2.28
21	2.18	2.22	2.25	2.29	2.32	2.35	2.39
22	2.29	2.32	2.36	2.40	2.43	2.47	2.50
23	2.39	2.43	2.47	2.50	2.54	2.58	2.62
24	2.49	2.53	2.57	2.61	2.65	2.69	2.73
25	2.60	2.64	2.68	2.72	2.76	2.80	2.84
26	2.70	2.74	2.79	2.83	2.87	2.91	2.96
27	2.81	2.85	2.89	2.94	2.98	3.02	3.07
28	2.91	2.95	3.00	3.05	3.09	3.14	3.18
29	3.01	3.06	3.11	3.15	3.20	3.25	3.29
30	3.12	3.16	3.21	3.26	3.31	3.36	3.41
31	3.22	3.27	3.32	3.37	3.42	3.47	3.52
32	3.32	3.37	3.43	3.48	3.53	3.58	3.63
33	3.42	3.48	3.53	3.59	3.64	3.69	3.75
34	3.53	3.58	3.64	3.69	3.75	3.80	3.86
35	3.63	3.69	3.74	3.80	3.86	3.91	3.97

TABLE 2.9 Barometer Temperature Correction—Metric Units (*Continued*)

B. Brass scale (*continued*)

Observed barometer height in millimeters								Temp. °C
710	720	730	740	750	760	770	780	
mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0
0.12	0.12	0.12	0.12	0.12	0.12	0.13	0.13	1
0.23	0.23	0.24	0.24	0.24	0.25	0.25	0.25	2
0.35	0.35	0.36	0.36	0.37	0.37	0.38	0.38	3
0.46	0.47	0.48	0.48	0.49	0.50	0.50	0.51	4
0.58	0.59	0.59	0.60	0.61	0.62	0.63	0.64	5
0.69	0.70	0.71	0.72	0.73	0.74	0.75	0.76	6
0.81	0.82	0.83	0.84	0.86	0.87	0.88	0.89	7
0.93	0.94	0.95	0.96	0.98	0.99	1.00	1.02	8
1.04	1.06	1.07	1.08	1.10	1.11	1.13	1.14	9
1.16	1.17	1.19	1.21	1.22	1.24	1.25	1.27	10
1.27	1.29	1.31	1.33	1.34	1.36	1.38	1.40	11
1.39	1.41	1.43	1.45	1.47	1.48	1.50	1.52	12
1.50	1.52	1.54	1.57	1.59	1.61	1.63	1.65	13
1.62	1.64	1.66	1.69	1.71	1.73	1.75	1.78	14
1.73	1.76	1.78	1.81	1.83	1.85	1.88	1.90	15
1.85	1.87	1.90	1.93	1.95	1.98	2.00	2.03	16
1.96	1.99	2.02	2.05	2.07	2.10	2.13	2.16	17
2.08	2.11	2.14	2.17	2.20	2.22	2.25	2.28	18
2.19	2.22	2.25	2.29	2.32	2.35	2.38	2.41	19
2.31	2.34	2.37	2.41	2.44	2.47	2.50	2.54	20
2.42	2.46	2.49	2.53	2.56	2.59	2.63	2.66	21
2.54	2.57	2.61	2.65	2.68	2.72	2.75	2.79	22
2.65	2.69	2.73	2.77	2.80	2.84	2.88	2.91	23
2.77	2.81	2.85	2.88	2.92	2.96	3.00	3.04	24
2.88	2.92	2.96	3.00	3.05	3.09	3.13	3.17	25
3.00	3.04	3.08	3.12	3.17	3.21	3.25	3.29	26
3.11	3.16	3.20	3.24	3.29	3.33	3.38	3.42	27
3.23	3.27	3.32	3.36	3.41	3.45	3.50	3.54	28
3.34	3.39	3.44	3.48	3.53	3.58	3.62	3.67	29
3.46	3.50	3.55	3.60	3.65	3.70	3.75	3.80	30
3.57	3.62	3.67	3.72	3.77	3.82	3.87	3.92	31
3.68	3.74	3.79	3.84	3.89	3.94	4.00	4.05	32
3.80	3.85	3.91	3.96	4.01	4.07	4.12	4.17	33
3.91	3.97	4.02	4.08	4.13	4.19	4.24	4.30	34
4.03	4.09	4.14	4.20	4.26	4.31	4.37	4.43	35

TABLE 2.9 Barometer Temperature Correction—Metric Units (*Continued*)

C. Correction of a barometer for capillarity (*Smithsonian Tables*)

Diameter of tube, millimeters	Height of meniscus in millimeters 0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8
	Correction to be added in millimeters							
4	0.83	1.22	1.54	1.98	2.37	. . . .	. . . .	. . . .
5	0.47	0.65	0.86	1.19	1.45	1.80	. . . .	. . . .
6	0.27	0.41	0.56	0.78	0.98	1.21	1.43	. . . .
7	0.18	0.28	0.40	0.53	0.67	0.82	0.97	1.13
8	. . . .	0.20	0.29	0.38	0.46	0.56	0.65	0.77
9	. . . .	0.15	0.21	0.28	0.33	0.40	0.46	0.52
10	. . . .	. . . .	0.15	0.20	0.25	0.29	0.33	0.37
11	. . . .	. . . .	0.10	0.14	0.18	0.21	0.24	0.27
12	. . . .	. . . .	0.07	0.10	0.13	0.15	0.18	0.19
13	. . . .	. . . .	0.04	0.07	0.10	0.12	0.13	0.14

TABLE 2.10 Barometric Latitude-Gravity Table—Metric Units

Smithsonian Tables.

The values in the table below are to be subtracted from the barometric reading for latitudes from 0 to 45° inclusive, and are to be added from 46 to 90°.

Deg. Lat.	Barometer readings, millimeters 680	700	720	740	760	780
	mm.	mm.	mm.	mm.	mm.	mm.
0	1.82	1.87	1.93	1.98	2.04	2.09
5	1.79	1.85	1.90	1.95	2.00	2.06
10	1.71	1.76	1.81	1.86	1.92	1.97
15	1.58	1.63	1.67	1.72	1.77	1.81
20	1.40	1.44	1.49	1.53	1.57	1.61
21	1.36	1.40	1.44	1.48	1.52	1.56
22	1.32	1.36	1.40	1.44	1.48	1.51
23	1.28	1.31	1.35	1.39	1.43	1.46
24	1.23	1.27	1.30	1.34	1.37	1.41
25	1.18	1.22	1.25	1.29	1.32	1.36
26	1.13	1.17	1.20	1.23	1.27	1.30
27	1.08	1.12	1.15	1.18	1.21	1.24
28	1.03	1.06	1.09	1.12	1.15	1.18
29	0.98	1.01	1.04	1.07	1.10	1.12
30	0.93	0.95	0.98	1.01	1.04	1.06
31	0.87	0.90	0.92	0.95	0.98	1.00
32	0.82	0.84	0.86	0.89	0.91	0.94
33	0.76	0.78	0.80	0.83	0.85	0.87
34	0.70	0.72	0.74	0.76	0.79	0.81
35	0.64	0.66	0.68	0.70	0.72	0.74
36	0.58	0.60	0.62	0.64	0.65	0.67
37	0.52	0.54	0.56	0.57	0.59	0.60
38	0.46	0.48	0.49	0.51	0.52	0.53

TABLE 2.10 Barometric Latitude-Gravity Table—Metric Units (*Continued*)

Deg. Lat.	Barometer readings, millimeters 680 mm.	700 mm.	720 mm.	740 mm.	760 mm.	780 mm.
39	0.40	0.42	0.43	0.44	0.45	0.46
40	0.34	0.35	0.36	0.37	0.38	0.39
41	0.28	0.29	0.30	0.30	0.31	0.32
42	0.22	0.22	0.23	0.24	0.24	0.25
43	0.16	0.16	0.16	0.17	0.17	0.18
44	0.09	0.10	0.10	0.10	0.10	0.11
45	0.03	0.03	0.03	0.03	0.03	0.04
46	0.03	0.03	0.03	0.03	0.04	0.04
47	0.09	0.10	0.10	0.10	0.10	0.11
48	0.16	0.16	0.17	0.17	0.18	0.18
49	0.22	0.23	0.23	0.24	0.25	0.25
50	0.28	0.29	0.30	0.31	0.31	0.32
51	0.34	0.35	0.36	0.37	0.38	0.39
52	0.40	0.42	0.43	0.44	0.45	0.46
53	0.46	0.48	0.49	0.51	0.52	0.53
54	0.52	0.54	0.56	0.57	0.59	0.60
55	0.58	0.60	0.62	0.64	0.65	0.67
56	0.64	0.66	0.68	0.70	0.72	0.74
57	0.70	0.72	0.74	0.76	0.78	0.80
58	0.76	0.78	0.80	0.82	0.85	0.87
59	0.81	0.84	0.86	0.89	0.91	0.93
60	0.87	0.89	0.92	0.94	0.97	1.00
61	0.92	0.95	0.98	1.00	1.03	1.06
62	0.97	1.00	1.02	1.05	1.08	1.11
63	1.03	1.06	1.09	1.12	1.15	1.18
64	1.08	1.11	1.14	1.17	1.20	1.23
65	1.13	1.16	1.19	1.22	1.26	1.29
66	1.17	1.21	1.24	1.28	1.31	1.35
67	1.22	1.25	1.29	1.33	1.36	1.40
68	1.26	1.30	1.34	1.37	1.41	1.45
69	1.31	1.34	1.38	1.42	1.46	1.50
70	1.35	1.39	1.43	1.47	1.51	1.55
72	1.42	1.47	1.51	1.55	1.59	1.63
75	1.53	1.57	1.62	1.66	1.71	1.75
80	1.66	1.71	1.76	1.81	1.86	1.90
85	1.74	1.79	1.84	1.90	1.95	2.00
90	1.77	1.82	1.87	1.93	1.98	2.03

TABLE 2.11 Barometric Correction for Gravity—Metric Units

The values in the table below are to be subtracted from the readings taken on a mercurial barometer to correct for the decrease in gravity with increase in altitude.

Height above sealevel meters	Observed barometer height in millimeters								
	400	450	500	550	600	650	700	750	800
	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.
100	....	....	....	....	....	....	0.02	0.02	0.02
200	....	....	....	....	....	....	0.04	0.05	0.05
300	....	....	....	....	....	....	0.07	0.07	0.07
400	....	....	....	....	....	....	0.09	0.10	0.10
500	....	....	....	....	....	....	0.11	0.12	0.13
600	....	....	....	....	....	0.12	0.13	0.14	....
700	....	....	....	....	....	0.14	0.15	0.16	....
800	....	....	....	....	....	0.16	0.18	0.19	....
900	....	....	....	....	....	0.18	0.20	0.22	....
1000	....	....	....	0.18	0.19	0.20	0.22	0.24	....
1100	....	....	....	0.19	0.21	0.22	0.24	....	....
1200	....	....	....	0.21	0.23	0.24	0.26	....	....
1300	....	....	....	0.22	0.24	0.26	0.29	....	....
1400	....	....	....	0.24	0.26	0.28	0.31	....	....
1500	....	....	0.24	0.26	0.28	0.30	0.33	....	....
1600	....	....	0.25	0.28	0.30	0.32	....	....	....
1700	....	....	0.27	0.30	0.32	0.34	....	....	....
1800	....	....	0.28	0.31	0.34	0.36	....	....	....
1900	....	....	0.30	0.33	0.36	0.39	....	....	....
2000	....	0.28	0.31	0.34	0.38	0.41	....	....	....
2100	....	0.30	0.33	0.36	0.40	....	....	....	....
2200	....	0.31	0.35	0.38	0.41	....	....	....	....
2300	....	0.32	0.36	0.40	0.43	....	....	....	....
2400	....	0.34	0.38	0.42	0.45	....	....	....	....
2500	0.31	0.35	0.39	0.43	0.47	....	....	....	....
2600	0.33	0.37	0.41	....	....	....	....	....	....
2800	0.35	0.40	0.44	....	....	....	....	....	....
3000	0.38	0.42	0.47	....	....	....	....	....	....
3200	0.40	0.46	....	....	....	....	....	....	....
3400	0.43	0.48	....	....	....	....	....	....	....

TABLE 2.12 Reduction of the Barometer to Sea Level—Metric Units

A barometer located at an elevation above sea level will show a reading lower than a barometer at sea level by an amount approximately 2.5 mm (0.1 in) for each 30.5 m (100 ft) of elevation. A closer approximation can be made by reference to the following tables, which take into account (1) the effect of altitude of the station at which the barometer is read, (2) the mean temperature of the air column extending from the station down to sea level, (3) the latitude of the station at which the barometer is read, and (4) the reading of the barometer corrected for its temperature, a correction which is applied only to mercurial barometers since the aneroid barometers are compensated for temperature effects.

Example. A barometer which has been corrected for its temperature reads 650 mm at a station whose altitude is 1350 m above sea level and at a latitude of 30°. The mean temperature (outdoor temperature) at the station is 20°C.

Table A (metric units) gives for these conditions a temperature-altitude factor of 135.2
The Latitude Factor Table gives for 135.2 at 30° lat. a correction of .... +0.17
Therefore, the corrected value of the temperature-altitude factor is 135.37

Entering Table B (metric units), with a temperature-altitude factor of 135.37 and a barometric reading of 650 mm (corrected for temperature), the correction is found to be 109.6

Accordingly the barometric reading reduced to sea level is 650 + 109.6 = 759.6 mm.

Latitude Factor–English or Metric Units. For latitudes 0°–45° add the latitude factor, for 45°–90° subtract the latitude factor, from the values obtained in Table A.

Temp.—Alt. Factor From Table A	Latitude				
	0°	10°	20°	30°	45°
50	0.1	0.1	0.1	0.1	0.0
100	0.3	0.3	0.2	0.1	0.0
150	0.4	0.4	0.3	0.2	0.0
200	0.5	0.5	0.4	0.3	0.0
250	0.7	0.6	0.5	0.3	0.0
300	0.8	0.8	0.6	0.4	0.0
350	0.9	0.9	0.7	0.5	0.0
	90°	80°	70°	60°	45°

*A. Values of the temperature-altitude factor for use in Table B.**

Altitude in Meters	Mean Temperature of Air Column in Centigrade Degrees										
	−16°	−8°	−4°	0°	6°	10°	14°	18°	20°	22°	26°
10	1.2	1.1	1.1	1.1	1.1	1.0	1.0	1.0	1.0	1.0	1.0
50	5.8	5.6	5.5	5.4	5.3	5.2	5.1	5.0	5.0	5.0	4.9
100	11.5	11.2	11.0	10.8	10.6	10.4	10.3	10.1	10.0	9.9	9.8
150	17.3	16.7	16.5	16.2	15.9	15.6	15.4	15.1	15.0	14.9	14.7
200	23.0	22.3	22.0	21.6	21.1	20.8	20.5	20.2	20.0	19.9	19.6
250	28.8	27.9	27.5	27.0	26.4	26.0	25.6	25.2	25.0	24.9	24.5
300	34.5	33.5	33.0	32.5	31.7	31.2	30.7	30.3	30.1	29.8	29.4
350	40.3	39.0	38.5	37.9	37.0	36.4	35.9	35.3	35.1	34.8	34.3
400	46.0	44.6	43.9	43.3	42.3	41.6	41.0	40.4	40.1	39.8	39.2
450	51.8	51.3	49.4	48.7	47.6	46.8	46.1	45.4	45.1	44.8	44.1
500	57.5	55.8	54.9	54.1	52.9	52.0	51.2	50.5	50.1	49.7	49.0
550	63.3	61.4	60.4	59.5	58.1	57.2	56.4	55.5	55.1	54.7	53.9
600	69.0	66.9	65.9	64.9	63.4	62.4	61.5	60.6	60.1	59.7	58.8
650	74.8	72.5	71.4	70.3	68.7	67.6	66.6	65.6	65.1	64.6	63.7

TABLE 2.12 Reduction of the Barometer to Sea Level—Metric Units (*Continued*)

Altitude in Meters	Mean Temperature of Air Column in Centigrade Degrees										
	−16°	−8°	−4°	0°	6°	10°	14°	18°	20°	22°	26°
700	80.6	78.1	76.9	75.7	74.0	72.9	71.7	70.7	70.1	69.6	68.6
750	86.3	83.7	82.4	81.1	79.3	78.1	76.9	75.7	75.1	74.6	73.5
800	92.1	89.2	87.9	86.5	84.6	83.3	82.0	80.8	80.1	79.6	78.4
850	97.8	94.8	93.4	92.0	89.8	88.5	87.1	85.8	85.2	84.5	83.3
900	103.6	100.4	98.9	97.4	95.1	93.7	92.2	90.8	90.2	89.5	88.2
950	109.3	106.0	104.4	102.8	100.4	98.9	97.4	95.9	95.2	94.5	93.1
1000	115.1	111.5	109.8	108.2	105.7	104.1	102.5	100.9	100.2	99.4	98.0
1050	120.8	117.1	115.3	113.6	111.0	109.3	107.6	106.0	105.2	104.4	102.9
1100	126.6	122.7	120.8	119.0	116.3	114.5	112.7	111.0	110.2	109.4	107.8
1150	132.3	128.3	126.3	124.4	121.6	119.7	117.9	116.1	115.2	114.4	112.7
1200	138.1	133.8	131.8	129.8	126.8	124.9	123.0	121.1	120.2	119.3	117.6
1250	143.8	139.4	137.3	135.2	132.1	130.1	128.1	126.2	125.2	124.3	122.5
1300	149.6	145.0	142.8	140.6	137.4	135.3	133.2	131.2	130.2	129.3	127.4
1350	155.3	150.6	148.3	146.0	142.7	140.5	138.4	136.3	135.2	134.2	132.3
1400	161.1	156.2	153.8	151.4	148.0	145.7	143.5	141.3	140.2	139.2	137.2
1450	166.8	161.7	159.3	156.8	153.3	150.9	148.6	146.4	145.3	144.2	142.1
1500	172.6	167.3	164.8	162.3	158.5	156.1	153.7	151.4	150.3	149.1	147.0
1550	178.3	172.9	170.2	167.7	163.8	161.3	158.8	156.4	155.3	154.1	151.8
1600	184.1	178.5	175.7	173.1	169.1	166.5	164.0	161.5	160.3	159.1	156.7
1650	189.8	184.0	181.2	178.5	174.4	171.7	169.1	166.5	165.3	164.1	161.6
1700	195.6	189.6	186.7	183.9	179.7	176.9	174.2	171.6	170.3	169.0	166.5
1750	201.4	195.2	192.2	189.3	185.0	182.1	179.3	176.6	175.3	174.0	171.4
1800	207.1	200.8	197.7	194.7	190.2	187.3	184.5	181.7	180.3	179.0	176.3
1850	212.9	206.3	203.2	200.1	195.5	192.5	189.6	186.7	185.3	183.9	181.2
1900	218.6	211.9	208.7	205.5	200.8	197.7	194.7	191.8	190.3	188.9	186.1
1950	224.4	217.5	214.2	210.9	206.1	202.9	199.8	196.8	195.3	193.9	191.0
2000	230.1	223.0	219.7	216.3	211.4	208.1	204.9	201.9	200.3	198.8	195.0
2050	235.9	228.6	225.1	221.7	216.7	213.3	210.1	206.9	205.3	203.8	200.8
2100	241.6	234.2	230.6	227.1	221.9	218.5	215.2	211.9	210.4	208.8	205.7
2150	247.4	239.8	236.1	232.5	227.2	223.7	220.3	217.0	215.4	213.8	210.6
2200	253.1	245.4	241.6	237.9	232.5	228.9	225.4	222.0	220.4	218.7	215.5
2250	258.9	250.9	247.1	243.4	237.8	234.1	230.6	227.1	225.4	223.7	220.4
2300	264.6	256.5	252.6	248.8	243.1	239.3	235.7	232.1	230.4	228.7	225.3
2350	270.4	262.1	258.1	254.2	248.3	244.5	240.8	237.2	235.4	233.6	230.2
2400	276.1	267.7	263.6	259.6	253.6	249.7	245.9	242.2	240.4	238.6	235.1
2450	281.9	273.2	269.1	265.0	258.9	254.9	251.0	247.3	245.4	243.6	240.0
2500	287.6	278.8	274.5	270.4	264.2	260.1	256.2	252.3	250.4	248.5	244.9
2550	293.4	284.4	280.0	275.8	269.5	265.3	261.3	257.3	255.4	253.5	249.8
2600	299.1	290.0	285.5	281.2	274.8	270.5	266.4	262.4	260.4	258.5	254.7
2650	304.9	295.5	291.0	286.6	280.0	275.7	271.5	267.4	265.4	263.4	259.6
2700	310.6	301.1	296.5	292.0	285.3	280.9	276.6	272.5	270.4	268.4	264.5
2750	316.4	306.7	302.0	297.4	290.6	286.1	281.8	277.5	275.4	273.4	269.4
2800	322.1	312.3	307.5	302.8	295.9	291.3	286.9	282.6	280.4	278.3	274.3
2850	327.9	317.8	313.0	308.2	301.2	296.5	292.0	287.6	285.4	283.3	279.2
2900	333.6	323.4	318.4	313.6	306.4	301.7	297.1	292.6	290.4	288.3	284.1
2950	339.4	329.0	323.9	319.0	311.7	306.9	302.2	297.7	295.5	293.3	289.0
3000	345.1	334.5	329.4	324.4	317.0	312.1	307.4	302.7	300.5	298.2	293.8

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* From *Smithsonian Meteorological Tables,* 3d ed., 1907.

TABLE 2.12 Reduction of the Barometer to Sea Level—Metric Units (*Continued*)

*B. Values in millimeters to be added.**

Temp. —Alt. Factor	Barometer Reading in Millimeters						
	790	770	750	730	710	690	670
1	0.9	0.9	0.9	0.8	0.8	0.8	
5	4.6	4.4	4.3	4.2	4.1	4.0	
10	9.1	8.9	8.7	8.5	8.2	8.0	
15	13.8	13.4	13.1	12.7	12.4	12.0	
20	18.4	17.9	17.5	17.0	16.5	16.1	
25		22.5	21.9	21.3	20.7	20.1	
30		27.1	26.4	25.7	25.0	24.2	
35		31.7	30.8	30.0	29.2	28.4	
40		36.3	35.3	34.4	33.5	32.5	31.6
45			39.9	38.8	37.8	36.7	35.6
	750	730	710	690	670	650	630
50	44.4	43.3	42.1	40.9	39.7		
55	49.0	47.7	46.4	45.1	43.8		
60	53.6	52.2	50.8	49.3	47.9		
65	58.3	56.7	55.2	53.6	52.1		
70		61.3	59.6	57.9	56.2		
75		65.8	64.0	62.2	60.4		
80		70.4	68.5	66.6	64.6	62.7	60.8
85		75.0	73.0	70.9	68.9	66.8	64.8
90			77.5	75.3	73.1	71.0	68.8
95			82.1	79.7	77.4	75.1	72.8
	710	690	670	650	630	610	
100	86.6	84.2	81.8	79.3	76.9		
105	91.2	88.7	86.1	83.5	81.0		
110	95.9	93.2	90.5	87.8	85.1		
115	100.5	97.7	94.8	92.0	89.2		
120		102.2	99.3	96.3	93.3		
125		106.8	103.7	100.6	97.5	94.4	
130		111.4	108.2	104.9	101.7	98.5	
135		116.0	112.7	109.3	105.9	102.6	
140		120.7	117.2	113.7	110.2	106.7	
145			121.7	118.1	114.5	110.8	
	670	650	630	610	590	570	
150	126.3	122.5	118.8	115.0			
155	130.9	127.0	123.1	119.2			
160	135.5	131.5	127.4	123.4			
165	140.2	136.0	131.8	127.6			
170		140.5	136.2	131.9	127.5	123.2	
175		145.1	140.6	136.2	131.7	127.2	
180		149.7	145.1	140.5	135.9	131.3	
185		154.3	149.5	144.8	140.0	135.3	
190		158.9	154.0	149.2	144.3	139.4	
195			158.6	153.5	148.5	143.5	

* From *Smithsonian Meteorological Tables*, 3d ed., 1907.

TABLE 2.12 Reduction of the Barometer to Sea Level—Metric Units (*Continued*)

*B. Values in millimeters to be added.**

Temp. —Alt. Factor	Barometer Reading in Millimeters					
	630	610	590	570	550	530
200	163.1	157.9	152.8	147.6		
205	167.7	162.4	157.1	151.7		
210	172.3	166.8	161.4	155.9		
215	176.9	171.3	165.7	160.1	154.5	148.9
220		175.8	170.1	164.3	158.5	152.8
225		180.4	174.5	168.5	162.6	156.7
230		184.9	178.9	172.8	166.7	160.7
235		189.5	183.3	177.1	170.9	164.7
240		194.1	187.8	181.4	175.0	168.7
245		198.8	192.3	185.7	179.2	172.7
	590	570	550	530	510	
250	196.8	190.1	183.4	176.8		
255	201.3	194.5	187.7	180.8		
260	205.9	198.9	191.9	185.0	178.0	
265	210.5	203.3	196.2	189.1	181.9	
270	215.1	207.8	200.5	193.2	185.9	
275	219.8	212.3	204.9	197.4	190.0	
280		216.8	209.2	201.6	194.0	
285		221.4	213.6	205.8	198.1	
290		225.9	218.0	210.1	202.1	
295		230.5	222.4	214.3	206.3	
	570	550	530	510	490	
300	235.1	226.9	218.6	210.4		
305	239.8	231.4	223.0	214.6	206.1	
310		235.9	227.3	218.7	210.1	
315		240.4	231.7	222.9	214.2	
320		245.0	236.1	227.2	218.3	
325		249.6	240.5	231.4	222.4	
330		254.2	244.9	235.7	226.5	
335		258.8	249.4	240.0	230.6	
340		263.5	253.9	244.4	234.8	
345			258.4	248.7	238.9	

* From *Smithsonian Meteorological Tables,* 3d ed., 1907.

TABLE 2.13 Viscosity Conversion Table

Centistokes to Saybolt, Redwood, and Engler units.

Poise = cgs unit of absolute viscosity   Centipoise = 0.01 poise

Stoke = cgs unit of kinematic viscosity   Centistoke = 0.01 stoke

Centipoises = centistokes × density (at temperature under consideration)

Reyn (1 lb · s per sq in) = 69×10^5 centpoises

Cf. *Jour. Inst. Pet. Tech.,* Vol. 22, p. 21 (1936); *Reports of A. S. T. M. Committee D-2, 1936 and 1937.*

The values of Saybolt Universal Viscosity at 100°F and at 210°F are taken directly from the comprehensive *ASTM Viscosity Table, Special Technical Publication No. 43A* (1953) by permission of the publishers, American Society for Testing Materials, 1916 Race St., Philadelphia 3, Pa.

	Saybolt Universal Viscosity at			Redwood Seconds at			Engler Degrees at all Temps.
Centistokes	100°F.	130°F.	210°F.	70°F.	140°F.	200°F.	
2.0	32.62	32.68	32.85	30.2	31.0	31.2	1.14
3.0	36.03	36.10	36.28	32.7	33.5	33.7	1.22
4.0	39.14	39.22	39.41	35.3	36.0	36.3	1.31
5.0	42.35	42.43	42.65	37.9	38.5	38.9	1.40
6.0	45.56	45.65	45.88	40.5	41.0	41.5	1.48
7.0	48.77	48.86	49.11	43.2	43.7	44.2	1.56
8.0	52.09	52.19	52.45	46.0	46.4	46.9	1.65
9.0	55.50	55.61	55.89	48.9	49.1	49.7	1.75
10.0	58.91	59.02	59.32	51.7	52.0	52.6	1.84
11.0	62.43	62.55	62.86	54.8	55.0	55.6	1.93
12.0	66.04	66.17	66.50	57.9	58.1	58.8	2.02
14.0	73.57	73.71	74.09	64.4	64.6	65.3	2.22
16.0	81.30	81.46	81.87	71.0	71.4	72.2	2.43
18.0	89.44	89.61	90.06	77.9	78.5	79.4	2.64
20.0	97.77	97.96	98.45	85.0	85.8	86.9	2.87
22.0	106.4	106.6	107.1	92.4	93.3	94.5	3.10
24.0	115.0	115.2	115.8	99.9	100.9	102.2	3.34
26.0	123.7	123.9	124.5	107.5	108.6	110.0	3.58
28.0	132.5	132.8	133.4	115.3	116.5	118.0	3.82
30.0	141.3	141.6	142.3	123.1	124.4	126.0	4.07
32.0	150.2	150.5	151.2	131.0	132.3	134.1	4.32
34.0	159.2	159.5	160.3	138.9	140.2	142.2	4.57
36.0	168.2	168.5	169.4	146.9	148.2	150.3	4.83
38.0	177.3	177.6	178.5	155.0	156.2	158.3	5.08
40.0	186.3	186.7	187.6	163.0	164.3	166.7	5.34
42.0	195.3	195.7	196.7	171.0	172.3	175.0	5.59
44.0	204.4	204.8	205.9	179.1	180.4	183.3	5.85
46.0	213.7	214.1	215.2	187.1	188.5	191.7	6.11
48.0	222.9	223.3	224.5	195.2	196.6	200.0	6.37
50.0	232.1	232.5	233.8	203.3	204.7	208.3	6.63
60.0	278.3	278.8	280.2	243.5	245.3	250.0	7.90
70.0	324.4	325.0	326.7	283.9	286.0	291.7	9.21
80.0	370.8	371.5	373.4	323.9	326.6	333.4	10.53
90.0	417.1	417.9	420.0	364.4	367.4	375.0	11.84
100.0*	463.5	464.4	466.7	404.9	408.2	416.7	13.16

* At higher values use the same ratio as above for 100 centistokes; ***e.g.,*** 102 centistokes = 102 × 4.635 Saybolt seconds at 100°F.

To obtain the Saybolt Universal viscosity equivalent to a kinematic viscosity determined at ***t***°F., multiply the equivalent Saybolt Universal viscosity at 100°F. by $1 + (t - 100)\ 0.000064$; ***e.g.,*** 10 centistokes at 210°F are equivalent to 58.91 × 1.0070, or 59.32 Saybolt Universal Viscosity at 210°F.

TABLE 2.14 Conversion of Weighings in Air to Weighings in Vacuo

If the mass of a substance in air is m_f, its density ρ_m, the density of weights used in making the weighing ρ_w, and the density* of air ρ_a, the true mass of the substance in vacuo, m_{vac}, is

$$m_{\text{vac}} = m_f + \rho_a m_f \left(\frac{1}{\rho_m} - \frac{1}{\rho_w} \right)$$

For most purposes it is sufficient to assume a density of 8.4 for brass weights, and a density of 0.0012 for air under ordinary conditions. The equation then becomes

$$m_{\text{vac}} = m_f + 0.0012 m_f \left(\frac{1}{\rho_m} - \frac{1}{8.4} \right)$$

The table which follows gives the values of *k* (buoyancy reduction factor), which is the correction necessary because of the buoyant effect of the air upon the object weighed; the table is computed for air with the density of 0.0012; *m* is the weight in grams of the object when weighed in air; weight of object reduced to “in vacuo” = $m + km/1000$.

	Buoyancy reduction factor, *k*			
Density of object weighed	Brass weights, density = 8.4	Pt or Pt-Ir weights, density = 21.5	Al or quartz weights, density = 2.7	Gold weights, density = 17
0.2	5.89	5.98	5.58	5.97
0.3	3.87	3.96	3.56	3.95
0.4	2.87	2.95	2.55	2.94
0.5	2.26	2.35	1.95	2.34
0.6	1.86	1.95	1.55	1.93
0.7	1.57	1.66	1.26	1.65
0.75	1.46	1.55	1.15	1.53
0.80	1.36	1.45	1.05	1.43
0.82	1.32	1.41	1.01	1.39
0.84	1.29	1.37	0.98	1.36
0.86	1.25	1.34	0.94	1.33
0.88	1.22	1.31	0.91	1.29
0.90	1.19	1.28	0.88	1.26
0.92	1.16	1.25	0.85	1.24
0.94	1.13	1.22	0.82	1.21
0.96	1.11	1.20	0.80	1.18
0.98	1.08	1.17	0.77	1.16
1.00	1.06	1.15	0.75	1.13
1.02	1.03	1.12	0.72	1.11
1.04	1.01	1.10	0.70	1.08
1.06	0.99	1.08	0.68	1.06
1.08	0.97	1.06	0.66	1.04
1.10	0.95	1.04	0.64	1.02
1.12	0.93	1.02	0.62	1.00
1.14	0.91	1.00	0.60	0.98
1.16	0.89	0.98	0.58	0.96
1.18	0.87	0.96	0.56	0.95
1.20	0.86	0.95	0.55	0.93
1.25	0.82	0.91	0.51	0.89
1.30	0.78	0.87	0.47	0.85

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* See Table 5.15, Specific Gravity of Air at Various Temperatures.

TABLE 2.14 Conversion of Weighings in Air to Weighings in Vacuo (*Continued*)

	Buoyancy reduction factor, *k*			
Density of object weighed	Brass weights, density = 8.4	Pt or Pt-Ir weights, density = 21.5	Al or quartz weights, density = 2.7	Gold weights, density = 17
1.35	0.75	0.83	0.44	0.82
1.40	0.71	0.80	0.40	0.79
1.50	0.66	0.74	0.35	0.73
1.6	0.61	0.69	0.30	0.68
1.7	0.56	0.65	0.25	0.64
1.8	0.52	0.61	0.21	0.60
1.9	0.49	0.58	0.18	0.56
2.0	0.46	0.54	0.15	0.53
2.2	0.40	0.49	0.09	0.48
2.4	0.36	0.44	0.05	0.43
2.6	0.32	0.41	0.01	0.39
2.8	0.29	0.37	−0.02	0.36
3.0	0.26	0.34	−0.05	0.33
3.5	0.20	0.29	−0.11	0.27
4	0.16	0.24	−0.15	0.23
5	0.10	0.18	−0.21	0.17
6	0.06	0.14	−0.25	0.13
7	0.03	0.12	−0.28	0.10
8	0.01	0.09	−0.30	0.08
9	−0.01	0.08	−0.32	0.06
10	−0.02	0.06	−0.33	0.05
12	−0.04	0.04	−0.35	0.03
14	−0.06	0.03	−0.37	0.02
16	−0.07	0.02	−0.38	0.00
18	−0.08	0.01	−0.39	0.00
20	−0.08	0.00	−0.39	−0.01
22	−0.09	0.00	−0.40	−0.02

TABLE 2.15 Hydrometer Conversion Table

This table gives the relation between density (c.g.s.) and degrees on the Baumé and Twaddell scales. The Twaddell scale is never used for densities less than unity. See also Sec. 2.1.2.1, Hydrometers.

Density	Degrees Baumé (NIST* scale)	Degrees Baumé (A.P.I.† scale)
0.600	103.33	104.33
0.605	101.40	102.38
0.610	99.51	100.47
0.615	97.64	98.58
0.620	95.81	96.73
0.625	94.00	94.90
0.630	92.22	93.10
0.635	90.47	91.33
0.640	88.75	89.59
0.645	87.05	87.88
0.650	85.38	86.19
0.655	83.74	84.53
0.660	82.12	82.89
0.665	80.52	81.28
0.670	78.95	79.69
0.675	77.41	78.13
0.680	75.88	76.59
0.685	74.38	75.07
0.690	72.90	73.57
0.695	71.43	72.10
0.700	70.00	70.64
0.705	68.57	69.21
0.710	67.18	67.80
0.715	65.80	66.40
0.720	64.44	65.03
0.725	63.10	63.67
0.730	61.78	62.34
0.735	60.48	61.02
0.740	59.19	59.72
0.745	57.92	58.43
0.750	56.67	57.17
0.755	55.43	55.92
0.760	54.21	54.68
0.765	53.01	53.47
0.770	51.82	52.27
0.775	50.65	51.08
0.780	49.49	49.91
0.785	48.34	48.75
0.790	47.22	47.61
0.795	46.10	46.49
0.800	45.00	45.38
0.805	43.91	44.28
0.810	42.84	43.19
0.815	41.78	42.12
0.820	40.73	41.06
0.825	39.70	40.02
0.830	38.68	38.98
0.835	37.66	37.96
0.840	36.67	36.95
0.845	35.68	35.96
0.850	34.71	34.97
0.855	33.74	34.00
0.860	32.79	33.03
0.865	31.85	32.08
0.870	30.92	31.14
0.875	30.00	30.21
0.880	29.09	29.30
0.885	28.19	28.39
0.890	27.30	27.49
0.895	26.42	26.60
0.900	25.56	25.72
0.905	24.70	24.85
0.910	23.85	23.99
0.915	23.01	23.14
0.920	22.17	22.30
0.925	21.35	21.47
0.930	20.54	20.65
0.935	19.73	19.84
0.940	18.94	19.03
0.945	18.15	18.24
0.950	17.37	17.45
0.955	16.60	16.67
0.960	15.83	15.90
0.965	15.08	15.13
0.970	14.33	14.38
0.975	13.59	13.63
0.980	12.86	12.89
0.985	12.13	12.15
0.990	11.41	11.43
0.995	10.70	10.71
1.000	10.00	10.00

DENSITIES GREATER THAN UNITY

Density	Degrees Baumé (NIST* scale)	Degrees Twaddell
1.00	0.00	0
1.01	1.44	2
1.02	2.84	4

* NIST, National Institute for Science and Technology (formerly the National Bureau of Standards, U.S.).

† A.P.I. is the American Petroleum Institute.

TABLE 2.15 Hydrometer Conversion Table (*Continued*)

Density	Degrees Baumé (NIST* scale)	Degrees Twaddell
1.03	4.22	6
1.04	5.58	8
1.05	6.91	10
1.06	8.21	12
1.07	9.49	14
1.08	10.78	16
1.09	11.97	18
1.10	13.18	20
1.11	14.37	22
1.12	15.54	24
1.13	16.68	26
1.14	17.81	28
1.15	18.91	30
1.16	20.00	32
1.17	21.07	34
1.18	22.12	36
1.19	23.15	38
1.20	24.17	40
1.21	25.16	42
1.22	26.15	44
1.23	27.11	46
1.24	28.06	48
1.25	29.00	50
1.26	29.92	52
1.27	30.83	54
1.28	31.72	56
1.29	32.60	58
1.30	33.46	60
1.31	34.31	62
1.32	35.15	64
1.33	35.98	66
1.34	36.79	68
1.35	37.59	70
1.36	38.38	72
1.37	39.16	74
1.38	39.93	76
1.39	40.68	78
1.40	41.43	80
1.41	42.16	82
1.42	42.89	84
1.43	43.60	86
1.44	44.31	88
1.45	45.00	90
1.46	45.68	92
1.47	46.36	94
1.48	47.03	96
1.49	47.68	98
1.50	48.33	100
1.51	48.97	102
1.52	49.60	104
1.53	50.23	106
1.54	50.84	108
1.55	51.45	110
1.56	52.05	112
1.57	52.64	114
1.58	53.23	116
1.59	53.80	118
1.60	54.38	120
1.61	54.94	122
1.62	55.49	124
1.63	56.04	126
1.64	56.58	128
1.65	57.12	130
1.66	57.65	132
1.67	58.17	134
1.68	58.69	136
1.69	59.20	138
1.70	59.71	140
1.71	60.20	142
1.72	60.70	144
1.73	61.18	146
1.74	61.67	148
1.75	62.14	150
1.76	62.61	152
1.77	63.08	154
1.78	63.54	156
1.79	63.99	158
1.80	64.44	160
1.81	64.89	162
1.82	65.31	164
1.83	65.77	166
1.84	66.20	168
1.85	66.62	170
1.86	67.04	172
1.87	67.46	174
1.88	67.87	176
1.89	68.28	178
1.90	68.68	180
1.91	69.08	182
1.92	69.48	184
1.93	69.87	186
1.94	70.26	188
1.95	70.64	190
1.96	71.02	192
1.97	71.40	194
1.98	71.77	196
1.99	72.14	198
2.00	72.50	200

* NIST, National Institute for Science and Technology (formerly the National Bureau of Standards, U.S.).

TABLE 2.16 Pressure Conversion Chart

psi	Inches H_2O at 4°C	Inches Hg at 0°C	mmH_2O at 4°C	mmHg at 0°C	atm	Pascals ($N \cdot m^{-2}$)
0.01	0.2768	0.0204	7.031	0.517	0.0007	68.95
0.02	0.5536	0.0407	14.06	1.034	0.0014	137.90
0.03	0.8304	0.0611	21.09	1.551	0.0020	206.8
0.04	1.107	0.0814	28.12	2.068	0.0027	275.8
0.05	1.384	0.1018	35.15	2.586	0.0034	344.7
0.06	1.661	0.1222	42.18	3.103	0.0041	413.7
0.07	1.938	0.1425	49.22	3.620	0.0048	482.6
0.08	2.214	0.1629	56.25	4.137	0.0054	551.6
0.09	2.491	0.1832	63.28	4.654	0.0061	620.5
0.10	2.768	0.2036	70.31	5.171	0.0068	689.5
0.20	5.536	0.4072	140.6	10.34	0.0136	1 379.9
0.30	8.304	0.6108	210.9	15.51	0.0204	2 068.5
0.40	11.07	0.8144	281.2	20.68	0.0272	2 758
0.50	13.84	1.018	351.5	25.86	0.0340	3 447
0.60	16.61	1.222	421.8	31.03	0.0408	4 137
0.70	19.38	1.425	492.2	36.20	0.0476	4 826
0.80	22.14	1.629	562.5	41.37	0.0544	5 516
0.90	24.91	1.832	632.8	46.54	0.0612	6 205
1.00	27.68	2.036	703.1	51.71	0.0689	6 895
2.00	55.36	4.072	1 072	103.4	0.1361	13 790
3.00	83.04	6.108	2 109	155.1	0.2041	20 684
4.00	110.7	8.144	2 812	206.8	0.2722	27 579
5.00	138.4	10.18	3 515	258.6	0.3402	34 474
6.00	166.1	12.22	4 218	310.3	0.4083	41 369
7.00	193.8	14.25	4 922	362.0	0.4763	48 263
8.00	221.4	16.29	5 625	413.7	0.5444	55 158
9.00	249.1	18.32	6 328	465.4	0.6124	62 053
10.0	276.8	20.36	7 031	517.1	0.6805	68 948
14.7	406.9	29.93	10 332	760.0	1.000	101 325
15.0	415.2	30.54	10 550	775.7	1.021	103 421
20.0	553.6	40.72	14 060	1 034	1.361	137 895
25.0	692.0	50.90	17 580	1 293	1.701	172 369
30.0	830.4	61.08	21 090	1 551	2.041	206 843
40.0	1 107	81.44	28 120	2 068	2.722	275 790
50.0	1 384	101.8	35 150	2 586	3.402	344 738
60.0	1 661	122.2	42 180	3 103	4.083	413 685
70.0	1 938	142.5	49 220	3 620	4.763	482 633
80.0	2 214	162.9	56 250	4 137	5.444	551 581
90.0	2 491	183.2	63 280	4 654	6.124	620 528
100.0	2 768	203.6	70 307	5 171	6.805	689 476
150.0	4 152	305.4		7 757	10.21	1 034 214
200.0	5 536	407.2		10 343	13.61	1 378 951
250.0	6 920	509.0			17.01	1 723 689
300.0	8 304	610.8			20.41	2 068 427
400.0					27.22	2 757 903
500.0					34.02	3 447 379

1 bar = 10^5 pascal.

TABLE 2.17 Corrections to Be Added to Molar Values to Convert to Molal

	Aqueous solution			
Temperature, °C	$\Delta G°$ $\mathrm{J \cdot mol^{-1}}$	$\Delta H°$ $\mathrm{J \cdot mol^{-1}}$	$\Delta S°$ $\mathrm{J \cdot deg^{-1} \cdot mol^{-1}}$	$\Delta C°_p$ $\mathrm{J \cdot deg^{-1} \cdot mol^{-1}}$
0	0.4	−42.7	−0.17	55.2
10	0.8	58.1	0.21	45.6
20	4.2	148.1	0.50	38.9
30	10.9	230.5	0.79	35.1
40	20.1	313.4	1.09	33.0
50	32.2	397.9	1.34	32.6
60	46.8	482.4	1.59	32.2

TABLE 2.18 Molar Equivalent of One Liter of Gas at Various Temperatures and Pressures

The values in this table, which give the number of moles in 1 liter of gas, are based on the properties of an "ideal" gas and were calculated by use of the formula:

$$\text{Moles/liter} = \frac{P}{760} \times \frac{273}{T} \times \frac{1}{22.40}$$

where P is the pressure in millimeters of mercury and T is the temperature in kelvins ($= t$°C + 273).

To convert to moles per cubic foot multiply the values in the table by 28.316.

Pressure mm of mercury	Temperature °C					
	10°	12°	14°	16°	18°	20°
655	0.03712	0.03686	0.03660	0.03634	0.03610	0.03585
660	3731	3714	3688	3662	3637	3612
665	3768	3742	3716	3690	3665	3640
670	3796	3770	3744	3718	3692	3667
675	3825	3798	3772	3745	3720	3695
680	0.03853	0.03826	0.03800	0.03773	0.03747	0.03694
685	3881	3854	3827	3801	3775	3749
690	3910	3882	3855	3829	3802	3776
695	3938	3910	3883	3856	3830	3804
700	3967	3939	3911	3884	3858	3831
702	0.03978	0.03950	0.03922	0.03895	0.03869	0.03842
704	3989	3961	3934	3906	3880	3853
706	4000	3972	3945	3917	3891	3864
708	4012	3984	3956	3929	3902	3875
710	4023	3995	3967	3940	3913	3886
712	0.04035	0.04006	0.03978	0.03951	0.03924	0.03897
714	4046	4018	3989	3962	3935	3908
716	4057	4029	4001	3973	3946	3919
718	4068	4040	4012	3984	3957	3930
720	4080	4051	4023	3995	3968	3941

TABLE 2.18 Molar Equivalent of One Liter of Gas at Various Temperatures and Pressures (*Continued*)

Pressure mm of mercury	Temperature °C					
	10°	12°	14°	16°	18°	20°
722	0.04091	0.04063	0.04034	0.04006	0.03979	0.03952
724	4103	4074	4045	4017	3990	3963
726	4114	4085	4057	4028	4001	3973
728	4125	4096	4068	4040	4012	3984
730	4136	4108	4079	4051	4023	3995
732	0.04148	0.04119	0.04090	0.04062	0.04034	0.04006
734	4159	4130	4101	4073	4045	4017
736	4171	4141	4112	4084	4056	4028
738	4182	4153	4124	4095	4067	4039
740	4193	4164	4135	4106	4078	4050
742	0.04204	0.04175	0.04146	0.04117	0.04089	0.04061
744	4216	4186	4157	4128	4100	4072
746	4227	4198	4168	4139	4111	4038
748	4239	4209	4179	4151	4122	4094
750	4250	4220	4191	4162	4133	4105
752	0.04261	0.04231	0.04202	0.04173	0.04144	0.04116
754	4273	4243	4213	4184	4155	4127
756	4284	4254	4224	4195	4166	4138
758	4295	4265	4235	4206	4177	4149
760	4307	4276	4247	4217	4188	4160
762	0.04318	0.04287	0.04258	0.04228	0.04199	0.04171
764	4329	4299	4269	4239	4210	4181
766	4341	4310	4280	4250	4221	4192
768	4352	4321	4291	4262	4232	4203
770	4363	4333	4302	4273	4243	4214
772	0.04375	0.04344	0.04314	0.04284	0.04254	0.04225
774	4386	4355	4325	4295	4265	4236
776	4397	4366	4336	4306	4276	4247
778	4409	4378	4347	4317	4287	4258
780	4420	4389	4358	4328	4298	4269

Pressure mm of mercury	Temperature °C					
	22°	24°	26°	28°	30°	32°
655	0.03561	0.03537	0.03515	0.03490	0.03467	0.03444
660	3588	3564	3541	3516	3493	3470
665	3614	3591	3568	3543	3520	3496
670	3642	3618	3595	3569	3546	3523
675	3669	3645	3622	3596	3572	3549
680	0.03697	0.03672	0.03649	0.03623	0.03599	0.03575
685	3724	3699	3676	3649	3625	3602
690	3751	3726	3702	3676	3652	3628
695	3778	3753	3729	3703	3678	3654
700	3805	3780	3756	3729	3705	3680

TABLE 2.18 Molar Equivalent of One Liter of Gas at Various Temperatures and Pressures (*Continued*)

Pressure mm of mercury	Temperature °C					
	22°	24°	26°	28°	30°	32°
702	0.03816	0.03790	0.03767	0.03740	0.03715	0.03691
704	3827	3801	3777	3750	3726	3701
706	3838	3812	3788	3761	3736	3712
708	3849	3823	3799	3772	3747	3722
710	3860	3834	3810	3783	3758	3733
712	0.03870	0.03844	0.03820	0.03793	0.03768	0.03744
714	3881	3855	3831	3804	3779	3754
716	3892	3866	3842	3815	3789	3765
718	3902	3877	3853	3825	3800	3775
720	3914	3888	3863	3836	3811	3786
722	0.03925	0.03898	0.03874	0.03847	0.03821	0.03796
724	3936	3909	3885	3857	3832	3807
726	3947	3920	3896	3868	3842	3817
728	3957	3931	3906	3878	3853	3828
730	3968	3941	3917	3889	3863	3838
732	0.03979	0.03952	0.03928	0.03900	0.03874	0.03849
734	3990	3963	3938	3910	3885	3859
736	4001	3974	3949	3921	3895	3870
738	4012	3985	3960	3932	3906	3880
740	4023	3995	3971	3942	3916	3891
742	0.04033	0.04006	0.03981	0.03953	0.03927	0.03901
744	4044	4017	3992	3964	3938	3912
746	4055	4028	4003	3974	3948	3922
748	4066	4039	4014	3985	3959	3933
750	4077	4049	4024	3996	3969	3943
752	0.04088	0.04060	0.04035	0.04006	0.03980	0.03954
754	4099	4071	4046	4017	3991	3964
756	4110	4082	4056	4028	4001	3975
758	4121	4093	4067	4038	4012	3985
760	4131	4103	4078	4049	4022	3996
762	0.04142	4114	4089	4060	4033	4006
764	4153	4125	4099	4070	4043	4017
766	4164	4136	4110	4081	4054	4027
768	4175	4147	4121	4092	4065	4038
770	4186	4158	4132	4102	4075	4048
772	0.04197	0.04168	0.04142	0.04113	0.04086	0.04059
774	4207	4179	4153	4124	4096	4070
776	4218	4190	4164	4134	4107	4080
778	4229	4201	4175	4145	4117	4091
780	4240	4211	4185	4155	4128	4101

TABLE 2.19 Factors for Reducing Gas Volumes to Normal (Standard) Temperature and Pressure (760 mmHg)

Examples: (a) 20 mL of dry gas at 22°C and 730 mm = 20 × 0.8888 = 17.78 mL at 0°C and 760 mm. *(b)* 20 mL of a gas over water at 22° and 730 mm = 20 × (factor corrected for aqueous tension; i.e., 730 − 19.8 or 710.2 mm) = 20 mL of dry gas at 22° and 710.2 mm = 20 × 0.86475 = 17.30 mL at 0°C and 760 mm. Mass in milligrams of 1 mL of gas at S.T.P.: acetylene, 1.173; carbon dioxide, 1.9769; hydrogen, 0.0899; nitric oxide (NO), 1.3402; nitrogen, 1.25057; oxygen, 1.42904.

Pressure mm of mercury	Temperature °C							
	10°	11°	12°	13°	14°	15°	16°	17°
670	0.8504	0.8474	0.8445	0.8415	0.8386	0.8357	0.8328	0.8299
672	0.8530	0.8500	0.8470	0.8440	0.8411	0.8382	0.8353	0.8324
674	0.8555	0.8525	0.8495	0.8465	0.8436	0.8407	0.8377	0.8349
676	0.8580	0.8550	0.8520	0.8490	0.8461	0.8431	0.8402	0.8373
678	0.8606	0.8576	0.8545	0.8516	0.8486	0.8456	0.8427	0.8398
680	0.8631	0.8601	0.8571	0.8541	0.8511	0.8481	0.8452	0.8423
682	0.8657	0.8626	0.8596	0.8566	0.8536	0.8506	0.8477	0.8448
684	0.8682	0.8651	0.8621	0.8591	0.8561	0.8531	0.8502	0.8472
686	0.8707	0.8677	0.8646	0.8616	0.8586	0.8556	0.8527	0.8497
688	0.8733	0.8702	0.8672	0.8641	0.8611	0.8581	0.8551	0.8522
690	0.8758	0.8727	0.8697	0.8666	0.8636	0.8606	0.8576	0.8547
692	0.8784	0.8753	0.8722	0.8691	0.8661	0.8631	0.8601	0.8572
694	0.8809	0.8778	0.8747	0.8717	0.8686	0.8656	0.8626	0.8596
696	0.8834	0.8803	0.8772	0.8742	0.8711	0.8681	0.8651	0.8621
698	0.8860	0.8828	0.8798	0.8767	0.8736	0.8706	0.8676	0.8646
700	0.8885	0.8854	0.8823	0.8792	0.8761	0.8731	0.8700	0.8671
702	0.8910	0.8879	0.8848	0.8817	0.8786	0.8756	0.8725	0.8695
704	0.8936	0.8904	0.8873	0.8842	0.8811	0.8781	0.8750	0.8720
706	0.8961	0.8930	0.8898	0.8867	0.8836	0.8806	0.8775	0.8745
708	0.8987	0.8955	0.8924	0.8892	0.8861	0.8831	0.8800	0.8770
710	0.9012	0.8980	0.8949	0.8917	0.8886	0.8856	0.8825	0.8794
712	0.9037	0.9006	0.8974	0.8943	0.8911	0.8880	0.8850	0.8819
714	0.9063	0.9031	0.8999	0.8968	0.8936	0.8905	0.8875	0.8844
716	0.9088	0.9056	0.9024	0.8993	0.8961	0.8930	0.8899	0.8869
718	0.9114	0.9081	0.9050	0.9018	0.8987	0.8955	0.8924	0.8894
720	0.9139	0.9107	0.9075	0.9043	0.9012	0.8980	0.8949	0.8918
722	0.9164	0.9132	0.9100	0.9068	0.9037	0.9005	0.8974	0.8943
724	0.9190	0.9157	0.9125	0.9093	0.9062	0.9030	0.8999	0.8968
726	0.9215	0.9183	0.9151	0.9118	0.9087	0.9055	0.9024	0.8993
728	0.9241	0.9208	0.9176	0.9144	0.9112	0.9080	0.9049	0.9017
730	0.9266	0.9233	0.9201	0.9169	0.9137	0.9105	0.9073	0.9042
732	0.9291	0.9259	0.9226	0.9194	0.9162	0.9130	0.9098	0.9067
734	0.9317	0.9284	0.9251	0.9219	0.9187	0.9155	0.9123	0.9092
736	0.9342	0.9309	0.9277	0.9244	0.9212	0.9180	0.9148	0.9117
738	0.9368	0.9334	0.9302	0.9269	0.9237	0.9205	0.9173	0.9141
740	0.9393	0.9360	0.9327	0.9294	0.9262	0.9230	0.9198	0.9166
742	0.9418	0.9385	0.9352	0.9319	0.9287	0.9255	0.9223	0.9191
744	0.9444	0.9410	0.9377	0.9345	0.9312	0.9280	0.9248	0.9216
746	0.9469	0.9436	0.9403	0.9370	0.9337	0.9305	0.9272	0.9240
748	0.9494	0.9461	0.9428	0.9395	0.9362	0.9329	0.9297	0.9265

TABLE 2.19 Factors for Reducing Gas Volumes to Normal (Standard) Temperature and Pressure (*Continued*)

Pressure mm of mercury	Temperature °C							
	10°	11°	12°	13°	14°	15°	16°	17°
750	0.9520	0.9486	0.9453	0.9420	0.9387	0.9354	0.9322	0.9290
752	0.9545	0.9511	0.9478	0.9445	0.9412	0.9379	0.9347	0.9315
754	0.9571	0.9537	0.9504	0.9470	0.9437	0.9404	0.9372	0.9339
756	0.9596	0.9562	0.9529	0.9495	0.9462	0.9429	0.9397	0.9364
758	0.9621	0.9587	0.9554	0.9520	0.9487	0.9454	0.9422	0.9389
760	0.9647	0.9613	0.9579	0.9546	0.9512	0.9479	0.9446	0.9414
762	0.9672	0.9638	0.9604	0.9571	0.9537	0.9504	0.9471	0.9439
764	0.9698	0.9663	0.9630	0.9596	0.9562	0.9529	0.9496	0.9463
766	0.9723	0.9689	0.9655	0.9620	0.9587	0.9554	0.9521	0.9488
768	0.9748	0.9714	0.9680	0.9646	0.9612	0.9579	0.9546	0.9513
770	0.9774	0.9739	0.9705	0.9671	0.9637	0.9604	0.9571	0.9538
772	0.9799	0.9764	0.9730	0.9696	0.9662	0.9629	0.9596	0.9562
774	0.9825	0.9790	0.9756	0.9721	0.9687	0.9654	0.9620	0.9587
776	0.9850	0.9815	0.9781	0.9746	0.9712	0.9679	0.9645	0.9612
778	0.9875	0.9840	0.9806	0.9772	0.9737	0.9704	0.9670	0.9637
780	0.9901	0.9866	0.9831	0.9797	0.9763	0.9729	0.9695	0.9662
782	0.9926	0.9891	0.9856	0.9822	0.9788	0.9754	0.9720	0.9686
784	0.9952	0.9916	0.9882	0.9847	0.9813	0.9778	0.9745	0.9711
786	0.9977	0.9942	0.9907	0.9872	0.9838	0.9803	0.9770	0.9736
788	1.0002	0.9967	0.9932	0.9897	0.9863	0.9828	0.9794	0.9761

Pressure mm of mercury	Temperature °C							
	18°	19°	20°	21°	22°	23°	24°	25°
670	0.8270	0.8242	0.8214	0.8186	0.8158	0.8131	0.8103	0.8076
672	0.8295	0.8267	0.8239	0.8211	0.8183	0.8155	0.8128	0.8100
674	0.8320	0.8291	0.8263	0.8235	0.8207	0.8179	0.8152	0.8124
676	0.8345	0.8316	0.8288	0.8259	0.8231	0.8204	0.8176	0.8149
678	0.8369	0.8341	0.8312	0.8284	0.8256	0.8228	0.8200	0.8173
680	0.8394	0.8365	0.8337	0.8308	0.8280	0.8252	0.8224	0.8197
682	0.8419	0.8390	0.8361	0.8333	0.8304	0.8276	0.8249	0.8221
684	0.8443	0.8414	0.8386	0.8357	0.8329	0.8301	0.8273	0.8245
686	0.8468	0.8439	0.8410	0.8382	0.8353	0.8325	0.8297	0.8269
688	0.8493	0.8464	0.8435	0.8406	0.8378	0.8349	0.8321	0.8293
690	0.8517	0.8488	0.8459	0.8430	0.8402	0.8373	0.8345	0.8317
692	0.8542	0.8513	0.8484	0.8455	0.8426	0.8398	0.8369	0.8341
694	0.8567	0.8537	0.8508	0.8479	0.8451	0.8422	0.8394	0.8366
696	0.8591	0.8562	0.8533	0.8504	0.8475	0.8446	0.8418	0.8390
698	0.8616	0.8587	0.8557	0.8528	0.8499	0.8471	0.8442	0.8414
700	0.8641	0.8611	0.8582	0.8553	0.8524	0.8495	0.8466	0.8438
702	0.8665	0.8636	0.8606	0.8577	0.8547	0.8519	0.8490	0.8462
704	0.8690	0.8660	0.8631	0.8602	0.8572	0.8543	0.8515	0.8486
706	0.8715	0.8685	0.8655	0.8626	0.8597	0.8568	0.8539	0.8510
708	0.8740	0.8710	0.8680	0.8650	0.8621	0.8592	0.8563	0.8534

TABLE 2.19 Factors for Reducing Gas Volumes to Normal (Standard) Temperature and Pressure (*Continued*)

Pressure mm of mercury	Temperature °C							
	18°	19°	20°	21°	22°	23°	24°	25°
710	0.8764	0.8734	0.8704	0.8675	0.8645	0.8616	0.8587	0.8558
712	0.8789	0.8759	0.8729	0.8699	0.8670	0.8640	0.8611	0.8582
714	0.8814	0.8783	0.8753	0.8724	0.8694	0.8665	0.8636	0.8607
716	0.8838	0.8808	0.8778	0.8748	0.8718	0.8689	0.8660	0.8631
718	0.8863	0.8833	0.8802	0.8773	0.8743	0.8713	0.8684	0.8655
720	0.8888	0.8857	0.8827	0.8797	0.8767	0.8738	0.8708	0.8679
722	0.8912	0.8882	0.8852	0.8821	0.8792	0.8762	0.8732	0.8703
724	0.8937	0.8906	0.8876	0.8846	0.8816	0.8786	0.8757	0.8727
726	0.8962	0.8931	0.8901	0.8870	0.8840	0.8810	0.8781	0.8751
728	0.8986	0.8956	0.8925	0.8895	0.8865	0.8835	0.8805	0.8775
730	0.9011	0.8980	0.8950	0.8919	0.8889	0.8859	0.8829	0.8799
732	0.9036	0.9005	0.8974	0.8944	0.8913	0.8883	0.8853	0.8824
734	0.9060	0.9029	0.8999	0.8968	0.8938	0.8907	0.8877	0.8848
736	0.9085	0.9054	0.9023	0.8992	0.8962	0.8932	0.8902	0.8872
738	0.9110	0.9079	0.9048	0.9017	0.8986	0.8956	0.8926	0.8896
740	0.9135	0.9103	0.9072	0.9041	0.9011	0.8980	0.8950	0.8920
742	0.9159	0.9128	0.9097	0.9066	0.9035	0.9005	0.8974	0.8944
744	0.9184	0.9153	0.9121	0.9090	0.9059	0.9029	0.8998	0.8968
746	0.9209	0.9177	0.9146	0.9115	0.9084	0.9053	0.9023	0.8992
748	0.9233	0.9202	0.9170	0.9139	0.9108	0.9077	0.9047	0.9016
750	0.9258	0.9226	0.9195	0.9164	0.9132	0.9102	0.9071	0.9041
752	0.9283	0.9251	0.9219	0.9188	0.9157	0.9126	0.9095	0.9065
754	0.9307	0.9276	0.9244	0.9212	0.9181	0.9150	0.9119	0.9089
756	0.9332	0.9300	0.9268	0.9237	0.9206	0.9174	0.9144	0.9113
758	0.9357	0.9325	0.9293	0.9261	0.9230	0.9199	0.9168	0.9137
760	0.9381	0.9349	0.9317	0.9286	0.9254	0.9223	0.9192	0.9161
762	0.9406	0.9374	0.9342	0.9310	0.9279	0.9247	0.9216	0.9185
764	0.9431	0.9399	0.9366	0.9335	0.9303	0.9272	0.9240	0.9209
766	0.9456	0.9423	0.9391	0.9359	0.9327	0.9296	0.9265	0.9233
768	0.9480	0.9448	0.9415	0.9383	0.9352	0.9320	0.9289	0.9258
770	0.9505	0.9472	0.9440	0.9408	0.9376	0.9344	0.9313	0.9282
772	0.9530	0.9497	0.9464	0.9432	0.9400	0.9369	0.9337	0.9306
774	0.9554	0.9522	0.9489	0.9457	0.9425	0.9393	0.9361	0.9330
776	0.9579	0.9546	0.9514	0.9481	0.9449	0.9417	0.9385	0.9354
778	0.9604	0.9571	0.9538	0.9506	0.9473	0.9441	0.9410	0.9378
780	0.9628	0.9595	0.9563	0.9530	0.9498	0.9466	0.9434	0.9402
782	0.9653	0.9620	0.9587	0.9555	0.9522	0.9490	0.9458	0.9426
784	0.9678	0.9645	0.9612	0.9579	0.9546	0.9514	0.9482	0.9450
786	0.9702	0.9669	0.9636	0.9603	0.9571	0.9538	0.9506	0.9474
788	0.9727	0.9694	0.9661	0.9628	0.9595	0.9563	0.9531	0.9499

TABLE 2.19 Factors for Reducing Gas Volumes to Normal (Standard) Temperature and Pressure (*Continued*)

Pressure mm of mercury	Temperature °C							
	26°	27°	28°	29°	30°	31°	32°	33°
670	0.8049	0.8022	0.7996	0.7969	0.7943	0.7917	0.7891	0.7865
672	0.8073	0.8046	0.8020	0.7993	0.7967	0.7940	0.7914	0.7889
674	0.8097	0.8070	0.8043	0.8017	0.7990	0.7964	0.7938	0.7912
676	0.8121	0.8094	0.8067	0.8041	0.8014	0.7988	0.7962	0.7936
678	0.8145	0.8118	0.8091	0.8064	0.8038	0.8011	0.7985	0.7959
680	0.8169	0.8142	0.8115	0.8088	0.8061	0.8035	0.8009	0.7982
682	0.8193	0.8166	0.8139	0.8112	0.8085	0.8059	0.8032	0.8006
684	0.8217	0.8190	0.8163	0.8136	0.8109	0.8082	0.8056	0.8029
686	0.8241	0.8214	0.8187	0.8160	0.8133	0.8106	0.8079	0.8053
688	0.8265	0.8238	0.8211	0.8183	0.8156	0.8129	0.8103	0.8076
690	0.8289	0.8262	0.8234	0.8207	0.8180	0.8153	0.8126	0.8100
692	0.8313	0.8286	0.8258	0.8231	0.8204	0.8177	0.8150	0.8123
694	0.8338	0.8310	0.8282	0.8255	0.8227	0.8200	0.8174	0.8147
696	0.8362	0.8334	0.8306	0.8278	0.8251	0.8224	0.8197	0.8170
698	0.8386	0.8358	0.8330	0.8302	0.8275	0.8248	0.8221	0.8194
700	0.8410	0.8382	0.8354	0.8326	0.8299	0.8271	0.8244	0.8217
702	0.8434	0.8406	0.8378	0.8350	0.8322	0.8295	0.8268	0.8241
704	0.8458	0.8429	0.8401	0.8374	0.8346	0.8319	0.8291	0.8264
706	0.8482	0.8453	0.8425	0.8397	0.8370	0.8342	0.8315	0.8288
708	0.8506	0.8477	0.8449	0.8421	0.8393	0.8366	0.8338	0.8311
710	0.8530	0.8501	0.8473	0.8445	0.8417	0.8389	0.8362	0.8335
712	0.8554	0.8525	0.8497	0.8469	0.8441	0.8413	0.8386	0.8358
714	0.8578	0.8549	0.8521	0.8493	0.8465	0.8437	0.8409	0.8382
716	0.8602	0.8573	0.8545	0.8516	0.8488	0.8460	0.8433	0.8405
718	0.8626	0.8597	0.8569	0.8540	0.8512	0.8484	0.8456	0.8429
720	0.8650	0.8621	0.8592	0.8564	0.8536	0.8508	0.8480	0.8452
722	0.8674	0.8645	0.8616	0.8588	0.8559	0.8531	0.8503	0.8475
724	0.8698	0.8669	0.8640	0.8612	0.8583	0.8555	0.8527	0.8499
726	0.8722	0.8693	0.8664	0.8635	0.8607	0.8579	0.8550	0.8522
728	0.8746	0.8717	0.8688	0.8659	0.8631	0.8602	0.8574	0.8546
730	0.8770	0.8741	0.8712	0.8683	0.8654	0.8626	0.8598	0.8569
732	0.8794	0.8765	0.8736	0.8707	0.8678	0.8649	0.8621	0.8593
734	0.8818	0.8789	0.8759	0.8730	0.8702	0.8673	0.8645	0.8616
736	0.8842	0.8813	0.8783	0.8754	0.8725	0.8697	0.8668	0.8640
738	0.8866	0.8837	0.8807	0.8778	0.8749	0.8720	0.8692	0.8663
740	0.8890	0.8861	0.8831	0.8802	0.8773	0.8744	0.8715	0.8687
742	0.8914	0.8884	0.8855	0.8826	0.8796	0.8768	0.8739	0.8710
744	0.8938	0.8908	0.8879	0.8849	0.8820	0.8791	0.8762	0.8734
746	0.8962	0.8932	0.8903	0.8873	0.8844	0.8815	0.8786	0.8757
748	0.8986	0.8956	0.8927	0.8897	0.8868	0.8838	0.8809	0.8781
750	0.9010	0.8980	0.8950	0.8921	0.8891	0.8862	0.8833	0.8804
752	0.9034	0.9004	0.8974	0.8945	0.8915	0.8886	0.8857	0.8828
754	0.9058	0.9028	0.8998	0.8968	0.8939	0.8909	0.8880	0.8851
756	0.9082	0.9052	0.9022	0.8992	0.8962	0.8933	0.8904	0.8875
758	0.9106	0.9076	0.9046	0.9016	0.8986	0.8957	0.8927	0.8898

TABLE 2.19 Factors for Reducing Gas Volumes to Normal (Standard) Temperature and Pressure (*Continued*)

Pressure mm of mercury	Temperature °C							
	26°	27°	28°	29°	30°	31°	32°	33°
760	0.9130	0.9100	0.9070	0.9040	0.9010	0.8980	0.8951	0.8922
762	0.9154	0.9124	0.9094	0.9064	0.9034	0.9004	0.8974	0.8945
764	0.9178	0.9148	0.9118	0.9087	0.9057	0.9028	0.8998	0.8969
766	0.9202	0.9172	0.9141	0.9111	0.9081	0.9051	0.9021	0.8992
768	0.9227	0.9196	0.9165	0.9135	0.9105	0.9075	0.9045	0.9015
770	0.9251	0.9220	0.9189	0.9159	0.9128	0.9098	0.9069	0.9039
772	0.9275	0.9244	0.9213	0.9182	0.9152	0.9122	0.9092	0.9062
774	0.9299	0.9268	0.9237	0.9206	0.9176	0.9146	0.9116	0.9086
776	0.9323	0.9292	0.9261	0.9230	0.9200	0.9169	0.9139	0.9109
778	0.9347	0.9316	0.9285	0.9254	0.9223	0.9193	0.9163	0.9133
780	0.9371	0.9340	0.9308	0.9278	0.9247	0.9217	0.9186	0.9156
782	0.9395	0.9363	0.9332	0.9301	0.9271	0.9240	0.9210	0.9180
784	0.9419	0.9387	0.9356	0.9325	0.9294	0.9264	0.9233	0.9203
786	0.9443	0.9411	0.9380	0.9349	0.9318	0.9287	0.9257	0.9227
788	0.9467	0.9435	0.9404	0.9373	0.9342	0.9311	0.9281	0.9250

Pressure mm of mercury	Temperature °C		
	34°	35°	36°
670	0.7839	0.7814	0.7789
672	0.7863	0.7837	0.7812
674	0.7886	0.7861	0.7835
676	0.7910	0.7884	0.7858
678	0.7933	0.7907	0.7882
680	0.7956	0.7931	0.7905
682	0.7980	0.7954	0.7928
684	0.8003	0.7977	0.7951
686	0.8027	0.8001	0.7975
688	0.8050	0.8024	0.7998
690	0.8073	0.8047	0.8021
692	0.8097	0.8071	0.8044
694	0.8120	0.8094	0.8068
696	0.8144	0.8117	0.8091
698	0.8167	0.8141	0.8114
700	0.8190	0.8164	0.8137
702	0.8214	0.8187	0.8161
704	0.8237	0.8211	0.8184
706	0.8261	0.8234	0.8207
708	0.8284	0.8257	0.8230
710	0.8307	0.8281	0.8254
712	0.8331	0.8304	0.8277
714	0.8354	0.8327	0.8300
716	0.8378	0.8350	0.8323
718	0.8401	0.8374	0.8347
720	0.8424	0.8397	0.8370
722	0.8448	0.8420	0.8393
724	0.8471	0.8444	0.8416
726	0.8495	0.8467	0.8440
728	0.8518	0.8490	0.8463
730	0.8541	0.8514	0.8486
732	0.8565	0.8537	0.8509
734	0.8588	0.8560	0.8533
736	0.8612	0.8584	0.8556
738	0.8635	0.8607	0.8579
740	0.8658	0.8630	0.8602
742	0.8682	0.8654	0.8626
744	0.8705	0.8677	0.8649
746	0.8729	0.8700	0.8672
748	0.8752	0.8724	0.8695
750	0.8775	0.8747	0.8719
752	0.8799	0.8770	0.8742
754	0.8822	0.8794	0.8765
756	0.8846	0.8817	0.8788
758	0.8869	0.8840	0.8812
760	0.8892	0.8864	0.8835
762	0.8916	0.8887	0.8858
764	0.8939	0.8910	0.8881
766	0.8963	0.8934	0.8905
768	0.8986	0.8957	0.8928
770	0.9009	0.8980	0.8951
772	0.9033	0.9004	0.8974
774	0.9056	0.9027	0.8998
776	0.9080	0.9050	0.9021
778	0.9103	0.9074	0.9044
780	0.9127	0.9097	0.9067
782	0.9150	0.9120	0.9091
784	0.9173	0.9144	0.9114
786	0.9197	0.9167	0.9137
788	0.9220	0.9190	0.9160

TABLE 2.20 Values of Absorbance for Percent Absorption

To convert percent absorption (% *A*) to absorbance, find the present absorption to the nearest whole digit in the left-hand column; read across to the column located under the tenth of a percent desired, and read the value of absorbance. The value of absorbance corresponding to 26.8% absorption is thus 0.1355.

% *A*	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0.0	.0000	.0004	.0009	.0013	.0017	.0022	.0026	.0031	.0035	.0039
1.0	.0044	.0048	.0052	.0057	.0061	.0066	.0070	.0074	.0079	.0083
2.0	.0088	.0092	.0097	.0101	.0106	.0110	.0114	.0119	.0123	.0128
3.0	.0132	.0137	.0141	.0146	.0150	.0155	.0159	.0164	.0168	.0173
4.0	.0177	.0182	.0186	.0191	.0195	.0200	.0205	.0209	.0214	.0218
5.0	.0223	.0227	.0232	.0236	.0241	.0246	.0250	.0255	.0259	.0264
6.0	.0269	.0273	.0278	.0283	.0287	.0292	.0297	.0301	.0306	.0311
7.0	.0315	.0320	.0325	.0329	.0334	.0339	.0343	.0348	.0353	.0357
8.0	.0362	.0367	.0372	.0376	.0381	.0386	.0391	.0395	.0400	.0405
9.0	.0410	.0414	.0419	.0424	.0429	.0434	.0438	.0443	.0448	.0453
10.0	.0458	.0462	.0467	.0472	.0477	.0482	.0487	.0491	.0496	.0501
11.0	.0506	.0511	.0516	.0521	.0526	.0531	.0535	.0540	.0545	.0550
12.0	.0555	.0560	.0565	.0570	.0575	.0580	.0585	.0590	.0595	.0600
13.0	.0605	.0610	.0615	.0620	.0625	.0630	.0635	.0640	.0645	.0650
14.0	.0655	.0660	.0665	.0670	.0675	.0680	.0685	.0691	.0696	.0701
15.0	.0706	.0711	.0716	.0721	.0726	.0731	.0737	.0742	.0747	.0752
16.0	.0757	.0762	.0768	.0773	.0778	.0783	.0788	.0794	.0799	.0804
17.0	.0809	.0814	.0820	.0825	.0830	.0835	.0841	.0846	.0851	.0857
18.0	.0862	.0867	.0872	.0878	.0883	.0888	.0894	.0899	.0904	.0910
19.0	.0915	.0921	.0926	.0931	.0937	.0942	.0947	.0953	.0958	.0964
20.0	.0969	.0975	.0980	.0985	.0991	.0996	.1002	.1007	.1013	.1018
21.0	.1024	.1029	.1035	.1040	.1046	.1051	.1057	.1062	.1068	.1073
22.0	.1079	.1085	.1090	.1096	.1101	.1107	.1113	.1118	.1124	.1129
23.0	.1135	.1141	.1146	.1152	.1158	.1163	.1169	.1175	.1180	.1186
24.0	.1192	.1198	.1203	.1209	.1215	.1221	.1226	.1232	.1238	.1244
25.0	.1249	.1255	.1261	.1267	.1273	.1278	.1284	.1290	.1296	.1302
26.0	.1308	.1314	.1319	.1325	.1331	.1337	.1343	.1349	.1355	.1361
27.0	.1367	.1373	.1379	.1385	.1391	.1397	.1403	.1409	.1415	.1421
28.0	.1427	.1433	.1439	.1445	.1451	.1457	.1463	.1469	.1475	.1481
29.0	.1487	.1494	.1500	.1506	.1512	.1518	.1524	.1530	.1537	.1543
30.0	.1549	.1555	.1561	.1568	.1574	.1580	.1586	.1593	.1599	.1605
31.0	.1612	.1618	.1624	.1630	.1637	.1643	.1649	.1656	.1662	.1669
32.0	.1675	.1681	.1688	.1694	.1701	.1707	.1713	.1720	.1726	.1733
33.0	.1739	.1746	.1752	.1759	.1765	.1772	.1778	.1785	.1791	.1798
34.0	.1805	.1811	.1818	.1824	.1831	.1838	.1844	.1851	.1858	.1864
35.0	.1871	.1878	.1884	.1891	.1898	.1904	.1911	.1918	.1925	.1931
36.0	.1938	.1945	.1952	.1959	.1965	.1972	.1979	.1986	.1993	.2000
37.0	.2007	.2013	.2020	.2027	.2034	.2041	.2048	.2055	.2062	.2069
38.0	.2076	.2083	.2090	.2097	.2104	.2111	.2118	.2125	.2132	.2140
39.0	.2147	.2154	.2161	.2168	.2175	.2182	.2190	.2197	.2204	.2211
40.0	.2218	.2226	.2233	.2240	.2248	.2255	.2262	.2269	.2277	.2284
41.0	.2291	.2299	.2306	.2314	.2321	.2328	.2336	.2343	.2351	.2358

TABLE 2.20 Values of Absorbance for Percent Absorption (*Continued*)

% *A*	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
42.0	.2366	.2373	.2381	.2388	.2396	.2403	.2411	.2418	.2426	.2434
43.0	.2441	.2449	.2457	.2464	.2472	.2480	.2487	.2495	.2503	.2510
44.0	.2518	.2526	.2534	.2541	.2549	.2557	.2565	.2573	.2581	.2588
45.0	.2596	.2604	.2612	.2620	.2628	.2636	.2644	.2652	.2660	.2668
46.0	.2676	.2684	.2692	.2700	.2708	.2716	.2725	.2733	.2741	.2749
47.0	.2757	.2765	.2774	.2782	.2790	.2798	.2807	.2815	.2823	.2832
48.0	.2840	.2848	.2857	.2865	.2874	.2882	.2890	.2899	.2907	.2916
49.0	.2924	.2933	.2941	.2950	.2958	.2967	.2976	.2984	.2993	.3002
50.0	.3010	.3019	.3028	.3036	.3045	.3054	.3063	.3072	.3080	.3089
51.0	.3098	.3107	.3116	.3125	.3134	.3143	.3152	.3161	.3170	.3179
52.0	.3188	.3197	.3206	.3215	.3224	.3233	.3242	.3251	.3261	.3270
53.0	.3279	.3288	.3298	.3307	.3316	.3325	.3335	.3344	.3354	.3363
54.0	.3372	.3382	.3391	.3401	.3410	.3420	.3429	.3439	.3449	.3458
55.0	.3468	.3478	.3487	.3497	.3507	.3516	.3526	.3536	.3546	.3556
56.0	.3565	.3575	.3585	.3595	.3605	.3615	.3625	.3635	.3645	.3655
57.0	.3665	.3675	.3686	.3696	.3706	.3716	.3726	.3737	.3747	.3757
58.0	.3768	.3778	.3788	.3799	.3809	.3820	.3830	.3840	.3851	.3862
59.0	.3872	.3883	.3893	.3904	.3915	.3925	.3936	.3947	.3958	.3969
60.0	.3979	.3990	.4001	.4012	.4023	.4034	.4045	.4056	.4067	.4078
61.0	.4089	.4101	.4112	.4123	.4134	.4145	.4157	.4168	.4179	.4191
62.0	.4202	.4214	.4225	.4237	.4248	.4260	.4271	.4283	.4295	.4306
63.0	.4318	.4330	.4342	.4353	.4365	.4377	.4389	.4401	.4413	.4425
64.0	.4437	.4449	.4461	.4473	.4485	.4498	.4510	.4522	.4535	.4547
65.0	.4559	.4572	.4584	.4597	.4609	.4622	.4634	.4647	.4660	.4672
66.0	.4685	.4698	.4711	.4724	.4737	.4750	.4763	.4776	.4789	.4802
67.0	.4815	.4828	.4841	.4855	.4868	.4881	.4895	.4908	.4921	.4935
68.0	.4948	.4962	.4976	.4989	.5003	.5017	.5031	.5045	.5058	.5072
69.0	.5086	.5100	.5114	.5129	.5143	.5157	.5171	.5186	.5200	.5214
70.0	.5229	.5243	.5258	.5272	.5287	.5302	.5317	.5331	.5346	.5361
71.0	.5376	.5391	.5406	.5421	.5436	.5452	.5467	.5482	.5498	.5513
72.0	.5528	.5544	.5560	.5575	.5591	.5607	.5622	.5638	.5654	.5670
73.0	.5686	.5702	.5719	.5735	.5751	.5768	.5784	.5800	.5817	.5834
74.0	.5850	.5867	.5884	.5901	.5918	.5935	.5952	.5969	.5986	.6003
75.0	.6021	.6038	.6055	.6073	.6091	.6108	.6126	.6144	.6162	.6180
76.0	.6198	.6216	.6234	.6253	.6271	.6289	.6308	.6326	.6345	.6364
77.0	.6383	.6402	.6421	.6440	.6459	.6478	.6498	.6517	.6536	.6556
78.0	.6576	.6596	.6615	.6635	.6655	.6676	.6696	.6716	.6737	.6757
79.0	.6778	.6799	.6819	.6840	.6861	.6882	.6904	.6925	.6946	.6968
80.0	.6990	.7011	.7033	.7055	.7077	.7100	.7122	.7144	.7167	.7190
81.0	.7212	.7235	.7258	.7282	.7305	.7328	.7352	.7375	.7399	.7423
82.0	.7447	.7471	.7496	.7520	.7545	.7570	.7595	.7620	.7645	.7670
83.0	.7696	.7721	.7747	.7773	.7799	.7825	.7852	.7878	.7905	.7932
84.0	.7959	.7986	.8013	.8041	.8069	.8097	.8125	.8153	.8182	.8210
85.0	.8239	.8268	.8297	.8327	.8356	.8386	.8416	.8447	.8477	.8508
86.0	.8539	.8570	.8601	.8633	.8665	.8697	.8729	.8761	.8794	.8827
87.0	.8861	.8894	.8928	.8962	.8996	.9031	.9066	.9101	.9136	.9172
88.0	.9208	.9245	.9281	.9318	.9355	.9393	.9431	.9469	.9508	.9547
89.0	.9586	.9626	.9666	.9706	.9747	.9788	.9830	.9872	.9914	.9957

TABLE 2.21 Transmittance-Absorbance Conversion Table

From Meites, Handbook of Analytical Chemistry, *1963, McGraw-Hill Book Company; by permission.*

This table gives absorbance values to four significant figures corresponding to % transmittance values which are given to three significant figures. The values of % transmittance are given in the left-hand column and in the top row. For example, 8.4% transmittance corresponds to an absorbance of 1.076.

Interpolation is facilitated and accuracy is maximized if the % transmittance is between 1 and 10, by multiplying its value by 10, finding the absorbance corresponding to the result, and adding 1. For example, to find the absorbance corresponding to 8.45% transmittance, note that 84.5% transmittance corresponds to an absorbance of 0.0731, so that 8.45% transmittance corresponds to an absorbance of 1.0731. For % transmittance values between 0.1 and 1, multiply by 100, find the absorbance corresponding to the result, and add 2.

Conversely, to find the % transmittance corresponding to an absorbance between 1 and 2, subtract 1 from the absorbance, find the % transmittance corresponding to the result, and divide by 10. For example, an absorbance of 1.219 can best be converted to % transmittance by noting that an absorbance of 0.219 would correspond to 60.4% transmittance; dividing this by 10 gives the desired value, 6.04% transmittance. For absorbance values between 2 and 3, subtract 2 from the absorbance, find the % transmittance corresponding to the result, and divide by 100.

% Trans-mittance	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	. . . . . .	3.000	2.699	2.523	2.398	2.301	2.222	2.155	2.097	2.046
1	2.000	1.959	1.921	1.886	1.854	1.824	1.796	1.770	1.745	1.721
2	1.699	1.678	1.658	1.638	1.620	1.602	1.585	1.569	1.553	1.538
3	1.523	1.509	1.495	1.481	1.469	1.456	1.444	1.432	1.420	1.409
4	1.398	1.387	1.377	1.367	1.357	1.347	1.337	1.328	1.319	1.310
5	1.301	1.292	1.284	1.276	1.268	1.260	1.252	1.244	1.237	1.229
6	1.222	1.215	1.208	1.201	1.194	1.187	1.180	1.174	1.167	1.161
7	1.155	1.149	1.143	1.137	1.131	1.125	1.119	1.114	1.108	1.102
8	1.097	1.092	1.086	1.081	1.076	1.071	1.066	1.060	1.056	1.051
9	1.046	1.041	1.036	1.032	1.027	1.022	1.018	1.013	1.009	1.004
10	1.000	0.9957	0.9914	0.9872	0.9830	0.9788	0.9747	0.9706	0.9666	0.9626
11	0.9586	0.9547	0.9508	0.9469	0.9431	0.9393	0.9355	0.9318	0.9281	0.9245
12	0.9208	0.9172	0.9136	0.9101	0.9066	0.9031	0.8996	0.8962	0.8928	0.8894
13	0.8861	0.8827	0.8794	0.8761	0.8729	0.8697	0.8665	0.8633	0.8601	0.8570
14	0.8539	0.8508	0.8477	0.8447	0.8416	0.8386	0.8356	0.8327	0.8297	0.8268
15	0.8239	0.8210	0.8182	0.8153	0.8125	0.8097	0.8069	0.8041	0.8013	0.7986
16	0.7959	0.7932	0.7905	0.7878	0.7852	0.7825	0.7799	0.7773	0.7747	0.7721
17	0.7696	0.7670	0.7645	0.7620	0.7595	0.7570	0.7545	0.7520	0.7496	0.7471
18	0.7447	0.7423	0.7399	0.7375	0.7352	0.7328	0.7305	0.7282	0.7258	0.7235
19	0.7212	0.7190	0.7167	0.7144	0.7122	0.7100	0.7077	0.7055	0.7033	0.7011
20	0.6990	0.6968	0.6946	0.6925	0.6904	0.6882	0.6861	0.6840	0.6819	0.6799
21	0.6778	0.6757	0.6737	0.6716	0.6696	0.6676	0.6655	0.6635	0.6615	0.6596
22	0.6576	0.6556	0.6536	0.6517	0.6498	0.6478	0.6459	0.6440	0.6421	0.6402
23	0.6383	0.6364	0.6345	0.6326	0.6308	0.6289	0.6271	0.6253	0.6234	0.6216
24	0.6198	0.6180	0.6162	0.6144	0.6126	0.6108	0.6091	0.6073	0.6055	0.6038
25	0.6021	0.6003	0.5986	0.5969	0.5952	0.5935	0.5918	0.5901	0.5884	0.5867
26	0.5850	0.5834	0.5817	0.5800	0.5784	0.5766	0.5751	0.5735	0.5719	0.5702
27	0.5686	0.5670	0.5654	0.5638	0.5622	0.5607	0.5591	0.5575	0.5560	0.5544
28	0.5528	0.5513	0.5498	0.5482	0.5467	0.5452	0.5436	0.5421	0.5406	0.5391
29	0.5376	0.5361	0.5346	0.5331	0.5317	0.5302	0.5287	0.5272	0.5258	0.5243
30	0.5229	0.5214	0.5200	0.5186	0.5171	0.5157	0.5143	0.5129	0.5114	0.5100

TABLE 2.21 Transmittance-Absorbance Conversion Table (*Continued*)

% Trans- mittance	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
31	0.5086	0.5072	0.5058	0.5045	0.5031	0.5017	0.5003	0.4989	0.4976	0.4962
32	0.4949	0.4935	0.4921	0.4908	0.4895	0.4881	0.4868	0.4855	0.4841	0.4828
33	0.4815	0.4802	0.4789	0.4776	0.4763	0.4750	0.4737	0.4724	0.4711	0.4698
34	0.4685	0.4672	0.4660	0.4647	0.4634	0.4622	0.4609	0.4597	0.4584	0.4572
35	0.4559	0.4547	0.4535	0.4522	0.4510	0.4498	0.4486	0.4473	0.4461	0.4449
36	0.4437	0.4425	0.4413	0.4401	0.4389	0.4377	0.4365	0.4353	0.4342	0.4330
37	0.4318	0.4306	0.4295	0.4283	0.4271	0.4260	0.4248	0.4237	0.4225	0.4214
38	0.4202	0.4191	0.4179	0.4168	0.4157	0.4145	0.4134	0.4123	0.4112	0.4101
39	0.4089	0.4078	0.4067	0.4056	0.4045	0.4034	0.4023	0.4012	0.4001	0.3989
40	0.3979	0.3969	0.3958	0.3947	0.3936	0.3925	0.3915	0.3904	0.3893	0.3883
41	0.3872	0.3862	0.3851	0.3840	0.3830	0.3820	0.3809	0.3799	0.3788	0.3778
42	0.3768	0.3757	0.3747	0.3737	0.3726	0.3716	0.3706	0.3696	0.3686	0.3675
43	0.3665	0.3655	0.3645	0.3635	0.3625	0.3615	0.3605	0.3595	0.3585	0.3575
44	0.3565	0.3556	0.3546	0.3536	0.3526	0.3516	0.3507	0.3497	0.3487	0.3478
45	0.3468	0.3458	0.3449	0.3439	0.3429	0.3420	0.3410	0.3401	0.3391	0.3382
46	0.3372	0.3363	0.3354	0.3344	0.3335	0.3325	0.3316	0.3307	0.3298	0.3288
47	0.3279	0.3270	0.3261	0.3251	0.3242	0.3233	0.3224	0.3215	0.3206	0.3197
48	0.3188	0.3179	0.3170	0.3161	0.3152	0.3143	0.3134	0.3125	0.3116	0.3107
49	0.3098	0.3089	0.3080	0.3072	0.3063	0.3054	0.3045	0.3036	0.3028	0.3019
50	0.3010	0.3002	0.2993	0.2984	0.2976	0.2967	0.2958	0.2950	0.2941	0.2933
51	0.2924	0.2916	0.2907	0.2899	0.2890	0.2882	0.2874	0.2865	0.2857	0.2848
52	0.2840	0.2832	0.2823	0.2815	0.2807	0.2798	0.2790	0.2782	0.2774	0.2765
53	0.2757	0.2749	0.2741	0.2733	0.2725	0.2716	0.2708	0.2700	0.2692	0.2684
54	0.2676	0.2668	0.2660	0.2652	0.2644	0.2636	0.2628	0.2620	0.2612	0.2604
55	0.2596	0.2588	0.2581	0.2573	0.2565	0.2557	0.2549	0.2541	0.2534	0.2526
56	0.2518	0.2510	0.2503	0.2495	0.2487	0.2480	0.2472	0.2464	0.2457	0.2449
57	0.2441	0.2434	0.2426	0.2418	0.2411	0.2403	0.2396	0.2388	0.2381	0.2373
58	0.2366	0.2358	0.2351	0.2343	0.2336	0.2328	0.2321	0.2314	0.2306	0.2299
59	0.2291	0.2284	0.2277	0.2269	0.2262	0.2255	0.2248	0.2240	0.2233	0.2226
60	0.2218	0.2211	0.2204	0.2197	0.2190	0.2182	0.2175	0.2168	0.2161	0.2154
61	0.2147	0.2140	0.2132	0.2125	0.2118	0.2111	0.2104	0.2097	0.2090	0.2083
62	0.2076	0.2069	0.2062	0.2055	0.2048	0.2041	0.2034	0.2027	0.2020	0.2013
63	0.2007	0.2000	0.1993	0.1986	0.1979	0.1972	0.1965	0.1959	0.1952	0.1945
64	0.1938	0.1931	0.1925	0.1918	0.1911	0.1904	0.1898	0.1891	0.1884	0.1878
65	0.1871	0.1864	0.1858	0.1851	0.1844	0.1838	0.1831	0.1824	0.1818	0.1811
66	0.1805	0.1798	0.1791	0.1785	0.1778	0.1772	0.1765	0.1759	0.1752	0.1746
67	0.1739	0.1733	0.1726	0.1720	0.1713	0.1707	0.1701	0.1694	0.1688	0.1681
68	0.1675	0.1669	0.1662	0.1656	0.1649	0.1643	0.1637	0.1630	0.1624	0.1618
69	0.1612	0.1605	0.1599	0.1593	0.1586	0.1580	0.1574	0.1568	0.1561	0.1555
70	0.1549	0.1543	0.1537	0.1530	0.1524	0.1518	0.1512	0.1506	0.1500	0.1494
71	0.1487	0.1481	0.1475	0.1469	0.1463	0.1457	0.1451	0.1445	0.1439	0.1433
72	0.1427	0.1421	0.1415	0.1409	0.1403	0.1397	0.1391	0.1385	0.1379	0.1373
73	0.1367	0.1361	0.1355	0.1349	0.1343	0.1337	0.1331	0.1325	0.1319	0.1314

TABLE 2.21 Transmittance-Absorbance Conversion Table (*Continued*)

% Trans-mittance	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
74	0.1308	0.1302	0.1296	0.1290	0.1284	0.1278	0.1273	0.1267	0.1261	0.1255
75	0.1249	0.1244	0.1238	0.1232	0.1226	0.1221	0.1215	0.1209	0.1203	0.1198
76	0.1192	0.1186	0.1180	0.1175	0.1169	0.1163	0.1158	0.1152	0.1146	0.1141
77	0.1135	0.1129	0.1124	0.1118	0.1113	0.1107	0.1101	0.1096	0.1090	0.1085
78	0.1079	0.1073	0.1068	0.1062	0.1057	0.1051	0.1046	0.1040	0.1035	0.1029
79	0.1024	0.1018	0.1013	0.1007	0.1002	0.0996	0.0991	0.0985	0.0980	0.0975
80	0.0969	0.0964	0.0958	0.0953	0.0947	0.0942	0.0937	0.0931	0.0926	0.0921
81	0.0915	0.0910	0.0904	0.0899	0.0894	0.0888	0.0883	0.0878	0.0872	0.0867
82	0.0862	0.0857	0.0851	0.0846	0.0841	0.0835	0.0830	0.0825	0.0820	0.0814
83	0.0809	0.0804	0.0799	0.0794	0.0788	0.0783	0.0778	0.0773	0.0768	0.0762
84	0.0757	0.0752	0.0747	0.0742	0.0737	0.0731	0.0726	0.0721	0.0716	0.0711
85	0.0706	0.0701	0.0696	0.0691	0.0685	0.0680	0.0675	0.0670	0.0665	0.0660
86	0.0655	0.0650	0.0645	0.0640	0.0635	0.0630	0.0625	0.0620	0.0615	0.0610
87	0.0605	0.0600	0.0595	0.0590	0.0585	0.0580	0.0575	0.0570	0.0565	0.0560
88	0.0555	0.0550	0.0545	0.0540	0.0535	0.0531	0.0526	0.0521	0.0516	0.0511
89	0.0506	0.0501	0.0496	0.0491	0.0487	0.0482	0.0477	0.0472	0.0467	0.0462
90	0.0458	0.0453	0.0448	0.0443	0.0438	0.0434	0.0429	0.0424	0.0419	0.0414
91	0.0410	0.0405	0.0400	0.0395	0.0391	0.0386	0.0381	0.0376	0.0372	0.0367
92	0.0362	0.0357	0.0353	0.0348	0.0343	0.0339	0.0334	0.0329	0.0325	0.0320
93	0.0315	0.0311	0.0306	0.0301	0.0297	0.0292	0.0287	0.0283	0.0278	0.0273
94	0.0269	0.0264	0.0259	0.0255	0.0250	0.0246	0.0241	0.0237	0.0232	0.0227
95	0.0223	0.0218	0.0214	0.0209	0.0205	0.0200	0.0195	0.0191	0.0186	0.0182
96	0.0177	0.0173	0.0168	0.0164	0.0159	0.0155	0.0150	0.0146	0.0141	0.0137
97	0.0132	0.0128	0.0123	0.0119	0.0114	0.0110	0.0106	0.0101	0.0097	0.0092
98	0.0088	0.0083	0.0079	0.0074	0.0070	0.0066	0.0061	0.0057	0.0052	0.0048
99	0.0044	0.0039	0.0035	0.0031	0.0026	0.0022	0.0017	0.0013	0.0009	0.0004

TABLE 2.22 Wavenumber/Wavelength Conversion Table

This table is based on the conversion: wavenumber (in cm^{-1}) = 10 000/wavelength (in μm). For example, 15.4 μm is equal to 649 cm^{-1}.

Wavelength (μm)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9 cm^{-1}
1.0	10000	9091	8333	7692	7143	6667	6250	5882	5556	5263
2.0	5000	4762	4545	4348	4167	4000	3846	3704	3571	3448
3.0	3333	3226	3125	3030	2941	2857	2778	2703	2632	2564
4.0	2500	2439	2381	2326	2273	2222	2174	2128	2083	2041
5.0	2000	1961	1923	1887	1852	1818	1786	1754	1724	1695
6.0	1667	1639	1613	1587	1563	1538	1515	1493	1471	1449
7.0	1429	1408	1389	1370	1351	1333	1316	1299	1282	1266
8.0	1250	1235	1220	1205	1190	1176	1163	1149	1136	1124
9.0	1111	1099	1087	1075	1064	1053	1042	1031	1020	1010
10.0	1000	990	980	971	962	952	943	935	926	917
11.0	909	901	893	885	877	870	862	855	847	840
12.0	833	826	820	813	806	800	794	787	781	775
13.0	769	763	758	752	746	741	735	730	725	719
14.0	714	709	704	699	694	690	685	680	676	671
15.0	667	662	658	654	649	645	641	637	633	629
16.0	625	621	617	613	610	606	602	599	595	592
17.0	588	585	581	578	575	571	568	565	562	559
18.0	556	552	549	546	543	541	538	535	532	529
19.0	526	524	521	518	515	513	510	508	505	503
20.0	500	498	495	493	490	488	485	483	481	478
21.0	476	474	472	469	467	465	463	461	459	457
22.0	455	452	450	448	446	444	442	441	439	437
23.0	435	433	431	429	427	426	424	422	420	418
24.0	417	415	413	412	410	408	407	405	403	402
25.0	400	398	397	395	394	392	391	389	388	386
26.0	385	383	382	380	379	377	376	375	373	372
27.0	370	369	368	366	365	364	362	361	360	358
28.0	357	356	355	353	352	351	350	348	347	346
29.0	345	344	342	341	340	339	338	337	336	334
30.0	333	332	331	330	329	328	327	326	325	324
31.0	323	322	321	319	318	317	316	315	314	313
32.0	313	312	311	310	309	308	307	306	305	304
33.0	303	302	301	300	299	299	298	297	296	295
34.0	294	293	292	292	291	290	289	288	287	287
35.0	286	285	284	283	282	282	281	280	279	279
36.0	278	277	276	275	274	274	273	272	272	271
37.0	270	270	269	268	267	267	266	265	265	264
38.0	263	262	262	261	260	260	259	258	258	257
39.0	256	256	255	254	254	253	253	252	251	251
40.0	250									

2.2 MATHEMATICAL TABLES

2.2.1 Logarithms

2.2.1.1 Properties and Uses

Definition of Logarithm. The *logarithm x* of the number N to the base b is the exponent of the power to which b must be raised to give N. That is,

$$\log_b N = x \qquad \text{or} \qquad b^x = N$$

The number N is positive and b may be any positive number except 1.

Properties of Logarithms

1. The logarithm of a product is equal to the sum of the logarithms of the factors; thus,

$$\log_b M \cdot N = \log_b M + \log_b N$$

2. The logarithm of a quotient is equal to the logarithm of the numerator minus the logarithm of the denominator; thus,

$$\log_b \frac{M}{N} = \log_b M - \log_b N$$

3. The logarithm of a power of a number is equal to the logarithm of the base multiplied by the exponent of the power; thus,

$$\log_b M^p = p \cdot \log_b M$$

4. The logarithm of a root of a number is equal to the logarithm of the number divided by the index of the root; thus

$$\log_b \sqrt[q]{M} = \frac{1}{q} \log_b M$$

Other properties of logarithms:

$$\log_b b = 1 \qquad \log_b \sqrt[q]{M^p} = \frac{p}{q} \log_b M$$

$$\log_b 1 = 0 \qquad \log_b N = \log_a N \cdot \log_b a = \frac{\log_a N}{\log_a b}$$

$$\log_b (b^N) = N \qquad b^{\log_b N} = N$$

Systems of Logarithms. There are two common systems of logarithms in use: (1) the *natural* (Napierian or hyperbolic) system which uses the base $e = 2.71828 \ldots$; (2) the *common* (Briggsian) system which uses the base 10.

We shall use the abbreviation $\log N \equiv \log_{10} N$ in this section.

Unless otherwise stated, tables of logarithms are always tables of common logarithms.

Characteristic of a Common Logarithm of a Number. Every real positive number has a real common logarithm such that if $a < b$, $\log a < \log b$. Neither zero nor any negative number has a real logarithm.

A common logarithm, in general, consists of an integer, which is called the *characteristic,* and a decimal (usually endless), which is called the *mantissa.* The characteristic of any number may be determined from the following rules:

Rule I. The characteristic of any number greater than 1 is one less than the number of digits before the decimal point.

*Rule II.** The characteristic of a number less than 1 is found by subtracting from 9 the number of ciphers between the decimal point and the first significant digit, and writing −10 after the result.

Thus the characteristic of log 936 is 2; the characteristic of log 9.36 is 0; of log 0.936 is 9 − 10; of log 0.00936 is 7 − 10.

Mantissa of a Common Logarithm of a Number. An important consequence of the use of base 10 is that the mantissa of a number is independent of the position of the decimal point. Thus 93 600, 93.600, 0.000 936, all have the same mantissa. Hence in Tables of Common Logarithms only mantissas are given. A five-place table gives the values of the mantissa correct to five places of decimals.

Since it is possible to obtain logarithms by using hand calculators, this Handbook contains no logarithm tables.

Helpful Hints

1. When connecting numbers to logarithms, use as many decimal places in the mantissa as there are significant digits in the number.

2. When finding the antilogarithm, keep as many significant digits as there are decimal places in the mantissa.

Examples: log 10.35 = 1.0149; antilog 0.065 = 1.16.

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* Some writers use a dash over the characteristic to indicate a negative value; for example,

$$\log 0.004657 = 7.6681 - 10 = \overline{3}.6681$$

TABLE 2.23 Derivatives and Differentiation

Rules for differentiation.

From Baumeister and Marks, *Standard Handbook for Mechanical Engineers,* 7th ed., McGraw-Hill Book Company, New York (1967); by permission.

To find the derivative of a given function at a given point: (1) If the function is given only by a curve, measure graphically the slope of the tangent at the point in question; (2) if the function is given by a mathematical expression, use the following rules for differentiation. These rules give, directly, the differential, dy, in terms of dx; to find the derivative, dy/dx, divide through by dx.

Here $u, v, w, \ldots$ represents any functions of a variable x, or may themselves be independent variables. a is a constant which does not change in values in the same discussion; $e = 2.71828$.

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1. $d(a + u) = du$

2. $d(au) = a\,du$

3. $d(u + v + w + \cdots) = du + dv + dw + \cdots$

4. $d(uv) = u\,dv + v\,du$

5. $d(uvw\ldots) = (uvw\ldots)\left(\dfrac{du}{u} + \dfrac{dv}{v} + \dfrac{dw}{w} + \cdots\right)$

6. $d\,\dfrac{u}{v} = \dfrac{v\,du - u\,dv}{v^2}$

7. $d(u^m) = mu^{m-1}\,du$

Thus, $d(u^2) = 2u\,du$; $d(u^3) = 3u^2\,du$; etc.

8. $d\sqrt{u} = \dfrac{du}{2\sqrt{u}}$

9. $d\left(\dfrac{1}{u}\right) = -\dfrac{du}{u^2}$

10. $d(e^u) = e^u\,du$

11. $d(a^u) = (\ln a)a^u\,du$

12. $d\ln u = \dfrac{du}{u}$

13. $d\log_{10} u = (\log_{10} e)\,\dfrac{du}{u} = (0.4343\ldots)\,\dfrac{du}{u}$

14. $d\sin u = \cos u\,du$

15. $d\csc u = -\cot u\csc u\,du$

16. $d\cos u = -\sin u\,du$

17. $d\sec u = \tan u\sec u\,du$

18. $d\tan u = \sec^2 u\,du$

19. $d\cot u = -\csc^2 u\,du$

20. $d\sin^{-1} u = \dfrac{du}{\sqrt{1 - u^2}}$

21. $d\csc^{-1} u = -\dfrac{du}{u\sqrt{u^2 - 1}}$

22. $d\cos^{-1} u = -\dfrac{du}{\sqrt{1 - u^2}}$

23. $d\sec^{-1} u = \dfrac{du}{u\sqrt{u^2 - 1}}$

24. $d\tan^{-1} u = \dfrac{du}{1 + u^2}$

25. $d\cot^{-1} u = -\dfrac{du}{1 + u^2}$

26. $d\ln\sin u = \cot u\,du$

27. $d\ln\tan u = \dfrac{2\,du}{\sin 2u}$

28. $d\ln\cos u = -\tan u\,du$

29. $d\ln\cot u = -\dfrac{2\,du}{\sin 2u}$

30. $d\sinh u = \cosh u\,du$

31. $d\,\mathrm{csch}\,u = -\mathrm{csch}\,u\coth u\,du$

32. $d\cosh u = \sinh u\,du$

33. $d\,\mathrm{sech}\,u = -\mathrm{sech}\,u\tanh u\,du$

34. $d\tanh u = \mathrm{sech}^2\,u\,du$

35. $d\coth u = -\mathrm{csch}^2\,u\,du$

36. $d\sinh^{-1} u = \dfrac{du}{\sqrt{u^2 + 1}}$

37. $d\,\mathrm{csch}^{-1}\,u = -\dfrac{du}{u\sqrt{u^2 + 1}}$

38. $d\cosh^{-1} u = \dfrac{du}{\sqrt{u^2 - 1}}$

39. $d\,\mathrm{sech}^{-1}\,u = -\dfrac{du}{u\sqrt{1 - u^2}}$

40. $d\tanh^{-1} u = \dfrac{du}{1 - u^2}$

41. $d\coth^{-1} u = \dfrac{du}{1 - u^2}$

42. $d(u^v) = (u^{v-1})(u\ln u\,dv + v\,du)$

TABLE 2.24 Integrals

From Baumeister and Marks, Standard Handbook for Mechanical Engineers, *7th ed., McGraw-Hill Book Company, New York (1967); by permission.*

An integral of $f(x)\,dx$ is any function whose differential is $f(x)\,dx$, and is denoted by $\int f(x)\,dx$. All the integrals of $f(x)\,dx$ are included in the expression $\int f(x)\,dx + C$, where $\int f(x)\,dx$ is any particular integral, and C is an arbitrary constant. The process of finding (when possible) an integral of a given function consists in recognizing by inspection a function which, when differentiated, will produce the given function; or in transforming the given function into a form in which such recognition is easy. The most common integrable forms are collected in the following brief table; for a more extended list, see Peirce, *Table of Integrals,* Ginn, or Dwight, *Table of Integrals and other Mathematical Data,* Macmillan.

General formulas

1. $\int a\,du = a\int du = au + C$

2. $\int (u + v)\,dx = \int u\,dx + \int v\,dx$

3. $\int u\,dv = uv - \int v\,du$

4. $\int f(x)\,dx = \int f[F(y)]F'(y)\,dy,\ x = F(y)$

5. $\int dy \int f(x, y)\,dx = \int dx \int f(x, y)\,dy$

Fundamental integrals

6. $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C$, when $n \neq -1$

7. $\int \dfrac{dx}{x} = \ln x + C = \ln cx$

8. $\int e^x\,dx = e^x + C$

9. $\int \sin x\,dx = -\cos x + C$

10. $\int \cos x\,dx = \sin x + C$

11. $\int \dfrac{dx}{\sin^2 x} = -\cot x + C$

12. $\int \dfrac{dx}{\cos^2 x} = \tan x + C$

13. $\int \dfrac{dx}{\sqrt{1 - x^2}} = \sin^{-1} x + C = -\cos^{-1} x + c$

14. $\int \dfrac{dx}{1 + x^2} = \tan^{-1} x + C = -\cot^{-1} x + c$

Rational functions

15. $\int (a + bx)^n\,dx = \dfrac{(a + bx)^{n+1}}{(n + 1)b} + C$

16. $\int \dfrac{dx}{a + bx} = \dfrac{1}{b}\ln(a + bx) + C = \dfrac{1}{b}\ln c(a + bx)$

17. $\int \dfrac{1}{x^2}\,dx = -\dfrac{1}{x} + C$

18. $\int \dfrac{dx}{(a + bx)^2} = -\dfrac{1}{b(a + bx)} + C$

19. $\int \dfrac{dx}{1 - x^2} = \tfrac{1}{2}\ln\dfrac{1 + x}{1 - x} + C = \tanh^{-1} x + C$, when $x < 1$

20. $\int \dfrac{dx}{x^2 - 1} = \tfrac{1}{2}\ln\dfrac{x - 1}{x + 1} + C = -\coth^{-1} x + C$, when $x > 1$

$$\left.\begin{aligned}
&\textbf{21.}\quad \int \frac{dx}{a + bx^2} = \frac{1}{\sqrt{ab}}\tan^{-1}\left(\sqrt{\frac{b}{a}}\,x\right) + C\\
&\textbf{22.}\quad \int \frac{dx}{a - bx^2} = \frac{1}{2\sqrt{ab}}\ln\frac{\sqrt{ab} + bx}{\sqrt{ab} - bx} + C\\
&\qquad\qquad\qquad = \frac{1}{\sqrt{ab}}\tanh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right) + C
\end{aligned}\right\}\ [a > 0,\ b > 0]$$

TABLE 2.24 Integrals (*Continued*)

Rational functions (*continued*)

23. $$\int \frac{dx}{a+2bx+cx^2} = \left.\frac{1}{\sqrt{ac-b^2}}\tan^{-1}\frac{b+cx}{\sqrt{ac-b^2}}+C\right\}\ [ac-b^2>0]$$

$$\left.\begin{aligned} &= \frac{1}{2\sqrt{b^2-ac}}\ln\frac{\sqrt{b^2-ac}-b-cx}{\sqrt{b^2-ac}+b+cx}+C \\ &= -\frac{1}{\sqrt{b^2-ac}}\tanh^{-1}\frac{b+cx}{\sqrt{b^2-ac}}+C \end{aligned}\right\}\ [b^2-ac>0]$$

24. $$\int \frac{dx}{a+2bx+cx^2} = -\frac{1}{b+cx}+C, \text{ when } b^2=ac$$

25. $$\int \frac{(m+nx)\,dx}{a+2bx+cx^2} = \frac{n}{2c}\ln(a+2bx+cx^2) + \frac{mc-nb}{c}\int \frac{dx}{a+2bx+cx^2}$$

26. In $\int \frac{f(x)\,dx}{a+2bx+cx^2}$, if $f(x)$ is a polynomial of higher than the first degree, divide by the denominator before integrating.

27. $$\int \frac{dx}{(a+2bx+cx^2)^p} = \frac{1}{2(ac-b^2)(p-1)} \times \frac{b+cx}{(a+2bx+cx^2)^{p-1}} + \frac{(2p-3)c}{2(ac-b^2)(p-1)}\int \frac{dx}{(a+2bx+cx^2)^{p-1}}$$

28. $$\int \frac{(m+nx)\,dx}{(a+2bx+cx^2)^p} = -\frac{n}{2c(p-1)} \times \frac{1}{(a+2bx+cx^2)^{p-1}} + \frac{mc-nb}{c}\int \frac{dx}{(a+2bx+cx^2)^p}$$

29. $$\int x^{m-1}(a+bx)^n\,dx = \frac{x^{m-1}(a+bx)^{n+1}}{(m+n)b} - \frac{(m-1)a}{(m+n)b}\int x^{m-2}(a+bx)^n\,dx$$

$$= \frac{x^m(a+bx)^n}{m+n} + \frac{na}{m+n}\int x^{m-1}(a+bx)^{n-1}\,dx$$

Irrational functions

30. $$\int \sqrt{a+bx}\,dx = \frac{2}{3b}\sqrt{(a+bx)^3}+C$$

31. $$\int \frac{dx}{\sqrt{a+bx}} = \frac{2}{b}\sqrt{a+bx}+C$$

32. $$\int \frac{(m+nx)\,dx}{\sqrt{a+bx}} = \frac{2}{3b^2}(3mb-2an+nbx)\sqrt{a+bx}+C$$

33. $\int \frac{dx}{(m+nx)\sqrt{a+bx}}$; substitute $y=\sqrt{a+bx}$, and use **21** and **22**

34. $\int \frac{f(x, \sqrt[n]{a+bx})}{F(x, \sqrt[n]{a+bx})}\,dx$; substitute $\sqrt[n]{a+bx}=y$

35. $$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}+C = -\cos^{-1}\frac{x}{a}+c$$

36. $$\int \frac{dx}{\sqrt{a^2+x^2}} = \ln\left(x+\sqrt{a^2+x^2}\right)+C = \sinh^{-1}\frac{x}{a}+c$$

37. $$\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left(x+\sqrt{x^2-a^2}\right)+C = \cosh^{-1}\frac{x}{a}+c$$

TABLE 2.24 Integrals (*Continued*)

Irrational functions (*continued*)

38. $$\int \frac{dx}{\sqrt{a+2bx+cx^2}} = \frac{1}{\sqrt{c}} \ln \left(b+cx+\sqrt{c}\,\sqrt{a+2bx+cx^2}\right) + C, \text{ when } c > 0$$
$$= \frac{1}{\sqrt{c}} \sinh^{-1} \frac{b+cx}{\sqrt{ac-b^2}} + C, \text{ when } ac - b^2 > 0$$
$$= \frac{1}{\sqrt{c}} \cosh^{-1} \frac{b+cx}{\sqrt{b^2-ac}} + C, \text{ when } b^2 - ac > 0$$
$$= \frac{-1}{\sqrt{-c}} \sin^{-1} \frac{b+cx}{\sqrt{b^2-ac}} + C, \text{ when } c < 0$$

39. $$\int \frac{(m+nx)\,dx}{\sqrt{a+2bx+cx^2}} = \frac{n}{c}\sqrt{a+2bx+cx^2} + \frac{mc-nb}{c} \int \frac{dx}{\sqrt{a+2bx+cx^2}}$$

40. $$\int \frac{x^m\,dx}{\sqrt{a+2bx+cx^2}} = \frac{x^{m-1}X}{mc} - \frac{(m-1)a}{mc} \int \frac{x^{m-2}\,dx}{X} - \frac{(2m-1)b}{mc} \int \frac{x^{m-1}\,dx}{X},$$
when $X = \sqrt{a+2bx+cx^2}$

41. $$\int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2} \ln \left(x+\sqrt{a^2+x^2}\right) + C$$
$$= \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2} \sinh^{-1} \frac{x}{a} + c$$

42. $$\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

43. $$\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2} \ln \left(x+\sqrt{x^2-a^2}\right) + C$$
$$= \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2} \cosh^{-1} \frac{x}{a} + c$$

44. $$\int \sqrt{a+2bx+cx^2}\,dx = \frac{b+cx}{2c}\sqrt{a+2bx+cx^2} + \frac{ac-b^2}{2c} \int \frac{dx}{\sqrt{a+2bx+cx^2}} + C$$

Transcendental functions

45. $$\int a^x\,dx = \frac{a^x}{\ln a} + C$$

46. $$\int x^n e^{ax}\,dx = \frac{x^n e^{ax}}{a}\left[1 - \frac{n}{ax} + \frac{n(n-1)}{a^2x^2} - \cdots \pm \frac{n!}{a^n x^n}\right] + C$$

47. $$\int \ln x\,dx = x \ln x - x + C$$

48. $$\int \frac{\ln x}{x^2}\,dx = -\frac{\ln x}{x} - \frac{1}{x} + C$$

49. $$\int \frac{(\ln x)^n}{x}\,dx = \frac{1}{n+1}(\ln x)^{n+1} + C$$

50. $$\int \sin^2 x\,dx = -\tfrac{1}{4}\sin 2x + \tfrac{1}{2}x + C = -\tfrac{1}{2}\sin x \cos x + \tfrac{1}{2}x + C$$

51. $$\int \cos^2 x\,dx = \tfrac{1}{4}\sin 2x + \tfrac{1}{2}x + C = \tfrac{1}{2}\sin x \cos x + \tfrac{1}{2}x + C$$

52. $$\int \sin mx\,dx = -\frac{\cos mx}{m} + C$$

53. $$\int \cos mx\,dx = \frac{\sin mx}{m} + C$$

TABLE 2.24 Integrals (*Continued*)

Transcendental functions (*continued*)

54. $$\int \sin mx \cos nx \, dx = -\frac{\cos(m+n)x}{2(m+n)} - \frac{\cos(m-n)x}{2(m-n)} + C$$

55. $$\int \sin mx \sin nx \, dx = \frac{\sin(m-n)x}{2(m-n)} - \frac{\sin(m+n)x}{2(m+n)} + C$$

56. $$\int \cos mx \cos nx \, dx = \frac{\sin(m-n)x}{2(m-n)} + \frac{\sin(m+n)x}{2(m+n)} + C$$

57. $$\int \tan x \, dx = -\ln \cos x + C$$

58. $$\int \cot x \, dx = \ln \sin x + C$$

59. $$\int \frac{dx}{\sin x} = \ln \tan \frac{x}{2} + C$$

60. $$\int \frac{dx}{\cos x} = \ln \tan \left(\frac{\pi}{4} + \frac{x}{2}\right) + C$$

61. $$\int \frac{dx}{1 + \cos x} = \tan \frac{x}{2} + C$$

62. $$\int \frac{dx}{1 - \cos x} = -\cot \frac{x}{2} + C$$

63. $$\int \sin x \cos x \, dx = \tfrac{1}{2} \sin^2 x + C$$

64. $$\int \frac{dx}{\sin x \cos x} = \ln \tan x + C$$

65.* $$\int \sin^n x \, dx = -\frac{\cos x \sin^{n-1} x}{n} + \frac{n-1}{n} \int \sin^{n-2} x \, dx$$

66.* $$\int \cos^n x \, dx = \frac{\sin x \cos^{n-1} x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \, dx$$

67. $$\int \tan^n x \, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \, dx$$

68. $$\int \cot^n x \, dx = -\frac{\cot^{n-1} x}{n-1} - \int \cot^{n-2} x \, dx$$

69. $$\int \frac{dx}{\sin^n x} = -\frac{\cos x}{(n-1) \sin^{n-1} x} + \frac{n-2}{n-1} \int \frac{dx}{\sin^{n-2} x}$$

70. $$\int \frac{dx}{\cos^n x} = \frac{\sin x}{(n-1) \cos^{n-1} x} + \frac{n-2}{n-1} \int \frac{dx}{\cos^{n-2} x}$$

71.* $$\begin{aligned} \int \sin^p x \cos^q x \, dx &= \frac{\sin^{p+1} x \cos^{q-1} x}{p+q} + \frac{q-1}{p+q} \int \sin^p x \cos^{q-2} x \, dx \\ &= -\frac{\sin^{p-1} x \cos^{q+1}}{p+q} + \frac{p-1}{p+q} \int \sin^{p-2} x \cos^q x \, dx \end{aligned}$$

72.* $$\int \sin^{-p} x \cos^q x \, dx = -\frac{\sin^{-p+1} x \cos^{q+1} x}{p-1} + \frac{p-q-2}{p-1} \int \sin^{-p+2} x \cos^q x \, dx$$

73.* $$\int \sin^p x \cos^{-q} x \, dx = \frac{\sin^{p+1} x \cos^{-q+1} x}{q-1} + \frac{q-p-2}{q-1} \int \sin^p x \cos^{-q+2} x \, dx$$

74. $$\begin{aligned} \int \frac{dx}{a + b \cos x} &= \frac{2}{\sqrt{a^2 - b^2}} \tan^{-1} \left(\sqrt{\frac{a-b}{a+b}} \tan \tfrac{1}{2}x\right) + C, \text{ when } a^2 > b^2 \\ &\left.\begin{aligned} &= \frac{1}{\sqrt{b^2 - a^2}} \ln \frac{b + a \cos x + \sin x \sqrt{b^2 - a^2}}{a + b \cos x} + C \\ &= \frac{2}{\sqrt{b^2 - a^2}} \tanh^{-1} \left(\sqrt{\frac{b-a}{b+a}} \tan \tfrac{1}{2}x\right) + C \end{aligned}\right\} [a^2 < b^2] \end{aligned}$$

75. $$\int \frac{\cos x \, dx}{a + b \cos x} = \frac{x}{b} - \frac{a}{b} \int \frac{dx}{a + b \cos x} + C$$

---

* If n, p, or q is an odd number, substitute $\cos x = z$ or $\sin x = z$

TABLE 2.24 Integrals (*Continued*)

Transcendental functions (*continued*)

76. $\int \frac{\sin x\, dx}{a + b \cos x} = -\frac{1}{b} \ln (a + b \cos x) + C$

77. $\int \frac{A + B \cos x + C \sin x}{a + b \cos x + c \sin x}\, dx = A \int \frac{dy}{a + p \cos y}$

$+ (B \cos u + C \sin u) \int \frac{\cos y\, dy}{a + p \cos y} - (B \sin u - C \cos u) \int \frac{\sin y\, dy}{a + p + \cos y},$

where $b = p \cos u$, $c = p \sin u$, and $x - u = y$

78. $\int e^{ax} \sin bx\, dx = \frac{a \sin bx - b \cos bx}{a^2 + b^2} e^{ax} + C$

79. $\int e^{ax} \cos bx\, dx = \frac{a \cos bx + b \sin bx}{a^2 + b^2} e^{ax} + C$

80. $\int \sin^{-1} x\, dx = x \sin^{-1} x + \sqrt{1 - x^2} + C$

81. $\int \cos^{-1} x\, dx = x \cos^{-1} x - \sqrt{1 - x^2} + C$

82. $\int \tan^{-1} x\, dx = x \tan^{-1} x - \tfrac{1}{2} \ln(1 + x^2) + C$

83. $\int \cot^{-1} x\, dx = x \cot^{-1} x + \tfrac{1}{2} \ln(1 + x^2) + C$

84. $\int \sinh x\, dx = \cosh x + C$

85. $\int \tanh x\, dx = \ln \cosh x + C$

86. $\int \cosh x\, dx = \sinh x + C$

87. $\int \coth x\, dx = \ln \sinh x + C$

88. $\int \operatorname{sech} x\, dx = 2 \tan^{-1} (e^x) + C$

89. $\int \operatorname{csch} x\, dx = \ln \tanh \left(\frac{x}{2}\right) + C$

90. $\int \sinh^2 x\, dx = \tfrac{1}{2} \sinh x \cosh x - \tfrac{1}{2}x + C$

91. $\int \cosh^2 x\, dx = \tfrac{1}{2} \sinh x \cosh + \tfrac{1}{2}x + C$

92. $\int \operatorname{sech}^2 x\, dx = \tanh x + C$

93. $\int \operatorname{csch}^2 x\, dx = -\coth x + C$

2.2.2 Surface Areas and Volumes*

Let a, b, c, d, and s denote lengths, A denote areas, and V denote volumes.

Triangle. $A = bh/2$, where b denotes the base and h the altitude.

Rectangle. $A = ab$, where a and b denote the lengths of the sides.

Parallelogram (opposite sides parallel). $A = ah = ab \sin \theta$, where a and b denote the sides, h the altitude, and θ the angle between the sides.

Trapezoid (four sides, two parallel). $A = \tfrac{1}{2}h(a + b)$, where a and b are the sides and h the altitude.

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* Adapted by permission from Burington, *Handbook of Mathematical Tables and Formulas,* 3d. ed., McGraw-Hill Book Company, New York (1959).

Regular Polygon of n *Sides* (Fig. 2.1)

$$A = \frac{1}{4} na^2 \operatorname{ctn} \frac{180°}{n}$$ where a is length of side

$$R = \frac{a}{2} \csc \frac{180°}{n}$$ where R is radius of circumscribed circle

$$r = \frac{a}{2} \operatorname{ctn} \frac{180°}{n}$$ where r is radius of inscribed circle

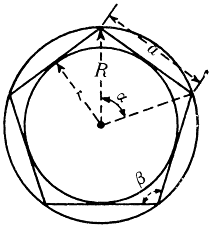

FIGURE 2.1

$$\alpha = \frac{360°}{n} = \frac{2\pi}{n} \text{ radians}$$

$$\beta = \left(\frac{n-2}{n}\right) \cdot 180° = \left(\frac{n-2}{n}\right) \pi \text{ radians}$$ where α and β are the angles indicated in Fig. 2.1

$$a = 2r \tan \frac{\alpha}{2} = 2R \sin \frac{\alpha}{2}$$

Circle (Fig. 2.2). Let

$C =$ circumference

$R =$ radius

$D =$ diameter

$A =$ area

$S =$ length of arc subtended by θ

$l =$ chord subtended by arc S

$h =$ rise

$\theta =$ central angle in radians

$$C = 2\pi R = \pi D \qquad \pi = 3.14159 \ . \ . \ .$$

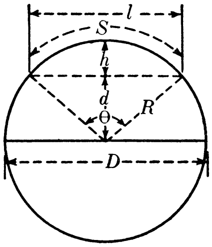

FIGURE 2.2

$$S = R\theta = \tfrac{1}{2}D\theta = D \cos^{-1} \frac{d}{R}$$

$$l = 2\sqrt{R^2 - d^2} = 2R \sin \frac{\theta}{2} = 2d \tan \frac{\theta}{2}$$

$$d = \tfrac{1}{2}\sqrt{4R^2 - l^2} = R \cos \frac{\theta}{2} = \tfrac{1}{2}l \operatorname{ctn} \frac{\theta}{2}$$

$$h = R - d$$

$$\theta = \frac{S}{R} = \frac{2S}{D} = 2 \cos^{-1} \frac{d}{R} = 2 \tan^{-1} \frac{l}{2d} = 2 \sin^{-1} \frac{l}{D}$$

$$A \text{ (circle)} = \pi R^2 = \tfrac{1}{4}\pi D^2$$

$$A \text{ (sector)} = \tfrac{1}{2}Rs = \tfrac{1}{2}R^2\theta$$

$$A \text{ (segment)} = A \text{ (sector)} - A \text{ (triangle)} = \tfrac{1}{2}R^2 (\theta - \sin \theta)$$

$$= R^2 \cos^{-1} \frac{R-h}{R} - (R-h)\sqrt{2Rh - h^2}$$

$$\text{Perimeter of } n\text{-sided regular polygon inscribed in a circle} = 2nR \sin \frac{\pi}{n}$$

$$\text{Area of inscribed polygon} = \tfrac{1}{2}nR^2 \sin \frac{2\pi}{n}$$

$$\text{Perimeter of } n\text{-sided regular polygon circumscribed about a circle} = 2nR \tan \frac{\pi}{n}$$

$$\text{Area of circumscribed polygon} = nR^2 \tan \frac{\pi}{n}$$

Radius of circle inscribed in a triangle of sides a, b, and c is

$$r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}} \qquad s = \tfrac{1}{2}(a+b+c)$$

Radius of circle circumscribed about a triangle is

$$R = \frac{abc}{4\sqrt{s(s-a)(s-b)(s-c)}}$$

Ellipse (Fig. 2.3). $A = \pi ab$, where a and b are lengths of semimajor and semiminor axes, respectively.

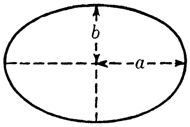

FIGURE 2.3

Parabola (Fig. 2.4)

$$A = \frac{2ld}{3}$$

$$\text{Height of } d_1 = \frac{d}{l^2}(l^2 - l_1^2)$$

$$\text{Width of } l_1 = l\sqrt{\frac{d-d_1}{d}}$$

$$\text{Length of arc} = l\left[1 + \frac{2}{3}\left(\frac{2d}{l}\right)^2 - \frac{2}{5}\left(\frac{2d}{l}\right)^4 + \cdots\right]$$

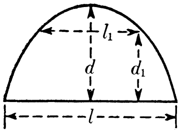

FIGURE 2.4

Area by Approximation (Fig. 2.5). If $y_0, y_1, y_2, \ldots, y_n$ are the length of a series of equally spaced parallel chords, and if h is their distance apart, the area enclosed by the boundary is given approximately by any one of the following formulae:

$$A_T = h[\tfrac{1}{2}(y_0 + y_n) + y_1 + y_2 + \cdots + y_{n-1}] \qquad \text{(Trapezoidal Rule)}$$

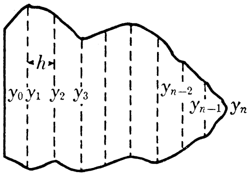

FIGURE 2.5

$$A_D = h[0.4(y_0 + y_n) + 1.1(y_1 + y_{n-1}) + y_2 + y_3 + \cdots + y_{n-2}] \qquad \text{(Durand's Rule)}$$

$$A_S = \tfrac{1}{3}h[(y_0 + y_n) + 4(y_1 + y_3 + \cdots + y_{n-1}) + 2(y_2 + y_4 + \cdots + y_{n-2})] \qquad (n \text{ even, Simpson's Rule})$$

In general, A_S gives the most accurate approximation.

The greater the value of n, the greater the accuracy of approximation.

Cube. $V = a^3$; $d = a\sqrt{2}$; total surface area $= 6a^2$, where a is length of side and d is length of diagonal.

Rectangular Parallelopiped. $V = abc$; $d = \sqrt{a^2 + b^2 + c^2}$; total surface area $= 2(ab + bc + ca)$, where a, b, and c are the lengths of the sides and d is length of diagonal.

Prism or Cylinder

$$V = (\text{area of base}) \cdot (\text{altitude})$$

$$\text{Lateral area} = (\text{perimeter of right section}) \cdot (\text{lateral edge})$$

Pyramid or Cone

$$V = \tfrac{1}{3}(\text{area of base}) \cdot (\text{altitude})$$

$$\text{Lateral area of regular pyramid} = \tfrac{1}{2}(\text{perimeter of base}) \cdot (\text{slant height})$$

Frustum of Pyramid or Cone. $V = \tfrac{1}{3}(A_1 + A_2 + \sqrt{A_1 \cdot A_2})h$, where h is the altitude and A_1 and A_2 are the areas of the bases.

$$\text{Lateral area of a regular figure} = \tfrac{1}{2}(\text{sum of perimeters of base}) \cdot (\text{slant height})$$

Prismoid

$$V = \frac{h}{6}(A_1 + A_2 + 4A_3)$$

where h = altitude, A_1 and A_2 are the areas of the bases, and A_3 is the area of the midsection parallel to bases.

Area of Surface and Volume of Regular Polyhedra of Edge l

Name	Type of surface	Area of surface	Volume
Tetrahedron	4 equilateral triangles	$1.73205l^2$	$0.11785l^3$
Hexahedron (cube)	6 squares	$6.00000l^2$	$1.00000l^3$
Octahedron	8 equilateral triangles	$3.46410l^2$	$0.47140l^3$
Dodecahedron	12 pentagons	$20.64578l^2$	$7.66312l^3$
Icosahedron	20 equilateral triangles	$8.66025l^2$	$2.18170l^3$

Sphere (Fig. 2.6)

$$A\ (\text{sphere}) = 4\pi R^2 = \pi D^2$$

$$A\ (\text{zone}) = 2\pi R h_1 = \pi D h_1$$

$$V\ (\text{sphere}) = \tfrac{4}{3}\pi R^3 = \tfrac{1}{6}\pi D^3$$

$$V\text{ (spherical sector)} = \tfrac{2}{3}\pi R^2 h_1 = \tfrac{1}{6}\pi D^2 h_1$$

$$V\text{ (spherical segment of one base)} = \tfrac{1}{6}\pi h_3(3r_3^2 + h_3^2)$$

$$V\text{ (spherical segment of two bases)} = \tfrac{1}{6}\pi h_2(3r_3^2 + 3r_2^2 + h_2^2)$$

$$A\text{ (lune)} = 2R^2\theta \qquad \text{where } \theta \text{ is angle in radians of lune}$$

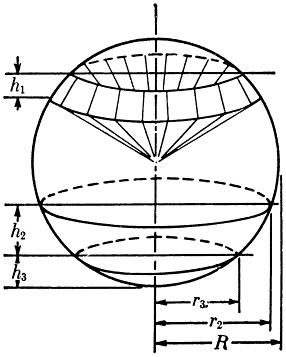

FIGURE 2.6

Ellipsoid. $V = \tfrac{4}{3}\pi abc$, where a, b, and c are the lengths of the semiaxes.

Torus (Fig. 2.7)

$$V = 2\pi^2 R r^2$$

$$\text{Area of surface} = S = 4\pi^2 R r$$

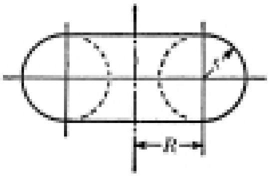

FIGURE 2.7

2.2.3 Trigonometric Functions of an Angle α

Let x be any angle whose initial side lies on the positive x axis and whose vertex is at the origin, and (x, y) be any point on the terminal side of the angle. (x is positive if measured along OX to the right, from the y axis; and negative, if measured along OX' to the left from the y axis. Likewise, y is positive if measured parallel to OY, and negative if measured parallel to OY'.) Let r be the positive distance from the origin to the point. The trigonometric functions of an angle are defined as follows:

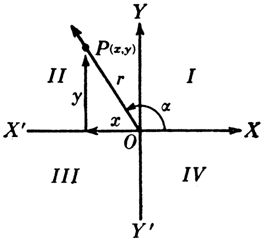

FIGURE 2.8

$$\text{sine } \alpha = \sin\alpha = \frac{y}{r}$$

$$\text{cosine } \alpha = \cos\alpha = \frac{x}{r}$$

$$\text{tangent } \alpha = \tan\alpha = \frac{y}{x}$$

$$\text{cotangent } \alpha = \text{ctn } \alpha = \cot\alpha = \frac{x}{y}$$

$$\text{secant } \alpha = \sec\alpha = \frac{r}{x}$$

$$\text{cosecant } x = \csc\alpha = \frac{r}{y}$$

$$\text{exsecant } x = \text{exsec } \alpha = \sec\alpha - 1$$

$$\text{versine } \alpha = \text{vers } \alpha = 1 - \cos\alpha$$

$$\text{coversine } \alpha = \text{covers } \alpha = 1 - \sin\alpha$$

$$\text{haversine } \alpha = \text{hav } \alpha = \tfrac{1}{2}\text{ vers } \alpha$$

2.2.3.1 Signs of the Functions

Quadrant	sin	cos	tan	ctn	sec	csc
I	+	+	+	+	+	+
II	+	−	−	−	−	+
III	−	−	+	+	−	−
IV	−	+	−	−	+	−

*2.2.3.2 Relations between the Functions of a Single Angle**

$$\sin^2 x + \cos^2 x = 1$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$$

$$1 + \tan^2 x = \sec^2 x = \frac{1}{\cos^2 x}$$

$$1 + \cot^2 x = \csc^2 x = \frac{1}{\sin^2 x}$$

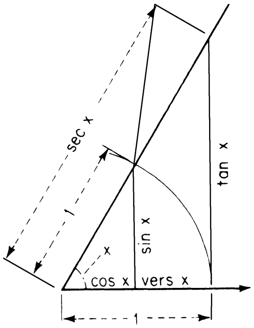

FIGURE 2.9

$$\sin x = \sqrt{1 - \cos^2 x} = \frac{\tan x}{\sqrt{1 + \tan^2 x}} = \frac{1}{\sqrt{1 + \cot^2 x}}$$

$$\cos x = \sqrt{1 - \sin^2 x} = \frac{1}{\sqrt{1 + \tan^2 x}} = \frac{\cot x}{\sqrt{1 + \cot^2 x}}$$

2.2.3.3 Functions of Negative Angles. $\sin(-x) = -\sin x$; $\cos(-x) = \cos x$; $\tan(-x) = -\tan x$.

2.2.3.4 Functions of the Sum and Difference of Two Angles

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\cot(x + y) = \frac{\cot x \cot y - 1}{\cot x + \cot y}$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

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* From Baumeister and Marks, *Standard Handbook for Mechanical Engineers,* 7th ed., McGraw-Hill Book Company, New York (1967); by permission.

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$\cot(x - y) = \frac{\cot x \cot y + 1}{\cot y - \cot x}$$

$$\sin x + \sin y = 2 \sin \tfrac{1}{2}(x + y) \cos \tfrac{1}{2}(x - y)$$

$$\sin x - \sin y = 2 \cos \tfrac{1}{2}(x + y) \sin \tfrac{1}{2}(x - y)$$

$$\cos x + \cos y = 2 \cos \tfrac{1}{2}(x + y) \cos \tfrac{1}{2}(x - y)$$

$$\cos x - \cos y = -2 \sin \tfrac{1}{2}(x + y) \sin \tfrac{1}{2}(x - y)$$

$$\tan x + \tan y = \frac{\sin(x + y)}{\cos x \cos y} \qquad \cot x + \cot y = \frac{\sin(x + y)}{\sin x \sin y}$$

$$\tan x - \tan y = \frac{\sin(x - y)}{\cos x \cos y} \qquad \cot x - \cot y = \frac{\sin(y - x)}{\sin x \sin y}$$

$$\sin^2 x - \sin^2 y = \cos^2 y - \cos^2 x = \sin(x + y) \sin(x - y)$$

$$\cos^2 x - \sin^2 y = \cos^2 y - \sin^2 x = \cos(x + y) \cos(x - y)$$

$$\sin(45° + x) = \cos(45° - x) \qquad \tan(45° + x) = \cot(45° - x)$$

$$\sin(45° - x) = \cos(45° + x) \qquad \tan(45° - x) = \cot(45° + x)$$

In the following transformations, a and b are supposed to be positive, $c = \sqrt{a^2 + b^2}$, $A =$ the positive acute angle for which $B = b/a$:

$$a \cos x + b \sin x = c \sin(A + x) = c \cos(B - x)$$

$$a \cos x - b \sin x = c \sin(A - x) = c \cos(B + x)$$

2.2.4 Expansion in Series*

The range of values of x for which each of the series is convergent is stated at the right of the series.

2.2.4.1 Exponential and Logarithmic Series

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots \qquad (-\infty < x < +\infty)$$

$$a^x = e^{mx} = 1 + \frac{m}{1!} x + \frac{m^2}{2!} x^2 + \frac{m^3}{3!} x^3 + \cdots \qquad (a > 0, -\infty < x < +\infty)$$

where $m = \ln a = 2.3026 \log_{10} a$.

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* From Baumeister and Marks, *Standard Handbook for Mechanical Engineers,* 7th ed., McGraw-Hill Book Company, New York (1967); by permission.

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} \cdots \qquad (-1 < x < +1)$$

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \cdots \qquad (-1 < x < +1)$$

$$\ln\left(\frac{1+x}{1-x}\right) = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \cdots\right) \qquad (-1 < x < +1)$$

$$\ln\left(\frac{x+1}{x-1}\right) = 2\left(\frac{1}{x} + \frac{1}{3x^3} + \frac{1}{5x^5} + \frac{1}{7x^7} + \cdots\right) \qquad (x < -1 \text{ or } +1 < x)$$

$$\ln x = 2\left[\frac{x-1}{x+1} + \frac{1}{3}\left(\frac{x-1}{x+1}\right)^3 + \frac{1}{5}\left(\frac{x-1}{x+1}\right)^5 + \cdots\right] \qquad (0 < x < \infty)$$

$$\ln(a+x) = \ln a + 2\left[\frac{x}{2a+x} + \frac{1}{3}\left(\frac{x}{2a+x}\right)^3 + \frac{1}{5}\left(\frac{x}{2a+x}\right)^5 + \cdots\right]$$

$$(0 < a < +\infty, \ -a < x < +\infty)$$

Series for the Trigonometric Functions. In the following formulas, *all angles must be expressed in radians.* If D = the number of degrees in the angle, and x = its radian measure, then $x = 0.017453D$.

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \qquad (-\infty < x < +\infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots \qquad (-\infty < x < +\infty)$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} + \cdots \qquad \left(-\frac{\pi}{2} < x < +\frac{\pi}{2}\right)$$

$$\cot x = \frac{1}{x} - \frac{x}{3} - \frac{x^3}{45} - \frac{2x^5}{945} - \frac{x^7}{4725} - \cdots \qquad (-\pi < x < +\pi)$$

$$\sin^{-1} y = y + \frac{y^3}{6} + \frac{3y^5}{40} + \frac{5y^7}{112} + \cdots \qquad (-1 \leqq y \leqq +1)$$

$$\tan^{-1} y = y - \frac{y^3}{3} + \frac{y^5}{5} - \frac{y^7}{7} + \cdots \qquad (-1 \leqq y \leqq +1)$$

$$\cos^{-1} y = \tfrac{1}{2}\pi - \sin^{-1} y \qquad \cot^{-1} y = \tfrac{1}{2}\pi - \tan^{-1} y$$

Reversing a Series. If $y = x + bx^2 + cx^3 + dx^4 + ex^5 + \cdots$, then $x = y - by^2 + (2b^2 - c)y^3 - (5b^3 - 5bc + d)y^4 + (14b^4 - 21b^2c + 6bd + 3c^2 - e)y^5 + \cdots$, provided the latter series is convergent.

Fourier's Series. Let $f(x)$ be a function which is finite in the interval from $x = -c$ to $x = +c$ and whose graph has finite arc length in that interval.* Then, for any value of x between $-c$ and c,

---

* If $x = x_0$ is a point of discontinuity, $f(x_0)$ is to be defined as $\tfrac{1}{2}[f_1(x_0) + f_2(x_0)]$, where $f_1(x_0)$ is the limit of $f(x)$ when x approaches x_0 from below, and $f_2(x_0)$ is the limit of $f(x)$ when x approaches x_0 from above.

$$f(x) = \tfrac{1}{2}a_0 + a_1 \cos \frac{\pi x}{c} + a_2 \cos \frac{2\pi c}{c} + a_3 \cos \frac{3\pi x}{c} + \cdots$$
$$+ b_1 \sin \frac{\pi x}{c} + b_2 \sin \frac{2\pi x}{c} + b_3 \sin \frac{3\pi x}{c} + \cdots$$

where the constant coefficients are determined as follows:

$$a_n = \frac{1}{c}\int_{-c}^{c} f(t) \cos \frac{n\pi t}{c}\, dt \qquad b_n = \frac{1}{c}\int_{-c}^{c} f(t) \sin \frac{n\pi t}{c}\, dt$$

In case the curve $y = f(x)$ is symmetrical with respect to the origin, the a's are all zero, and the series is a sine series. In case the curve is symmetrical with respect to the y axis, the b's are all zero, and a cosine series results. (In this case, the series will be valid not only for values of x between $-c$ and c, but also for $x = -c$ and $x = c$.) A Fourier series can always be integrated term by term; but the result of differentiating term by term may not be a convergent series.

TABLE 2.25 Some Constants

Constant	Number	Log_{10} of Number
Pi (π)	3.14159 26535 89793 23846	0.49714 98726 94133 85435
Napierian Base (e)	2.71828 18284 59045 23536	0.43429 448
$\text{M} = \log_{10}e$	0.43429 44819 03251 82765	9.63778 43113 00536 78912 − 10
$1 \div \text{M} = \log_e 10$	2.30258 50929 94045 68402	0.36221 569
$180 \div \pi =$ degrees in 1 radian	57.2957 795	1.75812 263
$\pi \div 180 =$ radians in 1°	0.01745 329	8.24187 737 − 10
$\pi \div 10800 =$ radians in 1′	0.00029 08882	6.46372 612 − 10
$\pi \div 648000 =$ radians in 1″	0.00000 48481 36811 095	4.68557 487 − 10

2.3 STATISTICS IN CHEMICAL ANALYSIS

2.3.1 Introduction

Each observation in any branch of scientific investigation is inaccurate to some degree. Often the accurate value for the concentration of some particular constituent in the analyte cannot be determined. However, it is reasonable to assume the accurate value exists, and it is important to estimate the limits between which this value lies. It must be understood that the statistical approach is concerned with the appraisal of experimental design and data. Statistical techniques can neither detect nor evaluate constant errors (bias); the detection and elimination of inaccuracy are analytical problems. Nevertheless, statistical techniques can assist considerably in determining whether or not inaccuracies exist and in indicating when procedural modifications have reduced them.

By proper design of experiments, guided by a statistical approach, the effects of experimental variables may be found more efficiently than by the traditional approach of holding all variables constant but one and systematically investigating each variable in turn. Trends in data may be sought to track down nonrandom sources of error.

2.3.2 Errors in Quantitative Analysis

Two broad classes of errors may be recognized. The first class, *determinate* or *systematic* errors, is composed of errors that can be assigned to definite causes, even though the cause may not have been located. Such errors are characterized by being unidirectional. The magnitude may be constant from sample to sample, proportional to sample size, or variable in a more complex way. An example is the error caused by weighing a hygroscopic sample. This error is always positive in sign; it increases with sample size but varies depending on the time required for weighing, with humidity and temperature. An example of a negative systematic error is that caused by solubility losses of a precipitate.

The second class, *indeterminate* or *random* errors, is brought about by the effects of uncontrolled variables. Truly random errors are as likely to cause high as low results, and a small random error is much more probable than a large one. By making the observation coarse enough, random errors would cease to exist. Every observation would give the same result, but the result would be less precise than the average of a number of finer observations with random scatter.

The *precision* of a result is its reproducibility; the *accuracy* is its nearness to the truth. A systematic error causes a loss of accuracy, and it may or may not impair the precision depending upon whether the error is constant or variable. Random errors cause a lowering of reproducibility, but by making sufficient observations it is possible to overcome the scatter within limits so that the accuracy may not necessarily be affected. Statistical treatment can properly be applied only to random errors.

2.3.3 Representation of Sets of Data

Raw data are collected observations that have not been organized numerically. An *average* is a value that is typical or representative of a set of data. Several averages can be defined, the most common being the arithmetic mean (or briefly, the mean), the median, the mode, and the geometric mean.

The *mean* of a set of N numbers, $x_1, x_2, x_3, \ldots, x_N$, is denoted by $\overline{x}$ and is defined as:

$$\overline{x} = \frac{x_1 + x_2 + x_3 + \cdots + x_N}{N} \tag{2.4}$$

It is an estimation of the unknown true value μ of an infinite population. We can also define the *sample variance* s^2 as follows:

$$s^2 = \frac{\sum_{i=1}^{N}(x_i - \overline{x})^2}{N - 1} \tag{2.5}$$

The values of $\overline{x}$ and s^2 vary from sample set to sample set. However, as N increases, they may be expected to become more and more stable. Their limiting values, for very large N, are numbers characteristic of the frequency distribution, and are referred to as the *population mean* and the *population variance*, respectively.

The *median* of a set of numbers arranged in order of magnitude is the middle value or the arithmetic mean of the two middle values. The median allows inclusion of all data in a set without undue influence from outlying values; it is preferable to the mean for small sets of data.

The *mode* of a set of numbers is that value which occurs with the greatest frequency (the most common value). The mode may not exist, and even if it does exist it may not be unique. The empirical relation that exists between the mean, the mode, and the median for unimodal frequency curves which are moderately asymmetrical is:

$$\text{Mean} - \text{mode} = 3(\text{mean} - \text{median}) \tag{2.6}$$

The *geometric mean* of a set of N numbers is the Nth root of the product of the numbers:

$$\sqrt[N]{x_1 x_2 x_3 \ . \ . \ . \ x_N} \tag{2.7}$$

The *root mean square* (RMS) or quadratic mean of a set of numbers is defined by:

$$\text{RMS} = \sqrt{\overline{x^2}} = \sqrt{\sum_{i=1}^{N} x_i^2/N} \tag{2.8}$$

2.3.4 The Normal Distribution of Measurements

The normal distribution of measurements (or the normal law of error) is the fundamental starting point for analysis of data. When a large number of measurements are made, the individual measurements are not all identical and equal to the accepted value μ, which is the mean of an infinite population or universe of data, but are scattered about μ, owing to random error. If the magnitude of any single measurement is the abscissa and the relative frequencies (i.e., the probability) of occurrence of different-sized measurements are the ordinate, the smooth curve drawn through the points (Fig. 2.10) is the *normal* or *Gaussian distribution curve* (also the *error curve* or *probability curve*). The term *error curve* arises when one considers the distribution of errors ($x - \mu$) about the true value.

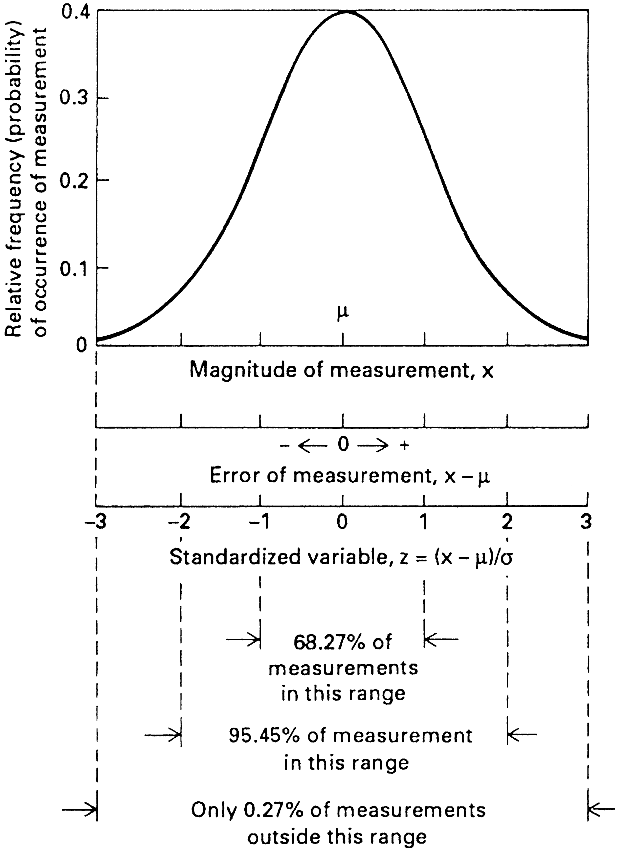

FIGURE 2.10 The Normal Distribution Curve.

The breadth or spread of the curve indicates the precision of the measurements and is determined by and related to the standard deviation, a relationship that is expressed in the equation for the normal curve (which is continuous and infinite in extent):

$$Y = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] \tag{2.9}$$

where σ is the standard deviation of the infinite population. The population mean μ expresses the magnitude of the quantity being measured. In a sense, σ measures the width of the distribution, and thereby also expresses the scatter or dispersion of replicate analytical results. When $(x - \mu)/\sigma$ is replaced by the standardized variable z, then:

$$Y = \frac{1}{\sqrt{2\pi}} e^{-(1/2)z^2} \tag{2.10}$$

The standardized variable (the z statistic) requires only the probability level to be specified. It measures the deviation from the population mean in units of standard deviation. Y is 0.399 for the most probable value, μ. In the absence of any other information, the normal distribution is assumed to apply whenever repetitive measurements are made on a sample, or a similar measurement is made on different samples.

Table 2.26*a* lists the height of an ordinate (Y) as a distance z from the mean, and Table 2.26*b* the area under the normal curve at a distance z from the mean, expressed as fractions of the total area, 1.000. Returning to Fig. 2.10, we note that 68.27% of the area of the normal distribution curve lies within 1 standard deviation of the center or mean value. Therefore, 31.73% lies outside those limits and 15.86% on each side. Ninety-five percent (actually 95.43%) of the area lies within 2 standard deviations, and 99.73% lies within 3 standard deviations of the mean. Often the last two areas are stated slightly different; viz. 95% of the area lies within 1.96σ (approximately 2σ) and 99% lies within approximately 2.5σ. The mean falls at exactly the 50% point for symmetric normal distributions.

Example 5 The true value of a quantity is 30.00, and σ for the method of measurement is 0.30. What is the probability that a single measurement will have a deviation from the mean greater than 0.45; that is, what percentage of results will fall outside the range 30.00 ± 0.45?

$$z = \frac{x-\mu}{\sigma} = \frac{0.45}{0.30} = 1.5$$

From Table 2.26*b* the area under the normal curve from -1.5σ to $+1.5\sigma$ is 0.866, meaning that 86.6% of the measurements will fall within the range 30.00 ± 0.45 and 13.4% will lie outside this range. Half of these measurements, 6.7%, will be less than 29.55; and a similar percentage will exceed 30.45. In actuality the uncertainty in z is about 1 in 15; therefore, the value of z could lie between 1.4 and 1.6; the corresponding areas under the curve could lie between 84% and 89%.

Example 6 If the mean value of 500 determinations is 151 and $\sigma = 15$, how many results lie between 120 and 155 (actually any value between 119.5 and 155.5)?

$$z = \frac{119.5 - 151}{15} = -2.10 \qquad \text{Area: } 0.482$$

$$z = \frac{155.5 - 151}{15} = 0.30 \qquad 0.118$$

$$\text{Total area: } 0.600$$

$$500(0.600) = 300 \text{ results}$$

TABLE 2.26*a* Ordinates (Y) of the Normal Distribution Curve at Values of z

z	0	1	2	3	4	5	6	7	8	9
0.0	0.3989	0.3989	0.3989	0.3988	0.3986	0.3984	0.3982	0.3980	0.3977	0.3973
0.1	0.3970	0.3965	0.3961	0.3956	0.3951	0.3945	0.3939	0.3932	0.3925	0.3918
0.2	0.3910	0.3902	0.3894	0.3885	0.3876	0.3867	0.3857	0.3847	0.3836	0.3825
0.3	0.3814	0.3802	0.3790	0.3778	0.3765	0.3752	0.3739	0.3725	0.3712	0.3697
0.4	0.3683	0.3668	0.3653	0.3637	0.3621	0.3605	0.3589	0.3572	0.3555	0.3538
0.5	0.3521	0.3503	0.3485	0.3467	0.3448	0.3429	0.3410	0.3391	0.3372	0.3352
0.6	0.3332	0.3312	0.3292	0.3271	0.3251	0.3230	0.3209	0.3187	0.3166	0.3144
0.7	0.3123	0.3101	0.3079	0.3056	0.3034	0.3011	0.2989	0.2966	0.2943	0.2920
0.8	0.2897	0.2874	0.2850	0.2827	0.2803	0.2780	0.2756	0.2732	0.2709	0.2685
0.9	0.2661	0.2637	0.2613	0.2589	0.2565	0.2541	0.2516	0.2492	0.2468	0.2444
1.0	0.2420	0.2396	0.2371	0.2347	0.2323	0.2299	0.2275	0.2251	0.2227	0.2203
1.1	0.2179	0.2155	0.2131	0.2107	0.2083	0.2059	0.2036	0.2012	0.1989	0.1965
1.2	0.1942	0.1919	0.1895	0.1872	0.1849	0.1826	0.1804	0.1781	0.1758	0.1736
1.3	0.1714	0.1691	0.1669	0.1647	0.1626	0.1604	0.1582	0.1561	0.1539	0.1518
1.4	0.1497	0.1476	0.1456	0.1435	0.1415	0.1394	0.1374	0.1354	0.1334	0.1315
1.5	0.1295	0.1276	0.1257	0.1238	0.1219	0.1200	0.1182	0.1163	0.1145	0.1127
1.6	0.1109	0.1092	0.1074	0.1057	0.1040	0.1023	0.1006	0.0989	0.0973	0.0957
1.7	0.0940	0.0925	0.0909	0.0893	0.0878	0.0863	0.0848	0.0833	0.0818	0.0804
1.8	0.0790	0.0775	0.0761	0.0748	0.0734	0.0721	0.0707	0.0694	0.0681	0.0669
1.9	0.0656	0.0644	0.0632	0.0620	0.0608	0.0596	0.0584	0.0573	0.0562	0.0551
2.0	0.0540	0.0529	0.0519	0.0508	0.0498	0.0488	0.0478	0.0468	0.0459	0.0449
2.1	0.0440	0.0431	0.0422	0.0413	0.0404	0.0396	0.0387	0.0379	0.0371	0.0363
2.2	0.0355	0.0347	0.0339	0.0332	0.0325	0.0317	0.0310	0.0303	0.0297	0.0290
2.3	0.0283	0.0277	0.0270	0.0264	0.0258	0.0252	0.0246	0.0241	0.0235	0.0229
2.4	0.0224	0.0219	0.0213	0.0208	0.0203	0.0198	0.0194	0.0189	0.0184	0.0180
2.5	0.0175	0.0171	0.0167	0.0163	0.0158	0.0154	0.0151	0.0147	0.0143	0.0139
2.6	0.0136	0.0132	0.0129	0.0126	0.0122	0.0119	0.0116	0.0113	0.0110	0.0107
2.7	0.0104	0.0101	0.0099	0.0096	0.0093	0.0091	0.0088	0.0086	0.0084	0.0081
2.8	0.0079	0.0077	0.0075	0.0073	0.0071	0.0069	0.0067	0.0065	0.0063	0.0061
2.9	0.0060	0.0058	0.0056	0.0055	0.0053	0.0051	0.0050	0.0048	0.0047	0.0046
3.0	0.0044	0.0043	0.0042	0.0040	0.0039	0.0038	0.0037	0.0036	0.0035	0.0034
3.1	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026	0.0025	0.0025
3.2	0.0024	0.0023	0.0022	0.0022	0.0021	0.0020	0.0020	0.0019	0.0018	0.0018
3.3	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014	0.0013	0.0013
3.4	0.0012	0.0012	0.0012	0.0011	0.0011	0.0010	0.0010	0.0010	0.0009	0.0009
3.5	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007	0.0007	0.0007	0.0006
3.6	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0004
3.7	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003	0.0003	0.0003	0.0003
3.8	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002	0.0002	0.0002	0.0002	0.0002
3.9	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001	0.0001

2.3.5 Standard Deviation as a Measure of Dispersion

Several ways may be used to characterize the spread or dispersion in the original data. The *range* is the difference between the largest value and the smallest value in a set of observations. However, almost always the most efficient quantity for characterizing variability is the *standard deviation* (also called the *root mean square*).

TABLE 2.26*b* Areas Under the Normal Distribution Curve from 0 to *z*

z	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

The standard deviation is the square root of the average squared differences between the individual observations and the population mean:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N} (x_i - \mu)^2}{N}} \tag{2.11}$$

The standard deviation σ may be estimated by calculating the standard deviation s drawn from a small sample set as follows:

$$s = \sqrt{\frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{N}} \qquad \text{or} \qquad s = \sqrt{\frac{x_1^2 + x_2^2 + \cdots - [(x_1 + x_2 + \cdots)^2]/N}{N - 1}} \tag{2.12}$$

where $x_\text{i} - \bar{x}$ represents the deviation of each number in the array from the arithmetic mean. Since two pieces of information, namely s and $\bar{x}$, have been extracted from the data, we are left with $N - 1$ *degrees of freedom* (df); that is, independent data points available for measurement of precision. If a relatively large sample of data corresponding to $N > 30$ is available, its mean can be taken as a measure of μ, and s as equal to σ.

So basic is the notion of a statistical *estimate* of a physical parameter that statisticians use Greek letters for the *parameters* and Latin letters for the estimates. For many purposes, one uses the *variance*, which for the sample is s^2 and for the entire populations is σ^2. The variance s^2 of a finite sample is an unbiased estimate of σ^2, whereas the standard deviation s is not an unbiased estimate of σ.

Because the standard deviation σ for the universe is a characteristic of the measuring procedure, it is possible to get a good estimate not only from a long series of repeated analyses of the same sample, but also by taking together several short series measured with slightly different samples of the same type. When a series of observations can be logically arranged into k subgroups, the variance is calculated by summing the squares of the deviations for each subgroup, and then adding all the k sums and dividing by $N - k$ because one degree of freedom is lost in each subgroup. It is not required that the number of repeated analyses in the different groups be the same. For two groups of observations consisting of N_A and N_B members of standard deviations s_A and s_B, respectively, the variance is given by:

$$s^2 = \frac{(N_A - 1)s_A^2 + (N_B - 1)s_B^2}{N_A + N_B - 2} \tag{2.13}$$

Another measure of dispersion is the *coefficient of variation*, which is merely the standard deviation expressed as a fraction of the arithmetic mean, viz., $s/\bar{x}$. It is useful mainly to show whether the relative or the absolute spread of values is constant as the values are changed.

2.3.6 Student's Distribution or *t* Test

In the next several sections, the theoretical distributions and tests of significance will be examined beginning with Student's distribution or t test. If the data contained only random (or chance) errors, the cumulative estimates $\bar{x}$ and s would gradually approach the limits μ and σ. The distribution of results would be normally distributed with mean μ and standard deviation σ. Were the true mean of the infinite population known, it would also have some symmetrical type of distribution centered around μ. However, it would be expected that the dispersion or spread of this dispersion about the mean would depend on the sample size.

The standard deviation of the distribution of means equals $\sigma/N^{1/2}$. Since σ is not usually known, its approximation for a finite number of measurements is overcome by the Student t test. It is a measure of error between μ and $\bar{x}$. The Student t takes into account both the possible variation of the value of $\bar{x}$ from μ on the basis of the expected variance $\sigma^2/N^{1/2}$ and the reliability of using s in place of σ. The distribution of the statistic is:

$$\pm t = \frac{\bar{x} - \mu}{s/\sqrt{N}} \qquad \text{or} \qquad \mu = \bar{x} \pm \frac{ts}{\sqrt{N}} \tag{2.14}$$

The distribution of the *t*-statistic $(\bar{x} - \mu)s$ is symmetrical about zero and is a function of the degrees of freedom. Limits assigned to the distance on either side of μ are called *confidence limits*. The percentage probability that μ lies within this interval is called the *confidence level*. The *level of significance* or *error probability* (100 − confidence level or 100 − α) is the percent probability that μ will lie outside the confidence interval, and represents the chances of being incorrect in stating that μ lies within the confidence interval. Values of t are in Table 2.27 for any desired degrees of freedom and various confidence levels.

An analytical procedure is often tested on materials of known composition. These materials may be pure substances, standard samples, or materials analyzed by some other more accurate method. Repeated determinations on a known material furnish data for both an estimate of the precision and a test for the presence of a constant error in the results. The standard deviation is found from Equation 12 (with the known composition replacing μ). A calculated value for t (Eq. 14) in excess of the appropriate value in Table 2.27 is interpreted as evidence of the presence of a constant error at the indicated level of significance.

TABLE 2.27 Percentile Values for Student *t* Distribution

df	$t_{0.995}$	$t_{0.99}$	$t_{0.975}$	$t_{0.95}$	$t_{0.90}$	$t_{0.80}$	$t_{0.75}$	$t_{0.70}$	$t_{0.60}$	$t_{0.55}$
1	63.66	31.82	12.71	6.31	3.08	1.376	1.000	0.727	0.325	0.158
2	9.92	6.96	4.30	2.92	1.89	1.061	0.816	0.617	0.289	0.142
3	5.84	4.54	3.18	2.35	1.64	0.978	0.765	0.584	0.277	0.137
4	4.60	3.75	2.78	2.13	1.53	0.941	0.741	0.569	0.271	0.134
5	4.03	3.36	2.57	2.02	1.48	0.920	0.727	0.559	0.267	0.132
6	3.71	3.14	2.45	1.94	1.44	0.906	0.718	0.553	0.265	0.131
7	3.50	3.00	2.36	1.90	1.42	0.896	0.711	0.549	0.263	0.130
8	3.36	2.90	2.31	1.86	1.40	0.889	0.706	0.546	0.262	0.130
9	3.25	2.82	2.26	1.83	1.38	0.883	0.703	0.543	0.261	0.129
10	3.17	2.76	2.23	1.81	1.37	0.879	0.700	0.542	0.260	0.129
11	3.11	2.72	2.20	1.80	1.36	0.876	0.697	0.540	0.260	0.129
12	3.06	2.68	2.18	1.78	1.36	0.873	0.695	0.539	0.259	0.128
13	3.01	2.65	2.16	1.77	1.35	0.870	0.694	0.538	0.259	0.128
14	2.98	2.62	2.14	1.76	1.34	0.868	0.692	0.537	0.258	0.128
15	2.95	2.60	2.13	1.75	1.34	0.866	0.691	0.536	0.258	0.128
16	2.92	2.58	2.12	1.75	1.34	0.865	0.690	0.535	0.258	0.128
17	2.90	2.57	2.11	1.74	1.33	0.863	0.689	0.534	0.257	0.128
18	2.88	2.55	2.10	1.73	1.33	0.862	0.688	0.534	0.257	0.127
19	2.86	2.54	2.09	1.73	1.33	0.861	0.688	0.533	0.257	0.127
20	2.84	2.53	2.09	1.72	1.32	0.860	0.687	0.533	0.257	0.127
21	2.83	2.52	2.08	1.72	1.32	0.859	0.686	0.532	0.257	0.127
22	2.82	2.51	2.07	1.72	1.32	0.858	0.686	0.532	0.256	0.127
23	2.81	2.50	2.07	1.71	1.32	0.858	0.685	0.532	0.256	0.127
24	2.80	2.49	2.06	1.71	1.32	0.857	0.685	0.531	0.256	0.127
25	2.79	2.48	2.06	1.71	1.32	0.856	0.684	0.531	0.256	0.127
26	2.78	2.48	2.06	1.71	1.32	0.856	0.684	0.531	0.256	0.127
27	2.77	2.47	2.05	1.70	1.31	0.855	0.684	0.531	0.256	0.127
28	2.76	2.47	2.05	1.70	1.31	0.855	0.683	0.530	0.256	0.127
29	2.76	2.46	2.04	1.70	1.31	0.854	0.683	0.530	0.256	0.127
30	2.75	2.46	2.04	1.70	1.31	0.854	0.683	0.530	0.256	0.127
40	2.70	2.42	2.02	1.68	1.30	0.851	0.681	0.529	0.255	0.126
60	2.66	2.39	2.00	1.67	1.30	0.848	0.679	0.527	0.254	0.126
120	2.62	2.36	2.98	1.66	1.29	0.845	0.677	0.526	0.254	0.126
∞	2.58	2.33	1.96	1.645	1.28	0.842	0.674	0.524	0.253	0.126

Example 7 A new method for the analysis of iron using pure FeO was replicated with five samples giving these results (in % Fe): 76.95, 77.02, 76.90, 77.20, and 77.50. Does a systematic error exist?

From Equation 4, $\bar{x}$ is 77.11; and from Equation 5, s is 0.24 for 4 degrees of freedom. Because σ is not known, the Student $t_{0.975}$ (2.78 for 4 degrees of freedom) is used to calculate the confidence interval at the 95% probability level.

$$\mu = \bar{x} \pm \frac{ts}{\sqrt{N}} = 77.11 \pm \frac{(2.78)\ (0.24)}{\sqrt{5}} = 77.11 \pm 0.30$$

We used a two-tailed test. Upon rereading the problem, we realize that this was pure FeO whose iron content was 77.60% so that $\mu = 77.60$ and the confidence interval does not include the known value. Since the FeO was a standard, a one-tailed test should have been used since only random values would be expected to exceed 77.60%. Now the Student t value of 2.13 (for $-t_{0.05}$) should have been used, and now the confidence interval becomes 77.11 ± 0.23. A systematic error is presumed to exist.

The t test can be applied to differences between pairs of observations. Perhaps only a single pair can be performed at one time, or possibly one wishes to compare two methods using samples of differing analytical content. It is still necessary that the two methods possess the same inherent standard deviation. An average difference $\bar{d}$ calculated, and individual deviations from $\bar{d}$ are used to evaluate the variance of the differences.

Example 8 From the following data do the two methods actually give concordant results?

Sample	Method *A*	Method *B*	Difference
1	33.27	33.04	$d_1 = 0.23$
2	51.34	50.96	$d_2 = 0.38$
3	23.91	23.77	$d_3 = 0.14$
4	47.04	46.79	$d_4 = 0.25$
			$\bar{d} = 0.25$

$$s_d = \frac{\sqrt{\sum (d - \bar{d})^2}}{N - 1} = 0.099$$

$$t = \frac{0.25}{0.099} \sqrt{4 - 1} = 4.30$$

From Table 2.27, $t_{0.975} = 3.18$ (at 95% probability) and $t_{0.995} = 5.84$ (at 99% probability). The difference between the two methods is probably significant.

If the t-value falls short of the formal significance level, this is not to be interpreted as proving the absence of a systematic error. Perhaps the data were insufficient in precision or in number to establish the presence of a constant error. Especially when the calculated value for t is only slightly short of the tabulated value, some additional data may suffice to build up the evidence for a constant error (or the lack thereof).

Should there be more than one known material, a weighted average of the individual differences ($\bar{x}$) should be taken. The value of s should be based on the combined estimate from the two or more materials (perhaps different primary standards for bases). Should the materials differ markedly in composition, a plot of the individual constant errors against composition should be made. If the constant error appear to depend upon the composition, they should not be pooled in a weighted average.

The t test is also used to judge whether a given lot of material conforms to a particular specification. If both plus and minus departures from the known value are to be guarded against, a two-tailed test is involved. If departures in only one direction are undesirable, then the 10% level values for t are appropriate for the 5% level in *one* direction. Similarly, the 2% level should be used to obtain the 1% level to test the departure from the known value in one direction only; these constitute a one-tailed test. More on this subject will be in the next section.

Sometimes just one determination is available on each of several known materials similar in composition. A single determination by each of two procedures (or two analysts) on a series of material may be used to test for a relative bias between the two methods, as in Example 2.4. Of course, the average difference does not throw any light on which procedure has the larger constant error. It only supplies a test as to whether the two procedures are in disagreement.

2.3.7 Hypotheses About Means

Statistical methods are frequently used to give a "yes" or "no" answer to a particular question concerning the significance of data. When performing hypothesis tests on real data, we cannot set an absolute cutoff as to where we can expect to find no values from the population against which we are testing data, but we can set a limit beyond which we consider it very unlikely to find a member of the population. If a measurement is made that does in fact fall outside the specified range, the probability of its happening by chance alone can be rejected; something beyond the randomness of the reference population must be operating. In other words, hypothesis testing is an attempt to determine whether a given measured statistic could have come from some hypothesized population.

In attempting to reach decisions, it is useful to make assumptions or guesses about the populations involved. Such assumptions, which may or may not be true, are called *statistical hypotheses* and in general are statements about the probability distributions of the populations. A common procedure is to set up a *null hypothesis,* denoted by H_0, which states that there is no significant difference between two sets of data or that a variable exerts no significant effect. Any hypothesis which differs from a null hypothesis is called an *alternative hypothesis*, denoted by H_1.

Our answer is qualified by a confidence level (or level of significance) indicating the degree of certainty of the answer. Generally confidence levels of 95% and 99% are chosen to express the probability that the answer is correct. These are also denoted as the 0.05 and 0.01 level of significance, respectively. When the hypothesis can be rejected at the 0.05 level of significance, but not at the 0.01 level, we can say that the sample results are probably significant. If, however, the hypothesis is also rejected at the 0.01 level, the results become highly significant.

The abbreviated table on the next page, which gives critical values of z for both one-tailed and two-tailed tests at various levels of significance, will be found useful for purposes of reference. Critical values of z for other levels of significance are found by the use of Table 2.26*b*. For a small number of samples we replace z, obtained from above or from Table 2.26*b*, by t from Table 2.27, and we replace σ by:

$$[\sqrt{N/(N-1)}]\, s$$

Level of significance, α	0.10	0.05	0.01	0.005	0.002
Critical values of z for one-tailed tests	-1.28 or 1.28	-1.645 or 1.645	-2.33 or 2.33	-2.58 or 2.58	-2.88 or 2.88
Critical values of z for two-tailed tests	-1.645 and 1.645	-1.96 and 1.96	-2.58 and 2.58	-2.81 and 2.81	-3.08 and 3.08

Procedures which enable us to decide whether to accept or reject hypotheses or to determine whether observed samples differ significantly from expected results are called *tests of hypotheses, tests of significance,* or *rules of decision.* For example, a set of z values outside the range -1.96 to 1.96 (at the 0.05 level of significance for a two-tailed test), constitute what is called the critical region or region of rejection of the hypothesis. The set of z results inside the range -1.96 to 1.96 could then be called the region of acceptance of the hypothesis.

Example 9 In the past a method gave $\mu = 0.050\%$. A recent set of 10 results gave $\bar{x} = 0.053\%$ and $s = 0.003\%$. Is everything satisfactory at a level of significance of 0.05? Of 0.01?

We wish to decide between the hypotheses:

H_0: $\mu = 0.050\%$ and the method is working properly, and

H_1: $\mu \neq 0.050\%$ and the method is not working properly.

A *two-tailed test* is required; that is, both tails on the distribution curve are involved:

$$t = \frac{0.053 - 0.050}{0.003}\sqrt{10 - 1} = -3.00$$

Enter Table 2.27 for nine degrees of freedom under the column headed $t_{0.975}$ for the 0.05 level of significance, and the column $t_{0.995}$ for the 0.01 level of significance. At the 0.05 level, accept H_0 if t lies inside the interval $-t_{0.975}$ to $t_{0.975}$, that is, within -2.26 and 2.26; reject otherwise. Since $t = -3.00$, we reject H_0. At the 0.01 level of significance, the corresponding interval is -3.25 to 3.25, which t lies within, indicating acceptance of H_0. Because we can reject H_0 at the 0.05 level but not at the 0.01 level of significance, we can say that the sample results are probably significant and that the method is working properly.

Let us digress a moment and consider when a two-tailed test is needed, and what a one-tailed test implies. We "assume" that the measurements can be described by the curve shown in Fig. 2.10. If so, then 95% of the time a sample from the specified population will fall within the indicated range and 5% of the time it will fall outside; 2.5% of the time it is outside on the high side of the range, and 2.5% of the time it is below the low side of the range. Our assumption implies that if μ does not equal the hypothesized value, the probability of its being above the hypothesized value is equal to the probability of its being below the hypothesized value.

There will be incidences when the foregoing assumptions for a two-tailed test will not be true. Perhaps some physical situation prevents μ from ever being less than the hypothesized value; it can only be equal or greater. No results would ever fall below the low end of the confidence interval; only the upper end of the distribution is operative. Now random samples will exceed the upper bound only 2.5% of the time, not the 5% specified in two-tail testing. Thus, where the possible values are restricted, what was supposed to be a hypothesis test at the 95% confidence level is actually being performed at a 97.5% confidence level. Stated in another way, 95% of the population data lie within the interval below $\mu + 1.65\sigma$ and 5% lie above. Of course, the opposite situation might also occur and only the lower end of the distribution is operative.

Example 10 Six samples from a bulk chemical shipment averaged 77.50% active ingredient with $s = 1.45\%$. The manufacturer claimed 80.00%. Can this claim be supported?

A one-tailed test is required since the alternative hypothesis states that the population parameter is equal to or less than the hypothesized value.

$$t = \frac{77.50 - 80.00}{1.45}\sqrt{6 - 1} = 3.86$$

Since $t_{0.95} = -2.01$, and $t_{0.99} = -3.36$, the hypothesis is rejected at both the 0.05 and the 0.01 levels of significance. It is extremely unlikely that the claim is justified.

2.3.8 The Chi-square (χ^2) Distribution

The χ^2 distribution describes the behavior of variances. Actually there is not a single χ^2 distribution but a whole set of distributions. Each distribution depends upon the number of degrees of freedom (designated variously as *df, d.f.,* or *f*) in that distribution. Table 2.28 is laid out so that the horizontal axis is labeled with probability levels, while the vertical axis is listed in descending order of increasing number of degrees of freedom. The entries increase both as you read down and across the table. Although Table 2.28 does not display the values for the mid-range of the distributions, at the 50% point of each distribution, the expected value of χ^2 is equal to the degrees of freedom. Estimates of the variance are uncertain when based only on a few degrees of freedom. With the 10 samples in Example 11, the standard deviation can vary by a large factor purely by random chance alone. Even 31 samples gives a spread of standard deviation of 2.6 at the 95% confidence level.

Understanding the χ^2 distribution allows us to calculate the expected values of random variables that are normally and independently distributed. In least squares multiple regression, or in calibration work in general, there is a basic assumption that the error in the response variable is random and normally distributed, with a variance that follows a χ^2 distribution.

Confidence limits for an estimate of the variance may be calculated as follows. For each group of samples a standard deviation is calculated. These estimates of σ possess a distribution called the χ^2 distribution:

$$\chi^2 = \frac{s^2}{\sigma^2/df} \tag{2.15}$$

The upper and lower confidence limits for the standard deviation are obtained by dividing $(N - 1)s^2$ by two entries taken from Table 2.28. The estimate of variance at the 90% confidence limits is for use in the entries $\chi^2_{0.05}$ and $\chi^2_{0.95}$ (for 5% and 95%) with N degrees of freedom.

Example 11 The variance obtained for 10 samples is $(0.65)^2$. σ^2 is known to be $(0.75)^2$. How reliable is s^2 as an estimate of σ^2?

$$\frac{s^2\,(N - 1)}{\chi^2_{0.975}} < \sigma^2 < \frac{s^2\,(N - 1)}{\chi^2_{0.025}}$$

$$\frac{(0.65)^2\,(10 - 1)}{19.02} < \sigma^2 < \frac{(0.65)^2\,(10 - 1)}{2.70}$$

$$0.20 < \sigma^2 < 1.43$$

Thus, only one time in 40 will $9s^2/\sigma^2$ be less than 2.70 by chance alone. Similarly, only one time

TABLE 2.28 Percentile Values for the Chi-square (χ^2) Distribution

df	$\chi^2_{0.995}$	$\chi^2_{0.99}$	$\chi^2_{0.975}$	$\chi^2_{0.95}$	$\chi^2_{0.90}$	$\chi^2_{0.75}$	$\chi^2_{0.50}$	$\chi^2_{0.25}$	$\chi^2_{0.10}$	$\chi^2_{0.05}$	$\chi^2_{0.025}$	$\chi^2_{0.01}$	$\chi^2_{0.005}$
1	7.88	6.63	5.02	3.84	2.71	1.32	0.455	0.102	0.0158	0.0039	0.0010	0.0002	0.0000
2	10.6	9.21	7.38	5.99	4.61	2.77	1.39	0.575	0.211	0.103	0.0506	0.0201	0.0100
3	12.8	11.3	9.35	7.81	6.25	4.11	2.37	1.21	0.584	0.352	0.216	0.115	0.072
4	14.9	13.3	11.1	9.49	7.78	5.39	3.36	1.92	1.06	0.711	0.484	0.297	0.207
5	16.7	15.1	12.8	11.1	9.24	6.63	4.35	2.67	1.61	1.15	0.831	0.554	0.412
6	18.5	16.8	14.4	12.6	10.6	7.84	5.35	3.45	2.20	1.64	1.24	0.872	0.676
7	20.3	18.5	16.0	14.1	12.0	9.04	6.35	4.25	2.83	2.17	1.69	1.24	0.989
8	22.0	20.1	17.5	15.5	13.4	10.2	7.34	5.07	3.49	2.73	2.18	1.65	1.34
9	23.6	21.7	19.0	16.9	14.7	11.4	8.34	5.90	4.17	3.33	2.70	2.09	1.73
10	25.2	23.2	20.5	18.3	16.0	12.5	9.34	6.74	4.87	3.94	3.25	2.56	2.16
11	26.8	24.7	21.9	19.7	17.3	13.7	10.3	7.58	5.58	4.57	3.82	3.05	2.60
12	28.3	26.2	23.3	21.0	18.5	14.8	11.3	8.44	6.30	5.23	4.40	3.57	3.07
13	29.8	27.7	24.7	22.4	19.8	16.0	12.3	9.30	7.04	5.89	5.01	4.11	3.57
14	31.3	29.1	26.1	23.7	21.1	17.1	13.3	10.2	7.79	6.57	5.63	4.66	4.07
15	32.8	30.6	27.5	25.0	22.3	18.2	14.3	11.0	8.55	7.26	6.26	5.23	4.60
16	34.3	32.0	28.8	26.3	23.5	19.4	15.3	11.9	9.31	7.96	6.91	5.81	5.14
17	35.7	33.4	30.2	27.6	24.8	20.5	16.3	12.8	10.1	8.67	7.56	6.41	5.70
18	37.2	34.8	31.5	28.9	26.0	21.6	17.3	13.7	10.9	9.39	8.23	7.01	6.26
19	38.6	36.2	32.9	30.1	27.2	22.7	18.3	14.6	11.7	10.1	8.91	7.63	6.84
20	40.0	37.6	34.2	31.4	28.4	23.8	19.3	15.5	12.4	10.9	9.59	8.26	7.43
21	41.4	38.9	35.5	32.7	29.6	24.9	20.3	16.3	13.2	11.6	10.3	8.90	8.03
22	42.8	40.3	36.8	33.9	30.8	26.0	21.3	17.2	14.0	12.3	11.0	9.54	8.64
23	44.2	41.6	38.1	35.2	32.0	27.1	22.3	18.1	14.8	13.1	11.7	10.2	9.26
24	45.6	43.0	39.4	36.4	33.2	28.2	23.3	19.0	15.7	13.8	12.4	10.9	9.89
25	46.9	44.3	40.6	37.7	34.4	29.3	24.3	19.9	16.5	14.6	13.1	11.5	10.5
26	48.3	45.6	41.9	38.9	35.6	30.4	25.3	20.8	17.3	15.4	13.8	12.2	11.2
27	49.6	47.0	43.2	40.1	36.7	31.5	26.3	21.7	18.1	16.2	14.6	12.9	11.8
28	51.0	48.3	44.5	41.3	37.9	32.6	27.3	22.7	18.9	16.9	15.39	13.6	12.5
29	52.3	49.6	45.7	42.6	39.1	33.7	28.3	23.6	19.8	17.7	16.0	14.3	13.1
30	53.7	50.9	47.0	43.8	40.3	34.8	29.3	24.5	20.6	18.5	16.8	15.0	13.8
40	66.8	63.7	59.3	55.8	51.8	45.6	39.3	33.7	29.1	26.5	24.4	22.2	20.7
50	79.5	76.2	71.4	67.5	63.2	56.3	49.3	42.9	37.7	34.8	32.4	29.7	28.0
60	92.0	88.4	83.3	79.1	74.4	67.0	59.3	52.3	46.5	43.2	40.5	37.5	35.5
70	104.2	100.4	95.0	90.5	85.5	77.6	69.3	61.7	55.3	51.7	48.8	45.4	43.3
80	116.3	112.3	106.6	101.9	96.6	88.1	79.3	71.1	64.3	60.4	57.2	53.5	51.2
90	128.3	124.1	118.1	113.1	107.6	98.6	89.3	80.6	73.3	69.1	65.6	61.8	59.2
100	140.2	135.8	129.6	124.3	118.5	109.1	99.3	90.1	82.4	77.9	74.2	70.1	67.3

in 40 will $9s^2/\sigma^2$ be greater than 19.02. Consequently, it is not unlikely that s^2 is a reliable estimate of σ^2.

Stated differently:

$$\text{Upper limit:} \qquad \sigma^2 = 9s^2/2.7 = 3.3s^2$$

$$\text{Lower limit:} \qquad \sigma^2 = 9s^2/19.02 = 0.48s^2$$

Ten measurements give an estimate of σ^2 that may be as much as 3.3 times or only about one-half the true variance.

2.3.9 The *F* Statistic

The *F* statistic, along with the *z*, *t*, and χ^2 statistics, constitute the group that are thought of as fundamental statistics. Collectively they describe all the relationships that can exist between means and standard deviations. To perform an *F* test, we must first verify the randomness and independence of the errors. If $\sigma_1^2 = \sigma_2^2$, then s_1^2/s_2^2 will be distributed properly as the *F* statistic. If the calculated *F* is outside the confidence interval chosen for that statistic, then this is evidence that $\sigma_1^2 \neq \sigma_2^2$.

The *F* statistic describes the distribution of the ratios of variances of two sets of samples. It requires three table labels: the probability level and the two degrees of freedom. Since the *F* distribution requires a three-dimensional table which is effectively unknown, the *F* tables are presented as large sets of two-dimensional tables. The *F* distribution in Table 2.29 has the different numbers of degrees of freedom for the denominator variance placed along the vertical axis, while in each table the two horizontal axes represent the numerator degrees of freedom and the probability level. Only two probability levels are given in Table 2.29: the upper 5% points ($F_{0.95}$) and the upper 1% points ($F_{0.99}$). More extensive tables of statistics will list additional probability levels, and they should be consulted when needed.

It is possible to compare the means of two relatively small sets of observations when the variances within the sets can be regarded as the same, as indicated by the *F* test. One can consider the distribution involving estimates of the true variance. With s_1^2 determined from a group of N_1 observations and s_2^2 from a second group of N_2 observations, the distribution of the ratio of the sample variances is given by the *F* statistic:

$$F = \frac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2} \tag{2.16}$$

The larger variance is placed in the numerator. For example, the *F* test allows judgment regarding the existence of a significant difference in the precision between two sets of data or between two analysts. The hypothesis assumed is that both variances are indeed alike and a measure of the same σ.

The fact that each sample variance is related to its own population variance means that the sample variance being used for the calculation need not come from the same population. This is a significant departure from the assumptions inherent in the *z*, *t*, and χ^2 statistics.

Example 12 Suppose Analyst *A* made five observations and obtained a standard deviation of 0.06, where Analyst *B* with six observations obtained $s_B = 0.03$. The experimental variance ratio is:

$$F = \frac{(0.06)^2}{(0.03)^2} = 4.00$$

From Table 2.28 with four degrees of freedom for *A* and five degrees of freedom for *B*, the value of *F* would exceed 5.19 five percent of the time. Therefore, the null hypothesis is valid, and comparable skills are exhibited by the two analysts.

As applied in Example 12, the *F* test was one-tailed. The *F* test may also be applied as a two-tailed test in which the alternative to the null hypothesis is $\sigma_1^2 \neq \sigma_2^2$. This doubles the probability that the null hypothesis is invalid and has the effect of changing the confidence level, in the above example, from 95% to 90%.

If improvement in precision is claimed for a set of measurements, the variance for the set against which comparison is being made should be placed in the numerator, regardless of magnitude. An experimental *F* smaller than unity indicates that the claim for improved precision cannot be supported. The technique just given for examining whether the precision varies with the two different analytical procedures, also serves to compare the precision with different materials, or with different operators, laboratories, or sets of equipment.

TABLE 2.29 *F* Distribution

Interpolation should be performed using reciprocals of the degrees of freedom.

	Upper 5% points ($F_{0.95}$)																		
	Degrees of freedom for numerator																		
Degrees of freedom for denominator	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161	200	216	225	230	234	237	239	241	242	244	246	248	249	250	251	252	253	254
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

TABLE 2.29 *F* Distribution (*Continued*)

Upper 1% points ($F_{0.99}$)																			
Degrees of freedom for denominator	Degrees of freedom for numerator																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	4052	5000	5403	5625	5764	5859	5928	5982	6023	6056	6106	6157	6209	6235	6261	6287	6313	6339	6366
2	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.5	99.5	99.5	99.5	99.5	99.5
3	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	26.9	26.7	26.6	26.5	26.4	26.3	26.2	26.1
4	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	14.4	14.2	14.0	13.9	13.8	13.7	13.7	13.6	13.5
5	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02
6	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88
7	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65
8	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86
9	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31
10	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17
14	8.86	6.51	5.56	5.04	4.70	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.00
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75
17	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57
19	8.19	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21
25	7.77	5.57	4.68	4.18	3.86	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.62	2.53	2.45	2.36	2.27	2.17
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.00

2.3.10 Curve Fitting

Very often in practice a relationship is found (or known) to exist between two or more variables. It is frequently desirable to express this relationship in mathematical form by determining an equation connecting the variables.

The first step is the collection of data showing corresponding values of the variables under consideration. From a scatter diagram, a plot of Y (ordinate) versus X (abscissa), it is often possible to visualize a smooth curve approximating the data. For purposes of reference, several types of approximating curves and their equations are listed. All letters other than X and Y represent constants.

1. $Y = \mathrm{a}_0 + a_1X$	Straight line
2. $Y = \mathrm{a}_0 + a_1X + a_2X^2$	Parabola or quadratic curve
3. $Y = \mathrm{a}_0 + a_1X + a_2X^2 + a_3X^3$	Cubic curve
4. $Y = \mathrm{a}_0 + a_1X + a_2 + \cdot\ \cdot\ \cdot + a_n X^n$	*n*th degree curve

As other possible equations (among many) used in practice, these may be mentioned:

5. $Y = (a_0 + a_1X)^{-1}$ or $1/Y = a_0 + a_1X$	Hyperbola
6. $Y = ab^X$ or $\log Y = \log a + (\log b)_{\mathrm{X}}$	Exponential curve
7. $Y = aX^b$ or $\log Y = \log a + b \log \mathrm{X}$	Geometric curve
8. $Y = ab^X + g$	Modified exponential curve
9. $Y = aX^n + g$	Modified geometric curve

When we draw a scatter plot of all X versus Y data, we see that some sort of shape can be described by the data points. From the scatter plot we can take a basic guess as to which type of curve will best describe the X—Y relationship. To aid in the decision process, it is helpful to obtain scatter plots of transformed variables. For example, if a scatter plot of log Y versus X shows a linear relationship, the equation has the form of number 6 above, while if log Y versus log X shows a linear relationship, the equation has the form of number 7. To facilitate this we frequently employ special graph paper for which one or both scales are calibrated logarithmically. These are referred to as *semilog* or *log-log graph paper*, respectively.

2.3.10.1 The Least Squares or Best-fit Line. The simplest type of approximating curve is a straight line, the equation of which can be written as in form number 1 above. It is customary to employ the above definition when X is the independent variable and Y is the dependent variable.

To avoid individual judgment in constructing any approximating curve to fit sets of data, it is necessary to agree on a definition of a *best-fit line*. One could construct what would be considered the best-fit line through the plotted pairs of data points. For a given value of X_1, there will be a difference D_1 between the value Y_1 and the constituent value $\hat{Y}$ as determined by the calibration model. Since we are assuming that all the errors are in Y, we are seeking the best-fit line that minimizes the deviations in the Y direction between the experimental points and the calculated line. This condition will be met when the sum of squares for the differences, called residuals (or the sum of squares due to error),

$$\sum_{i=1}^{N} (Y_i - \hat{Y}_i)^2 \equiv \sum (D_1^2 + D_2^2 + \cdots + D_N^2)$$

is the least possible value when compared to all other possible lines fitted to that data. If the sum of squares for residuals is equal to zero, the calibration line is a perfect fit to the data. With a mathematical treatment known as linear regression, one can find the "best" straight line through these real world points by minimizing the residuals.

This calibration model for the best-fit fit line requires that the line pass through the "centroid" of the points ($\overline{X}$, $\overline{Y}$). It can be shown that:

$$b = \frac{\sum_i (X_i - \overline{X})(Y_i - \overline{Y})}{\sum_i (X_i - \overline{X})^2} \tag{2.17}$$

$$a = \overline{Y} - b\overline{X} \tag{2.18}$$

The line thus calculated is known as the line of regression of Y on X, that is, the line indicating how Y varies when X is set to chosen values.

If X is the dependent variable, the definition is modified by considering horizontal instead of vertical deviations. In general these two definitions lead to different least square curves.

Example 13 The following data were recorded for the potential E of an electrode, measured against the saturated calomel electrode, as a function of concentration C (moles liter^{-1}).

$-\log C$	E, mV	$-\log C$	E, mV
1.00	106	2.10	174
1.10	115	2.20	182
1.20	121	2.40	187
1.50	139	2.70	211
1.70	153	2.90	220
1.90	158	3.00	226

Fit the best straight line to these data; X_i represents $-\log C$, and Y_i represents E. We will perform the calculation manually, using the following tabular lay-out.

X_i	Y_i	$(X_i - \overline{X})$	$(X_i - \overline{X})^2$	$(Y_i - \overline{Y})$	$(X_i - \overline{X})(Y_i - \overline{Y})$
1.00	106	−0.975	0.951	−60	58.5
1.10	115	−0.875	0.766	−51	44.6
1.20	121	−0.775	0.600	−45	34.9
1.50	139	−0.475	0.226	−27	12.8
1.70	153	−0.275	0.076	−13	3.6
1.90	158	−0.075	0.006	−8	0.6
2.10	174	+0.125	0.016	8	1.0
2.20	182	0.225	0.051	16	3.6
2.40	187	0.425	0.181	21	8.9
2.70	211	0.725	0.526	45	32.6
2.90	220	0.925	0.856	54	50.0
3.00	226	1.025	1.051	60	61.5
ΣX_i 23.7	ΣY_i 1992	Σ 0	Σ 5.306	Σ 0	Σ 312.6
$\overline{X} = 1.975$	$\overline{Y} = 166$				

Now substituting the proper terms into Equation 17, the slope is:

$$b = \frac{312.6}{5.306} = 58.91$$

and from Equation 18, and substituting the “centroid” values of the points ($\overline{X}$, $\overline{Y}$), the intercept is:

$$a = 166 - 58.91(1.975) = 49.64$$

The best-fit equation is therefore:

$$E = 49.64 - 58.91 \log C$$

2.3.10.2 Errors in the Slope and Intercept of the Best-fit Line. Upon examination of the plot of pairs of data points, the calibration line, it will be obvious that the precision involved in analyzing an unknown sample will be considerably poorer than that indicated by replicate error alone. The scatter of these original points about the calibration line is a good measure of the error to be expected in analyzing an unknown sample. And this same error is considerably larger than the replication error because it will include other sources of variability due to a variety of causes. One possible source of variability might be the presence of different amounts of an extraneous material in the various samples used to establish the calibration curve. While this variability causes scatter about the calibration curve, it will not be reflected in the replication error of any one sample if the sample is homogeneous.

The scatter of the points around the calibration line or random errors are of importance since the best-fit line will be used to estimate the concentration of test samples by interpolation. The method used to calculate the random errors in the values for the slope and intercept is now considered. We must first calculate the standard deviation $s_{Y/X}$, which is given by:

$$s_{Y/X} = \sqrt{\frac{\sum_i (Y_\mathrm{i} - \hat{Y})^2}{N - 2}} \tag{2.19}$$

Equation 19 utilizes the *Y-residuals,* $Y_i - \hat{Y}$, where $\hat{Y}_i$ are the points on the calculated best-fit line or the fitted Y_i values. The appropriate number of degrees of freedom is $N - 2$; the minus 2 arises from the fact that linear calibration lines are derived from both a slope and an intercept which leads to a loss of two degrees of freedom.

Now we can calculate the standard deviations for the slope and the intercept. These are given by:

$$s_b = \frac{s_{Y/X}}{\sqrt{\sum_i (X_i - \overline{X})^2}} \tag{2.20}$$

$$s_a = s_{Y/X} \sqrt{\frac{\sum_i X_i^2}{N \sum_i (X_i - \overline{X})^2}} \tag{2.21}$$

The confidence limits for the slope are given by $b \pm t_b$, where the t-value is taken at the desired confidence level and $(N - 2)$ degrees of freedom. Similarly, the confidence limits for the intercept are given by $a \pm ts_a$. The closeness of $\hat{x}$ to x_i is answered in terms of a confidence interval for x_0 that extends from an upper confidence (UCL) to a lower confidence (LCL) level. Let us choose 95% for the confidence interval. Then, remembering that this is a two-tailed test (UCL and LCL), we obtain from a table of Student's t distribution the critical value of t_c ($t_{0.975}$) and the appropriate number of degrees of freedom.

Example 14 For the best-fit line found in Example 13, express the result in terms of confidence intervals for the slope and intercept. We will choose 95% for the confidence interval.

The standard deviation $s_{Y/X}$ is given by Equation 19, but first a supplementary table must be constructed for the Y residuals and other data which will be needed in subsequent equations.

$\hat{Y}$	$(Y_i - \hat{Y})$	$(Y_i - \hat{Y})^2$	X_i^2
108.6	2.55	6.50	1.00
114.4	−0.56	0.31	1.21
120.3	−0.67	0.45	1.44
138.0	−1.00	1.00	2.25
149.8	−3.21	10.32	2.89
161.6	3.57	12.94	3.61
173.4	−0.65	0.42	4.41
179.2	−2.76	7.61	4.84
191.0	4.02	16.16	5.76
208.7	−2.30	5.30	7.29
220.5	0.48	0.23	8.41
226.4	0.40	0.16	9.00
	Σ 61.20	Σ 52.11	

Now substitute the appropriate values into Equation 19 where there are $12 - 2 = 10$ degrees of freedom:

$$s_{X/Y} = \sqrt{\frac{61.20}{10}} = 2.47$$

We can now calculate s_b and s_a from Equations 20 and 21, respectively:

$$s_b = \frac{s_{Y/X}}{\sqrt{5.31}} = 1.07$$

and

$$s_a = 2.47\sqrt{\frac{52.11}{12(5.306)}} = 2.23$$

Now, using a two-tailed value for Student's t:

$$b \pm ts_b = 58.91 \pm 2.23(1.07) = 58.91 \pm 2.39$$

$$a \pm ts_a = 49.64 \pm 2.23(2.23) = 49.64 \pm 4.97$$

The best-fit equation expressed in terms of the confidence intervals for the slope and intercept is:

$$E = (49.6_4 \pm 5.0) - (58.9_1 \pm 2.43) \log C$$

To conclude the discussion about the best-fit line, the following relationship can be shown to exist among Y, $\hat{Y}$, and $\overline{Y}$:

$$\sum_{i=1}^{N} (Y_i - \overline{Y})^2 = \sum_{i=1}^{N} (\hat{Y}_i - \overline{Y})^2 + \sum_{i=1}^{N} (Y_i - \hat{Y}_i)^2 \qquad (2.22)$$

The term on the left-hand side is a constant and depends only on the constituent values provided by the reference laboratory and does not depend in any way upon the calibration. The two terms on the right-hand side of the equation show how this constant value is apportioned between the two quantities that are themselves summations, and are referred to as the sum of squares due to regression and the sum of squares due to error. The latter will be the smallest possible value that it can possibly be for the given data.

2.3.11 Control Charts

It is often important in practice to know when a process has changed sufficiently so that steps may be taken to remedy the situation. Such problems arise in quality control where one must, often quickly, decide whether observed changes are due to simple chance fluctuations or to actual changes in the amount of a constituent in successive production lots, mistakes of employees, etc. Control charts provide a useful and simple method for dealing with such problems.

The chart consists of a central line and two pairs of limit lines or simply of a central line and one pair of control limits. By plotting a sequence of points in order, a continuous record of the quality characteristic is made available. Trends in data or sudden lack of precision can be made evident so that the causes may be sought.

The control chart is set up to answer the question of whether the data are in statistical control, that is, whether the data may be retarded as random samples from a single population of data. Because of this feature of testing for randomness, the control chart may be useful in searching out systematic sources of error in laboratory research data as well as in evaluating plant-production or control-analysis data.[1]

To set up a control chart, individual observations might be plotted in sequential order and then compared with control limits established from sufficient past experience. Limits of $\pm 1.96\sigma$ corresponding to a confidence level of 95%, might be set for control limits. The probability of a future observation falling outside these limits, based on chance, is only 1 in 20. A greater proportion of scatter might indicate a nonrandom distribution (a systematic error). It is common practice with some users of control charts to set inner control limits, or warning limits, at $\pm 1.96\sigma$ and outer control limits of $\pm 3.00\sigma$. The outer control limits correspond to a confidence level of 99.8%, or a probability of 0.002 that a point will fall outside the limits. One-half of this probability corresponds to a high result and one-half to a low result. However, other confidence limits can be used as well; the choice in each case depends on particular circumstances.

Special attention should be paid to one-sided deviation from the control limits, because systematic errors more often cause deviation in one direction than abnormally wide scatter. Two systematic errors of opposite sign would of course cause scatter, but it is unlikely that both would have entered at the same time. It is not necessary that the control chart be plotted in a time sequence. In any

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[1] G. Wernimont, *Ind. Eng. Chem., Anal. Ed.* **18:**587 (1946); J. A. Mitchell, *ibid.* **19:**961 (1947).

situation where relatively large numbers of units or small groups are to be compared, the control chart is a simple means of indicating whether any unit or group is out of line. Thus laboratories, production machines, test methods, or analysts may be put arbitrarily into a horizontal sequence.

Usually it is better to plot the means of small groups of observations on a control chart, rather than individual observations. The random scatter of averages of pairs of observations is $1/(2)^{1/2} = 0.71$ as great as that of single observations, and the likelihood of two "wild" observations in the same direction is vanishing small. The groups of two to five observations should be chosen in such a way that only change variations operate within the group, whereas assignable causes are sought for variations between groups. If duplicate analyses are performed each day, the pairs form logical groups.

Some measure of dispersion of the subgroup data should also be plotted as a parallel control chart. The most reliable measure of scatter is the standard deviation. For small groups, the range becomes increasingly significant as a measure of scatter, and it is usually a simple matter to plot the range as a vertical line and the mean as a point on this line for each group of observations.

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