03年 06年，07年

已知刚塑性变形体中某点处平面应力张量为 $\begin{bmatrix} -60 & \sqrt{300} \\ \sqrt{300} & -30 \end{bmatrix}$ MPa 应变分量 $d\varepsilon_x = -\delta$

试求应变增量张量及塑性功增量密度。

解：用增量理论 $\frac{d\varepsilon_x}{\sigma_x'} = \cdots = d\lambda = \frac{3}{2}\frac{d\bar{\varepsilon}}{\bar{\sigma}}$ 或 $d\varepsilon_x = \frac{d\bar{\varepsilon}}{\bar{\sigma}}[\sigma_x - \frac{1}{2}(\sigma_y + \sigma_z)]$

应力张量 $\begin{bmatrix} -60 & \sqrt{300} & 0 \\ \sqrt{300} & -30 & 0 \\ 0 & 0 & 0 \end{bmatrix}$　　$\sigma_m = \frac{\sigma_x + \sigma_y}{3} = -30$

$\sigma_x' = \sigma_x - \sigma_m = -30$　　$\sigma_y' = \sigma_y - \sigma_m = 0$　　$\sigma_z' = \sigma_z - \sigma_m = 30$

$\frac{d\varepsilon_{ij}}{\sigma_{ij}'} = \frac{d\varepsilon_x}{\sigma_x'} \Rightarrow d\varepsilon_{ij} = \frac{\sigma_{ij}'}{\sigma_x'} d\varepsilon_x$

$\therefore d\varepsilon_y = 0$　　$d\varepsilon_z = \frac{30}{-30} \times (-\delta) = \delta$

又 $\tau_{yz} = \tau_{zx} = 0$　　$\therefore d\gamma_{yz} = d\gamma_{zx} = 0$

$d\gamma_{xy} = \frac{\sqrt{300}}{-30}(-\delta) = \frac{\delta}{\sqrt{3}}$　　$\therefore \begin{bmatrix} -\delta & \frac{\delta}{\sqrt{3}} & 0 \\ \frac{\delta}{\sqrt{3}} & 0 & 0 \\ 0 & 0 & \delta \end{bmatrix}$

$\bar{\sigma} = \frac{\sqrt{2}}{2}\sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6\tau_{xy}^2} = 60$

$d\bar{\varepsilon} = \sqrt{\frac{2}{3}(d\varepsilon_x^2 + d\varepsilon_y^2 + d\varepsilon_z^2 + 2d\gamma_{xy}^2)} = \frac{4}{3}\delta$

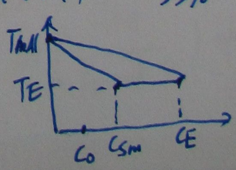
$dW^p = \bar{\sigma} d\bar{\varepsilon} = 60 \times \frac{4}{3}\delta = 80\delta$

03年 卷一　11年

水平浇注 Al-2.0% Cu 合金长圆棒形铸件，使其沿轴线单向凝固，冷却速度使得固-液界面保持为平面生长。设 Cu 在固-液界面固相侧中不扩散，在液相中为有限扩散。Al-Cu 二元状态图参数：Al 熔点 $T_{mAl} = 660°C$　Cu 在 Al 相中最大固溶度 $C_{sm} = 5.56\%$，共晶温度 $T_E = 548°C$，共晶点含 Cu 量 $C_E = 33\%$

解：$C_0 = 2.0\%$　　当 C_s 达到 C_{sm} 时，$C_L = C_E$

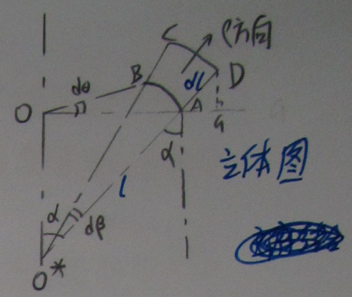
$\therefore k = \frac{C_{sm}}{C_E} = \frac{5.56\%}{33\%} = 16.8\%$　　m_L(液相线斜率) $= \frac{(660-548)°C}{33\%} = 339.4$



链熔点 $T_0 = T_{mAl} - C_0 \times \frac{T_m - T_E}{C_{sm}} = 507.7°C$

03.05 大圆筒拉深为小圆筒

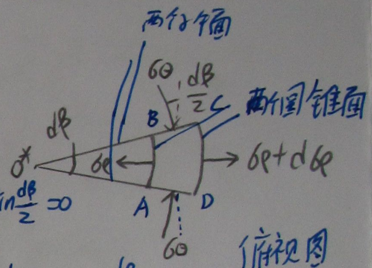
用两个夹角为dθ的子午面及两个圆筒件垂直轴线且垂直工件的圆锥面的圆锥面截取一个基元体ABCD，则 $OA=r$　$AG=dr$



由几何关系知

$O^*A = \frac{OA}{\sin\alpha}$　$\widehat{AB} = OA\,d\theta \simeq O^*A\,d\beta = l\,d\beta$

故 $d\beta = \sin\alpha\,d\theta$　又 $AD = dl = \frac{AG}{\sin\alpha} = \frac{dr}{\sin\alpha}$



沿ρ方向的静力平衡方程式为

$$(\sigma_\rho+d\sigma_\rho)(l+dl)d\beta\, t - \sigma_\rho l\, d\beta\, t + 2\sigma_\theta\, dl\, t \sin\frac{d\beta}{2} = 0$$

将上述关系代入得:

$$(\sigma_\rho+d\sigma_\rho)(r+dr)d\theta\, t - \sigma_\rho r\, d\theta\, t + \underline{2\sigma_\theta \frac{dr}{\sin\alpha} t \sin\frac{d\beta}{2}} = 0$$

$$2\sigma_\theta\, dr\, \frac{d\theta}{d\beta}\cdot\frac{d\beta}{2}$$

展开忽略高阶微量，得:

$$d\sigma_\rho r + \sigma_\rho dr + \sigma_\theta dr = 0$$

即　$d\sigma_\rho = -(\sigma_\rho+\sigma_\theta)\,dr$　由于σρ是拉应力，σθ是压应力

故　$\sigma_1 = \sigma_\rho$　$\sigma_3 = -\sigma_\theta$　近似塑性条件为

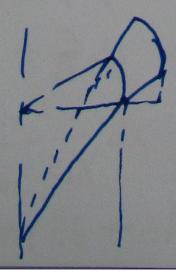
$$\sigma_1 - \sigma_3 = \sigma_\rho + \sigma_\theta = \beta S$$

故　$d\sigma_\rho = -\beta S \frac{dr}{r}$

$\sigma_\rho = -\beta S \ln r + C$　当 $r = \frac{D_0}{2}$ 时有 $\sigma_\rho = 0$

$\therefore C = \beta S \ln\frac{D_0}{2}$　　$\sigma_\rho = \beta S \ln\frac{D_0}{2r}$

拉深力P为　$P = \pi d_0 t\, \sigma_\rho|_{r=\frac{d_0}{2}} = \pi d_0 t\, \beta S \ln\frac{D_0}{d_0}$



①

三、由于是轴对称变形 则 $d\gamma_{r\theta} = d\gamma_{\theta z} = 0$　　✗ $\varepsilon_x = \frac{\partial u_x}{\partial x}$　$d\varepsilon_x = \frac{\partial (du_x)}{\partial x}$

由应力应变关系得 $\tau_{r\theta} = \tau_{\theta z} = 0$

$\therefore \sigma_{ij} = \begin{bmatrix} \sigma_r & 0 & \tau_{rz} \\ 0 & \sigma_\theta & 0 \\ \tau_{zr} & 0 & \sigma_z \end{bmatrix}$

$dW_P = \sigma_s d\bar{\varepsilon} = \sigma_s \cdot \delta \Rightarrow d\bar{\varepsilon} = \delta$　　轴对称 ∴ $d\varepsilon_r = d\varepsilon_\theta = \frac{\delta}{2}$

由小变形几何方程知：$d\varepsilon_r = \frac{\partial u_r}{\partial r} = \frac{\delta}{2}$

由 $d\varepsilon_r + d\varepsilon_\theta + d\varepsilon_z = 0 \Rightarrow d\varepsilon_z = -\delta$

$d\bar{\varepsilon} = \frac{\sqrt{2}}{3}\sqrt{(d\varepsilon_r - d\varepsilon_\theta)^2 + (d\varepsilon_\theta - d\varepsilon_z)^2 + (d\varepsilon_z - d\varepsilon_r)^2 + 6 d\gamma_{rz}^2} = \delta$

$\Rightarrow d\gamma_{rz} = 0$

由本构方程得：$d\varepsilon_{ij} = \frac{3}{2}\frac{d\bar{\varepsilon}}{\bar{\sigma}}\sigma'_{ij}$ 得：

$$\begin{cases} \sigma'_r = \sigma_r - \sigma_m = \frac{2}{3}\frac{\sigma_s}{\delta} d\varepsilon_r = \frac{1}{3}\sigma_s \\ \sigma'_\theta = \sigma_\theta - \sigma_m = \frac{1}{3}\sigma_s \\ \sigma'_z = \sigma_z - \sigma_m = -\frac{2}{3}\sigma_s \\ \tau_{rz} = \frac{2}{3}\frac{\sigma_s}{\delta} d\gamma_{rz} = 0 \end{cases}$$

又 $\sigma_r = \frac{1}{2}\sigma_s \Rightarrow \sigma_m = \frac{1}{6}\sigma_s$　∴ $\sigma_\theta = \frac{1}{2}\sigma_s$　$\sigma_z = -\frac{1}{2}\sigma_s$

∴ 该处应力张量为 $\sigma_{ij} = \begin{bmatrix} \frac{1}{2}\sigma_s & 0 & 0 \\ 0 & \frac{1}{2}\sigma_s & 0 \\ 0 & 0 & -\frac{1}{2}\sigma_s \end{bmatrix}$

四、解：(05年, 04年, 06年, 07年, 09年, 11年

凝固10%时 $f_S = 10\%$ $f_L = 90\%$ 根据夏尔公式

① $k = \frac{C_{sm}}{C_E} = \frac{30\%}{60\%} = 0.5$

$C_S^* = kC_0(1-f_S)^{k-1} = 0.5 \times 40\% \times (90\%)^{0.5-1} = 21.1\%$

$C_L^* = C_0 f_L^{k-1} = 42.2\%$

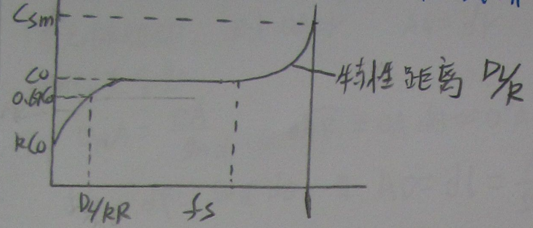
② 凝固完毕，共晶体所占比例为 $C_S^* = C_{sm}$ 时的 f_L

$C_{sm} = C_S^* = kC_0 f_L^{k-1} = 30\%$　得：$f_L = 44.4\%$

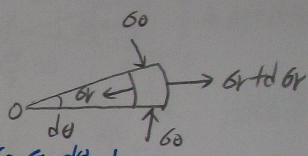
(3) 稳态结晶时固-液界面处液相侧Cu的含量和温度 $T_L(0)$;

稳态时 $C_S = C_0$　∴ $C_L = \frac{C_0}{k} = \frac{2.0\%}{0.168} = 12\%$

(4) 画出沿圆棒长度方向Cu的分布曲线,并标明各特征值.



04年B卷 一、圆板料拉深为圆筒件.



沿r方向列静平衡方程为

$$\Sigma F_r = (\sigma_r+d\sigma_r)(r+dr)\,d\theta\, t - \sigma_r r\, d\theta\, t + 2\sigma_\theta \sin\frac{d\theta}{2}\, dr\, t = 0$$

忽略高阶小量,化简得　$d\sigma_r = -(\sigma_r+\sigma_\theta)\frac{dr}{r}$

由塑性条件　$\sigma_1 = \sigma_r$　$\sigma_3 = -\sigma_\theta$　$\sigma_1 - \sigma_3 = \sigma_r + \sigma_\theta$

$\sigma_r + \sigma_\theta = \beta\sigma$ 代入得:

$d\sigma_r = -\beta\sigma\frac{dr}{r}$　　$\sigma_r = -\beta\sigma \ln r + C.$

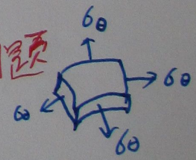
由不计摩擦,凸缘外缘处 $\sigma_r^{外} = 0$ 将该边界条件代入得

$\sigma^{外} = \sigma_r|_{r=R_0} = -\beta\sigma \ln R_0 + C = 0$　　$C = \beta\sigma \ln R_0$

∴ $\sigma_r = \beta\sigma \ln\frac{R_0}{r}$　则凸缘内缘处 $\sigma_r^{内} = \sigma_r|_{r=\frac{d_0}{2}} = \beta\sigma \ln\frac{2R}{d_0}$

∴ 拉深力 $F = \pi d_0 t\, \sigma^{内} = \pi d_0 t \beta\sigma \ln\frac{2R}{d_0}$

三、沿径向的静力平衡方程为　薄球壳问题



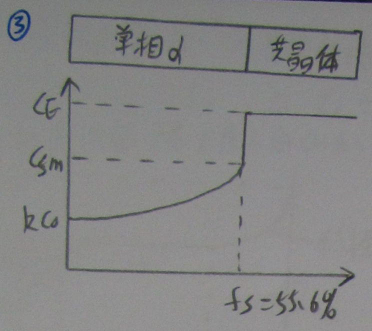
$$p\,(r\,d\theta)^2 - 4\sigma_\theta (r\,d\theta)\,t \cdot \sin\frac{d\theta}{2} = 0$$

$\sigma_\theta = \frac{r}{2t}p$　　$\sigma_1 = \sigma_2 = \sigma_\theta = \frac{r}{2t}p$　$\sigma_3 = \sigma_r = 0$

由Mises屈服准则　$\bar{\sigma} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2} = \sigma_s$

② $\frac{pr}{2t} = \sigma_s \Rightarrow p = \frac{2\sigma_s t}{r}$

③



④若为完全平衡凝固.

$$C_S^* = \frac{C_0 k}{1-f_S(1-k)} \qquad C_L^* = \frac{C_0}{k+f_L(1-k)}$$

共晶体数量即为 $C_S^* = C_{sm}$ 时的 f_L

$$C_{sm} = C_S^* = \frac{C_0 k}{k+f_L(1-k)} = 30\%$$

$$\Rightarrow f_L = 33.33\%$$

2005年.

一、从等效应力 $\bar\sigma$ 的定义出发证明 $\bar\sigma = \sqrt{\frac{3}{2}\sigma'_{ij}\sigma'_{ij}}$

证明: $\sigma_x - \sigma_y = (\sigma'_x + \sigma_m) - (\sigma'_y + \sigma_m) = \sigma'_x - \sigma'_y$

同理: $\sigma_y - \sigma_z = \sigma'_y - \sigma'_z \qquad \sigma_z - \sigma_x = \sigma'_z - \sigma'_x$

于是有 $(\sigma_x-\sigma_y)^2 + (\sigma_y-\sigma_z)^2 + (\sigma_z-\sigma_x)^2$

$$= 2\sigma_x'^2 + 2\sigma_y'^2 + 2\sigma_z'^2 - 2\sigma'_x\sigma'_y - 2\sigma'_y\sigma'_z - 2\sigma'_z\sigma'_x$$

$$= 3\sigma_x'^2 + 3\sigma_y'^2 + 3\sigma_z'^2 - \sigma'_x(\sigma'_x+\sigma'_y+\sigma'_z) - \sigma'_y(\sigma'_x+\sigma'_y+\sigma'_z) - \sigma'_z(\sigma'_x+\sigma'_y+\sigma'_z)$$

$$= 3\sigma_x'^2 + 3\sigma_y'^2 + 3\sigma_z'^2 - (\underbrace{\sigma'_x+\sigma'_y+\sigma'_z}_{=0})^2$$

$\bar\sigma$定义 $\bar\sigma^2 = \frac{1}{2}[(\sigma_x-\sigma_y)^2 + (\sigma_y-\sigma_z)^2 + (\sigma_z-\sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)]$

$$= \frac{1}{2}[3(\sigma_x'^2 + \sigma_y'^2 + \sigma_z'^2) + 6(\sigma_{xy}'^2 + \sigma_{yz}'^2 + \sigma_{zx}'^2)]$$ 自由指标表述

$$= \frac{3}{2}[\sigma_x'^2 + \sigma_y'^2 + \sigma_z'^2 + 2(\sigma_{xy}'^2 + \sigma_{yz}'^2 + \sigma_{zx}'^2)]$$ 求和

$$= \frac{3}{2}\sigma'_{ij}\sigma'_{ij} \qquad 故 \ \bar\sigma = \sqrt{\frac{3}{2}\sigma'_{ij}\sigma'_{ij}}$$

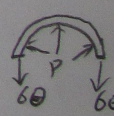
i, j为哑标 重复出现.

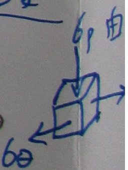
二、解：薄壁圆筒.(05,06,08)

(1) 圆筒：根据平衡条件可求得

$$\sigma_z = \frac{p\pi r^2}{2\pi r t} = \frac{pr}{2t} > 0$$

$$\sigma_\theta = \frac{p2r}{2t} = \frac{pr}{t} > 0$$





σ_ρ沿壁厚为线性分布，在内表面 $\sigma_\rho = -p$ 在外表面 $\sigma_\rho = 0$

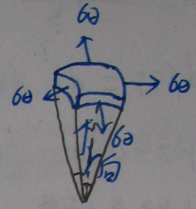
圆筒内表面先产生屈服，然后向外扩展，当外表面产生屈服时 整个圆筒就开始塑性变形

(2) $\therefore \sigma_1 = \sigma_\theta = \frac{pr}{t} \qquad \sigma_2 = \sigma_z = \frac{pr}{2t} \qquad \sigma_3 = \sigma_\rho = 0$

由Mises屈服准则

$$\bar{\sigma}^{(1)} = \frac{\sqrt{2}}{2}\sqrt{(\sigma_r-\sigma_\theta)^2+(\sigma_\theta-\sigma_z)^2+(\sigma_z-\sigma_r)^2}$$

$$= \frac{\sqrt{2}}{2}\sqrt{\sigma_\theta^2+\frac{1}{4}\sigma_\theta^2+\frac{1}{4}\sigma_\theta^2} = \frac{\sqrt{3}r}{2t}p$$



半球面：沿径向静力平衡方程为

$$p(rd\theta)^2 = 4\sigma_\theta(rd\theta)t\sin\frac{\theta}{2}$$

$$\Rightarrow \sigma_\theta = \frac{t}{2r}p$$

$\therefore \sigma_1=\sigma_2=\sigma_\theta \quad \sigma_3=\sigma_r=0$

见背面 →

$$\bar{\sigma}^{(2)} = \frac{\sqrt{2}}{2}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}$$

$$= \frac{\sqrt{2}}{2}\sqrt{\sigma_1^2+\sigma_1^2} = \sigma_1 = \sigma_\theta = \frac{r}{2t}p < \bar{\sigma}^{(1)}$$

因此圆筒先屈服　这时 $p = 2t\sigma_s/\sqrt{3}r$

(2) 设屈服时塑性功率密度为C，求对应应变速率张量

$$W = \sigma_s\dot{\bar{\varepsilon}} = C \Rightarrow \dot{\bar{\varepsilon}} = \frac{C}{\sigma_s}$$ → 等效应变速率

$$\sigma_m = \frac{1}{3}(\sigma_r+\sigma_\theta+\sigma_z) = \frac{1}{3}\left(\sigma_\theta+\frac{1}{2}\sigma_\theta\right) = \frac{1}{2}\sigma_\theta = \sigma_z$$

由表达式

和

$$\sigma_r' = \sigma_r-\sigma_m = -\frac{1}{2}\sigma_\theta$$

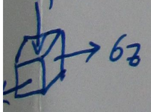
$$\sigma_\theta' = \sigma_\theta-\sigma_m = \frac{1}{2}\sigma_\theta$$

$$\sigma_z' = \sigma_z-\sigma_m = 0$$

又 $\bar{\sigma} = \frac{\sqrt{3}}{2}\sigma_\theta \Rightarrow \sigma_\theta = \frac{2}{\sqrt{3}}\sigma_s$

出现.

— σ_p 由此可知



$$\dot{\varepsilon}_r = \frac{3\dot{\bar{\varepsilon}}}{2\bar{\sigma}}\sigma_r' = \frac{3\dot{\bar{\varepsilon}}}{2\sigma_s}\left(-\frac{1}{2}\times\frac{2}{\sqrt{3}}\sigma_s\right) = \frac{\sqrt{3}C}{2\sigma_s}$$

$$\dot{\varepsilon}_\theta = \frac{3\dot{\bar{\varepsilon}}}{2\bar{\sigma}} = \frac{\sqrt{3}C}{2\sigma_s} \qquad \dot{\varepsilon}_z = 0$$

$$\dot{\gamma}_{rz} = 0$$

理想塑性材料 (非单值)　强化材料则可(单值)

13-12式
不可根据已知应力偏量来确定应变速度，但可根据应变速度来确定应力偏量

08年

假定某铁液凝固时的最大过冷度为100°C，并已知熔点 $T_m=1500°C$

凝固潜热 $L=15.17$ kJ/mol，界面张力 $\sigma_{CL}=7.9\times10^{-5}$ J/cm²，摩尔体积 6.9cm³

(1) 求铁液凝固时的 $\Delta G^*_{均}$ 和 $r^*_{均}$

(2) 若铁液中存在细小石墨颗粒，凝固铁相与石墨的润湿角θ为30°

计算 $\Delta G^*_{非}$ 和 $r^*_{非}$

(3) 比较以上两结果，说明什么问题。

解：(1) $r^*_{均}=\frac{2\sigma_{CL}V_S}{\Delta G_V}=\frac{2\sigma_{CL}}{L}\frac{T_mV_S}{\Delta T}=\frac{2\times7.9\times10^{-5}}{\frac{15.17\times10^{3}}{6.9}}\times\frac{1500+273}{100}\text{cm}=1.27\times10^{-6}\text{cm}$

$\Delta G^*_{均}=\frac{1}{3}A^*\sigma_{CL}=\frac{4}{3}\pi r^{*2}_{均}\sigma_{CL}=\frac{4}{3}\times3.14\times1.27^2\times10^{-12}\times7.9\times10^{-5}\text{J}=5.33\times10^{-6}\text{J}$

(2) $r^*_{非}=r^*_{均}=1.27\times10^{-6}\text{cm}$

$\Delta G^*_{非}=\Delta G^*_{均}f(\theta)=\Delta G^*_{均}\frac{2-3\cos\theta+\cos^3\theta}{4}=5.33\times10^{-6}\times\frac{2-3\times\frac{\sqrt3}{2}+\left(\frac{\sqrt3}{2}\right)^3}{4}=0.07\times10^{-6}\text{J}$

(3) $\Delta G^*_{非}<<\Delta G^*_{均}$ 异质形核所需的临界形核功远小于均质形核的。

09年

平面应变压缩实验。

由于是大应变，故须采用对数应变。 $b>>l>>h$，b方向应变忽略

$d\varepsilon_b=0$ 　 $d\varepsilon_h=\ln\frac{h}{h_0}=-\ln2$ 　 由 $d\varepsilon_l+d\varepsilon_h+d\varepsilon_b=0\Rightarrow d\varepsilon_l=\ln2$

故 $d\varepsilon_h=-d\varepsilon_l=-\ln2$

$d\bar\varepsilon=\frac{\sqrt2}{3}\sqrt{(\varepsilon_h-\varepsilon_b)^2+(\varepsilon_b-\varepsilon_l)^2+(\varepsilon_l-\varepsilon_h)^2}=\frac{2}{\sqrt3}\ln2$

由 $\varepsilon_{ij}=\frac{3}{2}\frac{d\bar\varepsilon}{\bar\sigma}\sigma'_{ij}$

$\sigma'_L=\frac{2\bar\sigma}{3d\bar\varepsilon}\varepsilon_L=\frac{2\sigma_s}{2\sqrt3\ln2}\cdot\ln2=\frac{1}{\sqrt3}\sigma_s$ 　 即 $\sigma_L-\sigma_m=\frac{1}{\sqrt3}\sigma_s$

又 $\sigma_L=0\Rightarrow\sigma_m=-\frac{1}{\sqrt3}\sigma_s$

$\sigma'_h=\sigma_h-\sigma_m=\frac{2\bar\sigma}{3d\bar\varepsilon}\varepsilon_h=-\frac{1}{\sqrt3}\sigma_s\Rightarrow\sigma_h=-\frac{2}{\sqrt3}\sigma_s$

07年，08年，10年

已知 $\sigma_{ij}=\begin{pmatrix}50 & 50 & 80\\ 50 & 0 & -75\\ 80 & -75 & -30\end{pmatrix}$ 求外法线 $l=m=\frac{1}{2}$ $n=\frac{1}{\sqrt{2}}$ 斜切面上全应力，正应力，切应力

解：$\sigma_N=S_x l+S_y m+S_z n=\sigma_x l^2+\sigma_y m^2+\sigma_z n^2+2\tau_{xy}lm+2\tau_{yz}mn+2\tau_{zx}nl=26.5\,MPa$

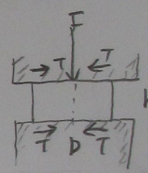
$$\left.\begin{aligned}S_x&=\sigma_x l+\tau_{yx}m+\tau_{zx}n=106.6\\ S_y&=\tau_{xy}l+\sigma_y m+\tau_{zy}n=-28.0\\ S_z&=\tau_{xz}l+\tau_{yz}m+\sigma_z n=-18.2\end{aligned}\right\}\ S=\sqrt{S_x^2+S_y^2+S_z^2}=111.8\,MPa$$

$\tau=\sqrt{S^2-\sigma^2}=108.6.$

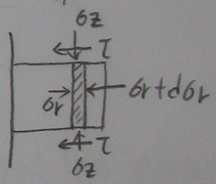
2. 屈服准则 (07年

3. 一圆柱体镦粗，侧面作用有均布压力σ_0，若均匀变形，为滑动摩擦 试用主应力法求单位流动压力P和镦粗力F

(07年



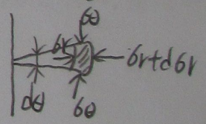
解：列平衡方程



$\Sigma F_r=\sigma_r r h d\theta+2\sigma_\theta h dr \sin\frac{d\theta}{2}-2\tau r d\theta dr-(\sigma_r+d\sigma_r)(r+dr)h d\theta=0$

$\sin\frac{d\theta}{2}=\frac{d\theta}{2}$，并略去二阶微量，得：

$\sigma_\theta h dr-2\tau r dr-\sigma_r h dr-rh d\sigma_r=0$



∵ 均匀变形 $d\varepsilon_\theta=d\varepsilon_r$；$\sigma_r=\sigma_\theta$

得：$d\sigma_r=-\frac{2\tau}{h}dr$　　$\tau=\mu\sigma_z$

根据屈服准则 $\sigma_z-\sigma_r=\sigma_s \Rightarrow d\sigma_z=d\sigma_r$

∴ $d\sigma_z=-\frac{2\mu\sigma_z}{h}dr \Rightarrow \sigma_z=C\exp\left(-\frac{2\mu r}{h}\right)$

边界条件：$r=\frac{D}{2}$时 $\sigma_r=\sigma_0$　　$C=(\sigma_0+\sigma_s)e^{\frac{\mu D}{h}}$

∴ $\sigma_z=(\sigma_0+\sigma_s)e^{\frac{\mu D-2\mu r}{h}}$

① 镦粗力 $F=\int_0^{\frac{D}{2}}\sigma_z dA=2\pi\int_0^{\frac{D}{2}}\sigma_z r dr=2\pi(\sigma_0+\sigma_s)\int_0^{\frac{D}{2}}re^{\frac{\mu D-2\mu r}{h}}dr$

$=-\frac{h\pi}{\mu}(\sigma_0+\sigma_s)\left[\frac{D}{2}+\frac{h}{2\mu}\left(1-e^{\frac{\mu D}{h}}\right)\right]$

08年

假定

凝固

(1)求铁

(2)若铁

计算

(3)比较

解：(1)

(2)

(3)

09年

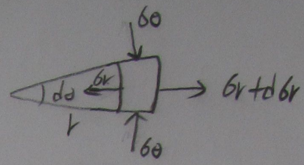
平面应变

由于是

06年　将圆板坯拉深为圆筒件　屈服应力 Y，压边力为 Q，摩擦系数 μ

求拉深 h 时拉深力　相当于平面应变问题

列平衡方程（为何没有 τ，看题干要求）



$$\Sigma F=(\sigma_r+d\sigma_r)(r+dr)d\theta\, t-\sigma_r\, r\, d\theta\, t$$
$$+2\sigma_\theta\, dr\, t\sin\frac{d\theta}{2}=0$$

$\sin\frac{d\theta}{2}=\frac{d\theta}{2}$ 同时略去二阶微量，化简为

$$\sigma_\theta t\,dr+d\sigma_r t r+\sigma_r t\,dr=0$$

由塑性条件　$\sigma_1=\sigma_r$　$\sigma_3=-\sigma_\theta$　$\sigma_1-\sigma_3=\sigma_r+\sigma_\theta=\beta Y$　代入得

$$d\sigma_r=-\frac{\beta Y}{r}dr$$

① $\sigma_r=-\beta Y\ln r$　　变量：h 的函数（由体积不变求）

边界条件　凸缘外缘处的 σ_r 与压边力 Q 引起的摩擦力作用等效

$\sigma_r^{外}=\frac{2\mu Q}{\pi D t}$　代入① 得：　$C=\frac{2\mu Q}{\pi D t}+\beta Y\ln\frac{D}{2}$

$$\therefore\ \sigma_r=\beta Y\ln\frac{D}{2r}+\frac{2\mu Q}{\pi D t}$$

在内缘处：

$$\sigma_r^{内}=\beta Y\ln\frac{D}{d}+\frac{2\mu Q}{\pi D t}$$

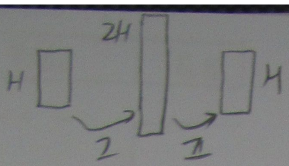
根据体积不变定律，可确定瞬时的 D：$\pi d h=\frac{\pi}{4}(D_0^2-D^2)$

$$\therefore\ D=\sqrt{D_0^2-4dh}$$

故 $P=\pi d t\cdot\sigma_r^{内}=\beta Y\frac{\pi d t}{2}\ln\left[\left(\frac{D_0}{d}\right)^2-\frac{4h}{d}\right]+\frac{2\mu d Q}{\sqrt{D_0^2-4dh}}$

其中 $\beta=1.1$

06年



试求这二个阶段的对数应变，等效对数应变及最终对数应变，等效对数应变.

$\varepsilon \rightarrow$ 工程应变

$\varepsilon^* \rightarrow$ 对数应变

解: (1) $\epsilon_z^{(I)} = \ln\frac{2H}{H} = \ln 2$

$\epsilon_r^{(I)} = \epsilon_\theta^{(I)}$　又 $\epsilon_r^{(I)} + \epsilon_\theta^{(I)} + \epsilon_z^{(I)} = 2\epsilon_r^{(I)} + \epsilon_z^{(I)} = 0$

$\Longrightarrow \epsilon_r^{(I)} = -\frac{1}{2}\ln 2$

$\bar{\epsilon}^{(I)} = \frac{\sqrt{2}}{3}\sqrt{(\epsilon_r^{(I)} - \epsilon_\theta^{(I)})^2 + (\epsilon_r^{(I)} - \epsilon_z^{(I)})^2 + (\epsilon_z^{(I)} - \epsilon_\theta^{(I)})^2} = \ln 2$

(2) $\epsilon_z^{(II)} = \ln\frac{H}{2H} = -\ln 2$

$\epsilon_r^{(II)} = \epsilon_\theta^{(II)} = \frac{1}{2}\ln 2$　　$\bar{\epsilon}^{(II)} = \ln 2$

(3) $\epsilon_r = \epsilon_\theta = \epsilon_r^{(I)} + \epsilon_r^{(II)} = -\frac{1}{2}\ln 2 + \frac{1}{2}\ln 2 = 0$

$\epsilon_z = \epsilon_z^{(I)} + \epsilon_z^{(II)} = 0$

$\bar{\epsilon} = \bar{\epsilon}^{(I)} + \bar{\epsilon}^{(II)} = 2\ln 2$

$\neq \frac{\sqrt{2}}{3}\sqrt{\quad}$

累积等效应变必须考虑加载变形履历

08年，10年

两端封闭的受内压P的薄壁圆筒，产生塑性变形时，轴向，径向，周向应变比例（设$\sigma_\rho=0$）

解: $\sigma_\theta = \frac{pr}{t}$　$\sigma_z = \frac{pr}{2t}$　$\sigma_\rho = 0$

$\sigma_m = \frac{1}{3}(\sigma_\theta + \sigma_z + \sigma_\rho) = \frac{pr}{2t}$

$\sigma_\rho' = \sigma_\rho - \sigma_m = -\frac{pr}{2t}$　$\sigma_\theta' = \sigma_\theta - \sigma_m = \frac{pr}{2t}$　$\sigma_z' = \sigma_z - \sigma_m = 0$

由 $d\varepsilon_{ij} = \sigma_{ij}' d\lambda$ 得

$d\varepsilon_\rho = -\frac{pr}{2t}d\lambda$　$d\varepsilon_\theta = \frac{pr}{2t}d\lambda$　$d\varepsilon_z = 0$

$\therefore d\varepsilon_\rho : d\varepsilon_\theta : d\varepsilon_z = -1:1:0$

偏

DB 应变增量 ⇒ OCDB 应力增量 ⇒ OAB 应变增量

$d\varepsilon_x = \delta\sigma_s^{\mathrm{I}} - \delta\sigma_s^{2}$

$\gamma = 0$

$d\sigma_x = \frac{d\bar{\sigma}}{d\bar{\varepsilon}} d\varepsilon_x = \frac{\sigma_s}{\sqrt{2}}$

$\tau_{xy} =$

$d\varepsilon_x = \frac{d\bar{\varepsilon}}{\bar{\sigma}} \sigma_x = \delta\sigma_s^{\mathrm{I}} - \delta\sigma_s^{2}$

$\gamma = \frac{3}{2}\frac{d\bar{\varepsilon}}{\bar{\sigma}}\tau = \frac{\sqrt{2}}{2} \times \frac{3}{2} \times \frac{\sqrt{2}(\delta\sigma_s^{\mathrm{I}} - \delta\sigma_s^{2})}{\sigma_s} \frac{1}{\sqrt{6}}\sigma_s$

$= \frac{\sqrt{6}}{4}(\sigma_s^{\mathrm{I}} - \sigma_s^{2})$

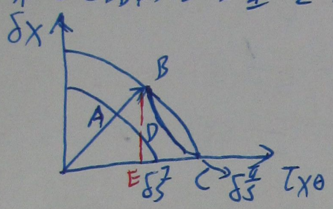
内弧线：初始屈服曲面　外弧线后继屈服曲面（材料经历塑性变形后的屈服曲面

OAB, OEC：简单加载

OEB, OECB：复杂加载

（达到外弧线后，若想再产生新的变形则必须超出外弧线）→ 强化材料

11年：圆筒拉扭中 $\frac{\sigma}{\tau}=\sqrt{3}$ 采用Mises准则 求路径OAB和OCDB的应变量 ε，γ



由 $\begin{cases} 2\sigma^2+6\tau^2=2\sigma_s^2 \\ \sigma=\sqrt{3}\tau \end{cases}$ 得：$\tau=\sqrt{\frac{1}{6}}\sigma_s$，$\sigma=\sqrt{\frac{1}{2}}\sigma_s$

OC应变增量 ⇒ OCDB应变增量

$d\varepsilon_x=\sigma_s^{II}-\sigma_s^{I}$　　$d\varepsilon_x=\sigma_s^{II}-$

$\gamma=0$　　$\gamma=0$

OAB的同OCDB应力状态相同

OCDB同OC应变状态相同.

OC应变状态：$d\varepsilon_x=\sigma_s^{II}-\sigma_s^{I}$　$\gamma=0$

故OCDB应变状态 $d\varepsilon_x=\sigma_s^{II}-\sigma_s^{I}$　$\gamma=0$

由 $d\varepsilon_x=\frac{d\bar{\varepsilon}}{\bar{\sigma}}\cdot\left[\sigma_x-\frac{1}{2}(\sigma_y+\sigma_z)\right]=\frac{d\bar{\varepsilon}}{\bar{\sigma}}\sqrt{\frac{1}{2}}\sigma_s=\sigma_s^{II}-\sigma_s^{I}$

$\therefore \frac{d\bar{\varepsilon}}{\bar{\sigma}}=\frac{(\sigma_s^{II}-\sigma_s^{I})\sqrt{2}}{\sigma_s}$

∴ OAB应力状态 $d\varepsilon_x=\frac{d\bar{\varepsilon}}{\bar{\sigma}}[\sigma_x]=\sigma_s^{II}-\sigma_s^{II}$

$\gamma=\frac{3}{2}\frac{d\bar{\varepsilon}}{\bar{\sigma}}\tau=\frac{\sqrt{2}(\sigma_s^{II}-\sigma_s^{I})}{2\sigma_s}\sqrt{\frac{1}{6}}\sigma_s$

$=\sqrt{12}(\sigma_s^{II}-\sigma_s^{I})$

内弧

OAB

OE

(这

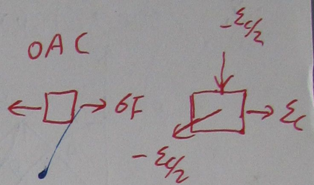
解: $\sigma_{ij}=\begin{bmatrix}0&0&0\\0&0&\tau_{\theta z}\\0&\tau_{z\theta}&\sigma_z\end{bmatrix}$ 由 $2\sigma_z^2+6\tau_{z\theta}^2=2\sigma_s^2$ 得: $\tau=\frac{1}{2}\sigma_s$

$d\varepsilon_\rho=\frac{d\bar\varepsilon}{\bar\sigma}[\sigma_\rho-\frac{1}{2}(\sigma_\theta+\sigma_z)]=\frac{d\bar\varepsilon}{\bar\sigma}[-\frac{1}{4}\sigma_s]$

$d\varepsilon_\theta=\frac{d\bar\varepsilon}{\bar\sigma}[\sigma_\theta-\frac{1}{2}(\sigma_\rho+\sigma_z)]=\frac{d\bar\varepsilon}{\bar\sigma}[-\frac{1}{4}\sigma_s]$

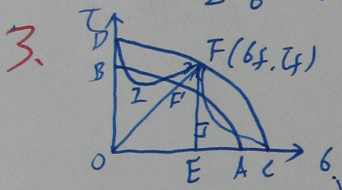
$d\varepsilon_z=\frac{d\bar\varepsilon}{\bar\sigma}[\sigma_z-\frac{1}{2}(\sigma_\rho+\sigma_\theta)]=\frac{d\bar\varepsilon}{\bar\sigma}[\frac{1}{2}\sigma_s]$

$d\gamma_{\theta z}=\frac{3}{2}\frac{d\bar\varepsilon}{\bar\sigma}\tau_{z\theta}=\frac{d\bar\varepsilon}{\bar\sigma}[\frac{3}{4}\sigma_s]$



OAC与OACJF等效应力相同，并未增加，故不能进一步变形

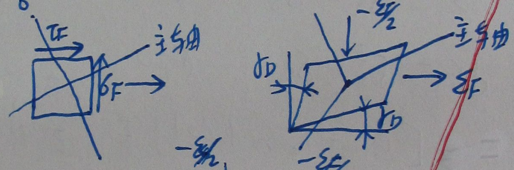
3.



OF'F是简单加载
OAC
OBD

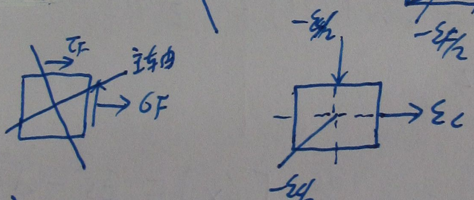
OF'F, OACJF, OBDIF 最终等效应变量相等

OF'F



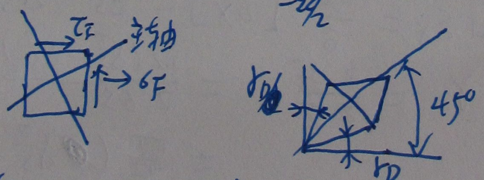
主轴重合

OACJF



应力改变了，应变未改变
主轴不重合

OBDIF



应力改变了，应变未改变
主轴不重合

OAC与OAC(E,J)F
OBD与OBDIF
相同应变状态对应不同应力状态 → 不同在σ和τ

OAC(E,J)F与OF'F → 相同应力状态对应不同应变状态. 不同在γ上

12年

1. 给出一应力张量 $\sigma_{ij}=\begin{bmatrix}-100 & 90 & 50\\ 0 & 0 & 0\\ 50 & 0 & 80\end{bmatrix}$ 求主应力及主方向.

解: $\tau_{xy}\neq\tau_{yx}$ 故用应力张量不变量来计算不行。要回到根本

设待求斜面上切应力 $\tau_N=0$，则正应力就是全应力，即 $S=\sigma_N$

由 $\begin{cases}S_x=\sigma_x l+\tau_{yx}m+\tau_{zx}n\\ S_y=\tau_{xy}l+\sigma_y m+\tau_{zy}n\\ S_z=\tau_{xz}l+\tau_{yz}m+\sigma_z n\end{cases}$ 又 $\begin{array}{l}S_x=Sl=\sigma_N l\\ S_y=Sm=\sigma_N m\\ S_z=Sn=\sigma_N n\end{array}$ 代入得

$$\left.\begin{array}{l}(\sigma_x-\sigma_N)l+\tau_{yx}m+\tau_{zx}n=0\\ \tau_{xy}l+(\sigma_y-\sigma_N)m+\tau_{zy}n=0\\ \tau_{xz}l+\tau_{yz}m+(\sigma_z-\sigma_N)n=0\end{array}\right\}$$ 是以 l,m,n 为未知数的齐次方程组 $(l^2+m^2+n^2=1)$

故 $$\begin{vmatrix}\sigma_x-\sigma_N & \tau_{yx} & \tau_{zx}\\ \tau_{xy} & \sigma_y-\sigma_N & \tau_{zy}\\ \tau_{xz} & \tau_{yz} & \sigma_z-\sigma_N\end{vmatrix}=\begin{vmatrix}-100-\sigma_N & 0 & 50\\ 90 & -\sigma_N & 0\\ 50 & 0 & 80-\sigma_N\end{vmatrix}=0$$

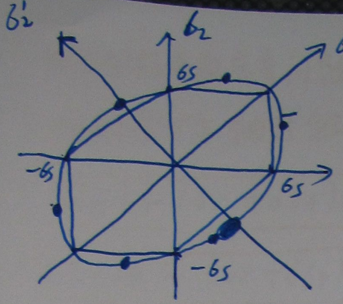
$\sigma_1=10\sqrt{106}-10$　　$\sigma_2=0$　　$\sigma_3=-10-10\sqrt{106}$ 代入原方程得:

对于 σ_1　$l_1=$　$m_1=$　$n_1=$

对于 σ_2　$l_2=$　$m_2=$　$n_2=$

对于 σ_3　$l_3=$　$m_3=$　$n_3=$

2. 在圆筒薄壁拉伸扭曲中，已知拉伸应力 $\sigma_z=\frac{\sigma_s}{2}$ 在屈服条件下计算所需切应力值，并导出应变增量公式.



Mises $\frac{\sigma_1'^2}{(\sqrt{2}\sigma_s)^2} + \frac{\sigma_2'^2}{(\sqrt{\frac{2}{3}}\sigma_s)^2} = 1$

Tresca: $|\sigma_1-\sigma_2| = \sigma_s$　$|\sigma_2|=\sigma_s$　$|\sigma_1|=\sigma_s$

任一平面力状态都可用 $\sigma_1-\sigma_2$ 平面上一点 P 表示 可用来判断是否弹塑性状态

六个交点处两个准则一致，都表两向主应力相等应力状态，四个为单向应力 另两个是 (σ_s, σ_s) $(-\sigma_s, -\sigma_s)$

两准则差别最大的六个点 中间应力等于平均应力，它们既表示平面应力状态又表示平面应变状态。

等效应力 $\bar{\sigma} = \frac{\sqrt{2}}{2}\sqrt{\cdots} = 435.9MPa$　又 $\bar{\sigma} = 200(1+\bar{\varepsilon})$

∴ $\bar{\varepsilon} = 1.18$

$\sigma_m = \frac{\sigma_1+\sigma_2+\sigma_3}{3}$　$\sigma' = \sigma - \sigma_m$　$\Rightarrow \begin{cases} \sigma_2' = -33.3MPa \\ \sigma_1' = 166.7MPa \\ \sigma_3' = -233.3MPa \end{cases}$

又 $\varepsilon_i = \frac{3\bar{\varepsilon}}{2\bar{\sigma}}\sigma_i'$ $\Rightarrow$ $\varepsilon_1 = 1.0828$　$\varepsilon_2 = -0.1352$　$\varepsilon_3 = -0.9472$

② $O \to A \to O \to B$

$O \to A$ 阶段.

$\bar{\sigma}^{(1)} = 217.9MPa$ $\Rightarrow$ $\overline{\Delta\varepsilon}^{(1)} = 0.0891$　$\sigma_m^{(1)} = -16.7MPa$

$\sigma_1^{(1)'} = -133.3MPa$　$\sigma_2^{(1)'} = 16.7MPa$　$\sigma_3^{(1)'} = 116.7MPa$

$\varepsilon_1^{(1)} = -0.0823$　$\varepsilon_2^{(1)} = 0.0103$　$\varepsilon_3^{(1)} = 0.0721$

又 $\varepsilon^{(2)} = \varepsilon + \varepsilon^{(1)}$

$\Rightarrow A \to B$ 过程　$\varepsilon_1^{(2)} = 1.0006$　$\varepsilon_2^{(2)} = -0.1244$　$\varepsilon_3^{(2)} = 0.8753$

则全过程主应变 $\varepsilon_i = \varepsilon_i^{(2)} + \varepsilon_i^{(1)}$

$\varepsilon_1 = -0.0823 + 1.0006 = 0.9183$

$\varepsilon_2 = -0.00136$

$\varepsilon_3 = 0.9474$

1、一高为H的长方体受压均匀变形，已知顶端质点的小量级压下量为u_0，底面质点静止不动。

解：$\varepsilon_z = -\frac{u_0}{H} = \frac{u_z}{z} \Rightarrow u_z = -\frac{z}{H}u_0$

设长方体长度方向位移量为u_x，宽度方向u_y，则由体积不变

$$\varepsilon_x+\varepsilon_y+\varepsilon_z=0$$

$$\varepsilon_x+\varepsilon_y=\frac{u_x}{x}+\frac{u_y}{y}=-\varepsilon_z=\frac{u_0}{H}$$

又 $\frac{u_x}{x}=\frac{u_y}{y}$ ∴ $\frac{u_x}{x}=\frac{u_y}{y}=-\frac{1}{2}\varepsilon_z=\frac{u_0}{2H}$

∴ $u_x=\frac{x}{2H}u_0$　$u_y=\frac{y}{2H}u_0$　$u_z=-\frac{z}{H}u_0$

小应变张量场

$$\varepsilon_x=\frac{\partial u_x}{\partial x}=\frac{u_0}{2H}\qquad \varepsilon_y=\frac{\partial u_y}{\partial y}=\frac{u_0}{2H}\qquad \varepsilon_z=\frac{\partial u_z}{\partial z}=-\frac{u_0}{H}$$

$$\gamma_{xy}=\frac{1}{2}\left[\frac{\partial u_y}{\partial x}+\frac{\partial u_x}{\partial y}\right]=0$$

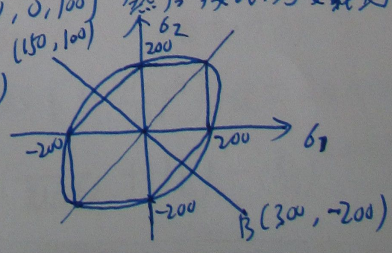
$$\therefore\ \varepsilon_{ij}=\begin{bmatrix}\frac{u_0}{2H} & 0 & 0\\ 0 & \frac{u_0}{2H} & 0\\ 0 & 0 & -\frac{u_0}{H}\end{bmatrix}$$

等效应变 $\bar{\varepsilon}=\frac{\sqrt{2}}{3}\sqrt{(\varepsilon_x-\varepsilon_y)^2+(\varepsilon_y-\varepsilon_z)^2+(\varepsilon_z-\varepsilon_x)^2+6(\gamma_{xy}^2+\gamma_{yz}^2+\gamma_{zx}^2)}=\frac{u_0}{H}$

2、一刚塑性硬化材料，其硬化曲线即等效应力一应变曲线为 $\bar{\sigma}=200(1+\bar{\varepsilon})$ MPa，质点承受两向应力，应力主轴始终不变，按下列两种加载路线分别求出最终的塑性全量主应变$\varepsilon_1,\varepsilon_2,\varepsilon_3$

① 主应力从0开始比例加载到最终主应力状态为(300, 0, -200) MPa

② 主应力从0开始按比例加载到(-150, 0, 100) 然后按比例变载到(300, 0, -200) MPa



解：两向应力状态屈服轨迹($\sigma_3=0$)

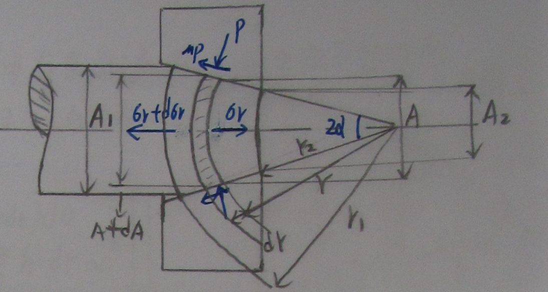
Mises $\sigma_1^2-\sigma_1\sigma_2+\sigma_2^2=\sigma_s^2$

⑧

09年

试采用球坐标系求圆柱锥体凹模拉拔圆柱坯料时的单位拉拔力。$\sigma_r+p=S$ 圆中F为轴向投影面积。

解：对锥形塑性应变区用球坐标建立坐标系
处于三向压应力状态，产生两向压缩一向拉伸变形，截取单元体.



$$\Sigma F_r=-(\sigma_r+d\sigma_r)\cdot\pi\left[(r+dr)\sin\alpha\right]^2+\sigma_r\cdot\pi(r\sin\alpha)^2$$
$$-\mu p\cdot 2\pi\left[\frac{r\sin\alpha+(r+dr)\sin\alpha}{2}\right]dr\cdot\cos\alpha-p\cdot 2\pi\left[\frac{r\sin\alpha+(r+dr)\sin\alpha}{2}\right]dr\sin\alpha=0$$

忽略高阶微分项，得：

$$r\,d\sigma_r+2(\sigma_r+p)dr=-2\mu p\,dr\cot\alpha$$

屈服准则 $\sigma_1=\sigma_r$　$\sigma_3=-p$　$\therefore\ \sigma_1-\sigma_3=\sigma_r+p=S$

$$\sigma_r=-(2S+2\mu p\cot\alpha)\ln r+C$$

边界条件：$r=r_1$ 时　$\sigma_r=-(2S+2\mu p\cot\alpha)\ln r_1+C=0$

$\therefore\ \sigma_r=(2S+2\mu p\cot\alpha)\ln\frac{r_1}{r}$　∵所需单位拉拔力为 $r=r_2$ 时

故 $r=r_2$ 时　$\sigma=(2S+2\mu p\cot\alpha)\ln\frac{r_1}{r_2}$　[p应该也随r而变化，故此处有问题]

注意：摩擦力条件，　Tresca　　Mises
常摩擦：$\tau=mk$　($\sigma_s=2k$　　$\sigma_s=\sqrt{3}k$)
滑动摩擦：$\tau=\mu\sigma_N$，有时用 $d\sigma_r=-dp$　化为p与r的微分方程
有时用 $\sigma_\theta=\sigma_r$　消去一个未知量

用 $d\sigma_r=-dp$ 可解得：　$\sigma=(2a+1)S\left[1-\left(\frac{r_2}{r_1}\right)^a\right]$　$a=2\mu\cot\alpha$

用 $\sigma_p=S-\sigma_r$ 得：　$\sigma_r=S\left(\frac{1}{\mu\cot\alpha}+1\right)\left[1-\left(\frac{r_2}{r_1}\right)^{2\mu\cot\alpha}\right]$

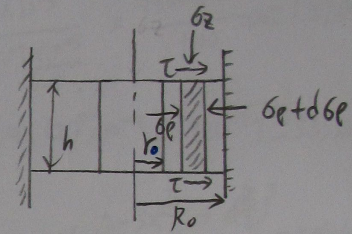
$$= \frac{\pi(D+d)h\gamma}{2\mu}\left(e^{\frac{\mu(D-d)}{2h}}-1\right)$$

单位压力 $p = \frac{P}{A} = \frac{P}{\frac{\pi}{4}(D^2-d^2)} = \frac{2h\gamma}{\mu(D-d)}\left(e^{\frac{\mu(D-d)}{2h}}-1\right)$

例：在模内压缩圆环，外径尺寸 R_0 不变，材料向里流动

假设接触面摩擦因素为 μ ，求接触面上正应力 σ_z

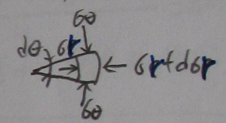
并指出此时 σ_z 峰值在何处。



解：列平衡微分方程

$\Sigma F_r = \sigma_r h r d\theta - (\sigma_r + d\sigma_r)h(r+dr)d\theta$
$+2\tau r d\theta dr + 2\sigma_\theta \cdot \sin\frac{d\theta}{2} h \cdot dr = 0$

化简得：同上一题 ①区一样



得：$\sigma_z = Ce^{\frac{2\mu}{h}r}$

边界条件 $r=\frac{d}{2}$ 时 $\sigma_r = \sigma_z - Y = C_1 e^{\frac{2\mu r_0}{h}} - Y = 0$

$C_1 = Y e^{\frac{-2\mu r_0}{h}}$

$\sigma_z = Y e^{\frac{2\mu(r-r_0)}{h}}$ 当 $r=R$ 时 σ_z 最大。

$\boxed{11年}$ 本构方程

圆柱体全粘着时接触面上摩擦切应力分布

$d\sigma_z = -\frac{2\tau}{h}dr$

滑动区：$\tau = \mu\sigma_z \Rightarrow \ln\sigma_z = -\frac{2\mu}{h}r + c$

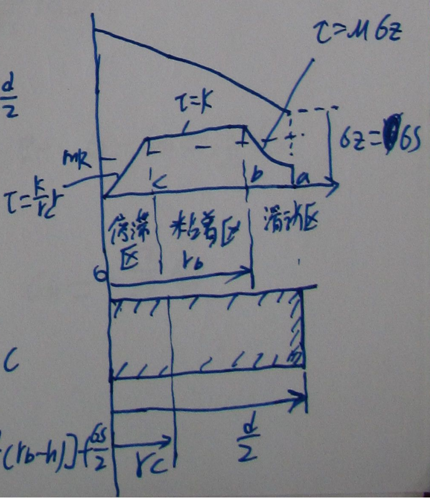
边界条件：$r=\frac{d}{2}$ 时 $\sigma_z = \sigma_s \Rightarrow c = \ln\sigma_s + \frac{2\mu}{h}\frac{d}{2}$

粘着区：$\tau = k = \frac{1}{2}\sigma_s \Rightarrow \sigma_z = -\frac{\sigma_s}{h}r + c$

边界条件：$r=r_b$ 时 $\mu\sigma_z = k = \frac{1}{2}\sigma_s$
$\Rightarrow C = \frac{\sigma_s}{2\mu} + \frac{\sigma_s}{h}r_b$

停滞区：$\tau = \frac{1}{2}\sigma_s\frac{r}{r_c} \Rightarrow \sigma_z = -\frac{\sigma_s r^2}{h r_c 2} + c$

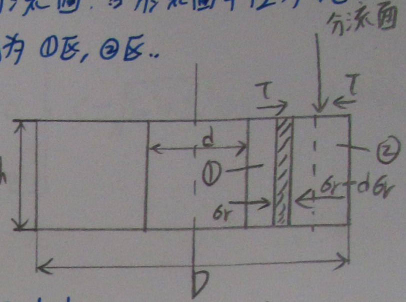
边界条件：$r = r_c \approx h \Rightarrow C = \frac{\sigma_s}{2\mu}[1+\frac{2\mu}{h}(r_b-h)] + \frac{\sigma_s}{2}$



08年

例：一圆环坯料受均匀压缩变形，坯料为理想刚塑性材料，工件与工具间为滑动摩擦，摩擦系数为μ，壁厚h，内外壁为自由表面，屈服强度为Y 求单位压力与总压力.

解：圆环均匀压缩变形，变形体中存在分流面，若分流面半径为 r_c
分流面两侧金属流动方向相反，分别为①区，②区..



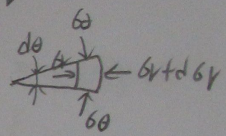
对①列平衡微分方程.

$$\Sigma F_r = \sigma_r h r d\theta - (\sigma_r + d\sigma_r) h (r+dr) d\theta + 2\tau r d\theta dr + 2\sigma_\theta \cdot \sin\frac{d\theta}{2} h \cdot dr = 0$$

化简忽略高阶微量得：

$$-\sigma_r h dr - d\sigma_r h r + 2\mu\sigma_z r dr + \sigma_\theta h dr = 0$$

由于圆环均匀压缩变形，$d\varepsilon_\theta = d\varepsilon_r$，$\sigma_\theta = \sigma_r$ 代入得



$$2\mu\sigma_z r dr = h r d\sigma_r$$

由屈服准则：$\sigma_z - \sigma_r = Y \Rightarrow d\sigma_z = d\sigma_r$ 代入得：

$$\frac{d\sigma_z}{\sigma_z} = \frac{2\mu r}{hr} dr = \frac{2\mu}{h} dr$$

$$\Rightarrow \sigma_z^{(1)} = C_1 e^{\frac{2\mu}{h} r}$$

边界条件：当 $r = \frac{d}{2}$ 时 $\sigma_r^{(1)} = \sigma_z - Y = C_1 e^{\frac{\mu d}{h}} - Y = 0$　$C_1 = Y e^{-\frac{\mu d}{h}}$

$$\therefore \sigma_z^{(1)} = Y \exp\left(\frac{2\mu r - \mu d}{h}\right)$$

对②列平衡微分方程

$$\Sigma F_r = \sigma_r h r d\theta - (\sigma_r + d\sigma_r) h (r+dr) d\theta - 2\tau r d\theta dr + 2\sigma_\theta \sin\frac{d\theta}{2} h dr = 0$$

同理得：$\sigma_z^{(2)} = Y \exp\left(\frac{\mu D - 2\mu r}{h}\right)$

求分流面位置：令 $\sigma_r^{(1)} = \sigma_r^{(2)}$ 即 $\sigma_z^{(1)} = \sigma_z^{(2)}$ 得：$r_c = \frac{D+d}{4}$

总压力 $P = \int_{\frac{d}{2}}^{r_c} \sigma_z^{(1)} dA + \int_{r_c}^{\frac{D}{2}} \sigma_z^{(2)} dA$

$$= \int_{\frac{d}{2}}^{\frac{D+d}{4}} Y e^{\frac{2\mu r - \mu d}{h}} 2\pi r dr + \int_{\frac{D+d}{4}}^{\frac{D}{2}} Y e^{\frac{\mu D - 2\mu r}{h}} 2\pi r dr$$