大工材料力学（土）

真题详解答案

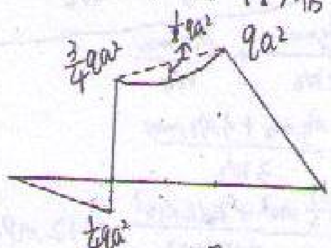
九八

一、解：由$\sum M_A=0$有：$F_B\cdot 2a+qa^2-qa\cdot\frac{3}{2}a-qa\cdot 3a=0$ 解得

$F_B=\frac{7}{4}qa$

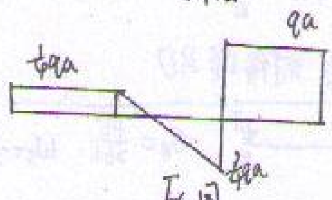
$\sum F_y=0$ 有 $F_A+F_B-2qa=0$ 得 $F_A=\frac{1}{4}qa$

求节点的M 有 $M_B=qa\cdot a=qa^2$（上拉），$M_C=F_A\cdot a=\frac{1}{4}qa^2$（下侧受拉）



M图

考点：M图与Fs图，必考!!



Fs图

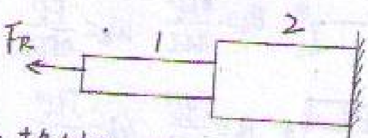
二、解：图示结构为一次超静定，受力如下图所示

考点：1. 超静定，步骤：①静力方面 ②几何方面 ③物理方面

2. 温度应力：$\Delta l_t=\alpha_l l\Delta t$

$F_N=\alpha_l EA\Delta t$

$\sigma=\alpha_l E\Delta t$



(1) 静力方面：$F_{N1}=F_{N2}=F_R$ ①

(2) 几何方面：杆的变形包括温度降低引起的变形Δl_t和由轴向压力引起的弹性变形 Δl_{F_1}，Δl_{F_2}

故其变形几何方程为 $\Delta l=\Delta l_t+\Delta l_{F_1}+\Delta l_{F_2}=0$ ②

(3) 物理方面 $\Delta l_{F_1}=\frac{F_R l}{EA_1}$，$\Delta l_{F_2}=\frac{F_R l}{EA_2}$，$\Delta l_t=-2\alpha\Delta t l$ ③

又$A_2=2A_1$ 联立①②③解得 $F_R=\frac{4\alpha\Delta t EA_1}{3}$

故 $\sigma_1=\frac{F_{N1}}{A_1}=\frac{4\alpha\Delta t E}{3}=\frac{4\times 12.5\times10^{-6}\times20\times200\times10^{3}}{3}=66.67mpa$

$\sigma_2=\frac{F_{N1}}{A_2}=\frac{4\alpha\Delta t E}{6}=\frac{4\times12.5\times10^{-6}\times20\times200\times10^{3}}{6}=33.33mpa$

C截面位移 $\Delta_C=\frac{F_{N1}l}{EA_1}+2\alpha\Delta t l=\frac{4\alpha\Delta t l}{3}-\alpha\Delta t l=0.0833\ mm$（右移）

三、解(1)依题图可得：$P_y = P\cos\theta = 5\times\frac{4}{5} = 4kN$

$P_z = P\sin\theta = 5\times\frac{3}{5} = 3kN$

此题可以为进行强度刚度校核!!

则 $M_{y\max} = \frac{1}{4}P_z l = \frac{9}{2}kN\cdot m$

$M_{z\max} = \frac{1}{4}P_y l = 6\ kN\cdot m$

则最大正应力为 $\sigma_{max} = \frac{M_{y\max}}{W_y} + \frac{M_{z\max}}{W_z}$ ①

又 $\frac{W_z}{W_y} = \frac{bh^2}{6}\Big/\frac{hb^2}{6} = \frac{4}{3}$ 代入①得 $\sigma_{max} = \frac{M_{y,max}}{\frac{3}{4}W_z} + \frac{M_{z,max}}{W_z}$

$= \frac{4M_{y,max} + 3M_{z,max}}{3W_z}$

$= \frac{4\times\frac{9}{2}\times10^3 + 3\times6\times10^3}{3\times\frac{0.15\times0.2^2}{6}} = 12mpa$

考点：

非对称性弯曲梁的正应力

$\sigma = \frac{M_y(zI_z - yI_{yz}) - M_z(yI_y - zI_{yz})}{I_y I_z - I_{yz}^2}$

中性轴与中性轴夹角 $\tan\theta = \frac{M_z I_y + M_y I_{yz}}{M_y I_z + M_z I_{yz}}$

$\sigma_{max} = \frac{M_{y,max}}{W_y} + \frac{M_{z,max}}{W_z}$

(2)

中性轴与y轴间的夹角θ满足为

$\tan\theta = \frac{M_z I_y}{M_y I_z} = \frac{6\times\frac{hb^3}{12}}{\frac{9}{2}\times\frac{bh^3}{12}} = \frac{3}{4}$，故可知中性轴对角线BD

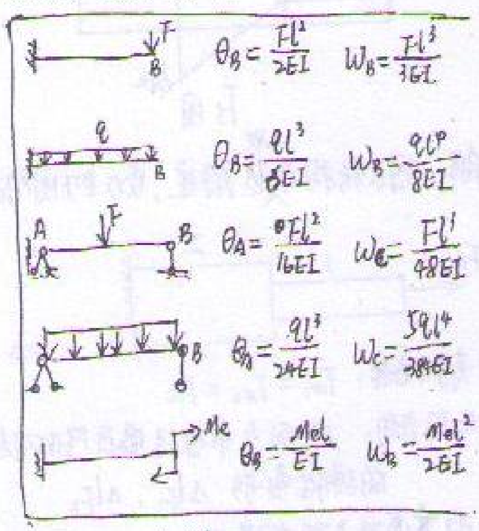
(3) $I_z = \frac{bh^3}{12} = \frac{0.15\times0.2^3}{12} = 1\times10^{-4}\ m^4$

$I_y = \frac{hb^3}{12} = \frac{0.2\times0.15^3}{12} = 5.625\times10^{-5}\ m^4$

$w_y = \frac{P_y l^3}{48EI_z} = \frac{4\times6^3\times10^3}{48\times10^{10}\times10^{-4}} = 18mm$

$w_z = \frac{P_z l^3}{48EI_y} = \frac{3\times10^3\times6^3}{48\times10^{10}\times5.625\times10^{-5}} = 24mm$

跨中总挠度为 $w_{max} = \sqrt{w_y^2 + w_z^2} = 30\ mm$



四、解(1) 依题图可知

如将题图截面变为工字形截面 让你如何求 τ(y) 参见教材 P131 P144-20.21

$\tau = \frac{F_S S_z^*}{I_z b}$ —— ①

又 $S_z^* = \int_{A^*} y\,dA = \int_y^{\frac{h}{2}} y_1 b\,dy_1 = b\frac{1}{2}y_1^2\Big|_y^{\frac{h}{2}} = \frac{b}{2}\left(\frac{h^2}{4} - y^2\right)$ —— ②

将②代入①得 $\tau(y) = \frac{F_S}{2I_z}\left(\frac{h^2}{4} - y^2\right)$

考点：1. $\tau = \frac{F_S S_z^*}{I_z b}$

其中 S_z^* 为横截面上距中性轴为y的横线以外部分的面积对中性轴的静矩

$S_z^* = \int_{A^*} y_1 dA = \int_y^{\frac{h}{2}} y_1 b\,dy_1 = \frac{b}{2}\left(\frac{h^2}{4} - y^2\right)$

2. 截面平均切应力 $\tau_平 = \frac{F_S}{A}$

(2)

截面的平均切应力 $\tau_平 = \frac{F_S}{A}$

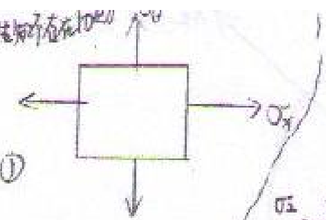
则有 $\frac{F_S}{bh} = \frac{F_S}{2I_z}\left(\frac{h^2}{4} - y^2\right)$ 解得 $y = \pm\frac{1}{\sqrt{12}}h = \pm0.289h$

补充知识点：最大切应力

矩形：$\tau_{max} = \frac{3}{2}\times\frac{F_S}{A}$　　工形：$\tau_{max} = \frac{F_S S_{z\,max}^*}{I_z d}$

环形：$\tau_{max} = 2\frac{F_S}{A}$　　 半圆：$\tau_{max} = \frac{4}{3}\frac{F_S}{A}$

五、解：(1) 在筒体上取单元体如图所示。由对称性知存在应力 σ_y



可由分离体的平衡条件确定，

$\sigma_x = \frac{PD}{4t}$, $\sigma_y = \frac{PD}{2t}$ --- ①

由广义胡克定律有：

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) \quad \text{--- ②}$$

将①代入②解得 $p = \frac{4tE\varepsilon_y}{D(2-\nu)}$

(2) 由广义胡克定律又有 $\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)$ --- ③

由②、③得 $\varepsilon_x = \frac{1-2\nu}{2-\nu}\varepsilon_y$

~~则 $\varepsilon_u = \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{\varepsilon_x-\varepsilon_y}{2}\cos(45°\times2) + \frac{\gamma_{xy}}{2}\sin45°\times2$~~

~~$= \frac{\varepsilon_x+\varepsilon_y}{2} = \frac{\frac{1-2\nu}{2-\nu}\varepsilon_y+\varepsilon_y}{2} =$~~

~~因为 $\sigma_\alpha = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos2\alpha - \tau_{xy}\sin2\alpha$~~

则 $\varepsilon_u = \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{\varepsilon_x-\varepsilon_y}{2}\cos(45°\times2) + \frac{\gamma_{xy}}{2}\sin45°\times2$

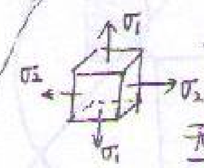
$= \frac{\varepsilon_x+\varepsilon_y}{2} = \frac{3(1-\nu)}{2(2-\nu)}\varepsilon_y$

(3) 依题意有：$\pi D + \pi D\varepsilon_y = \pi(D+\Delta D)$

则 $\Delta D = \varepsilon_y D$

考点：

封闭圆筒问题：



$\sigma_1 = \frac{PD}{2\delta}$ (环向) $\sigma_3 = 0$

$\sigma_2 = \frac{PD}{4\delta}$ (轴向)

无切应力!! 即切应变为0.

任意角度应变公式

$$\varepsilon_\alpha = \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{\varepsilon_x-\varepsilon_y}{2}\cos2\alpha + \frac{\gamma_{xy}}{2}\sin2\alpha$$

$$\frac{\Delta l}{l} = \varepsilon \Rightarrow \Delta l = \varepsilon l$$

此题可改为进行强度校核

$\sigma_{r3} = \sigma_1 - \sigma_3$

$\sigma_{r4} = \sqrt{\frac{1}{2}[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2]}$

例题见《指导》P86

广义胡克定律：

$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y+\sigma_z)]$ $\gamma_{xy} = \frac{\tau_{xy}}{G}$

$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x+\sigma_z)]$ $\gamma_{yz} = \frac{\tau_{yz}}{G}$

$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x+\sigma_y)]$ $\gamma_{zx} = \frac{\tau_{zx}}{G}$

六

1. (1) 求主应力，有 $\sigma_{1,3} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$

则 $\sigma_1 = \frac{\sigma}{2} + \sqrt{\frac{\sigma^2}{4}+\tau^2}$

$\sigma_3 = \frac{\sigma}{2} - \sqrt{\frac{\sigma^2}{4}+\tau^2}$

由第三强度理论有 $\sigma_{r3} = \sigma_1 - \sigma_3 = \sqrt{\sigma^2+4\tau^2} \le \sigma_s$

将 $\tau = 100MPa$, $\sigma_s = 240MPa$ 代入解得 $\sigma = 132.7MPa$

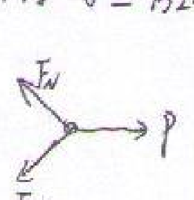
考点：$\sigma_{1,3} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$

只在一对截面上有应力。

第三强理论：$\sigma_{r3} = \sqrt{\sigma^2+4\tau^2}$

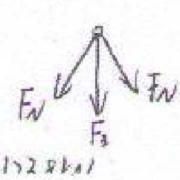
四……：$\sigma_{r4} = \sqrt{\sigma^2+3\tau^2}$

2. 解、如图取结点A分析：



由 $\Sigma F_x = 0$ 有 $2F_N\cos45° = P$ 得 $F_N = \frac{\sqrt{2}}{2}P$

再取结点B分析：$F_3 = 2F_N\cos45° = P$



分析图示结构可知 3杆最先失稳，

则 $F_{cr} = \frac{\pi^2EI}{(\mu l)^2} = \frac{\pi^2EI}{(\sqrt{2}a)^2} = 123.8kN$ 则 $P = 123.8kN$

压杆稳定：$F_{cr} = \frac{\pi^2EI}{(\mu l)^2}$

两端铰：$\mu = 1$

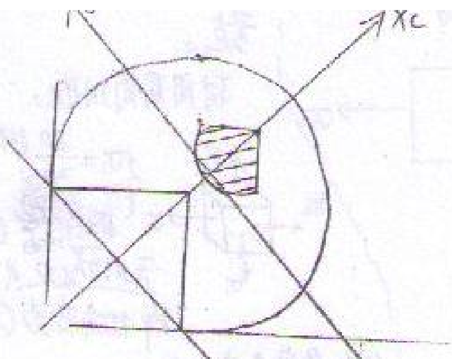
一固一铰：$\mu = 0.7$

两固：$\mu = 0.5$

一固一自由：$\mu = 2$

两固但可横移：$\mu = 1$

3.



4. 解：圆截面为实心时有 $\tau=\frac{T}{W_p}=\frac{T}{\pi D^3/16}=\frac{16T}{\pi D^3}$ —— ①

当截面如图所示时有 $\tau=\frac{T_1}{W_{p1}}=\frac{T_1}{\frac{\pi D^3(1-\alpha^4)}{16}}=\frac{16T_1}{\pi D^3(1-\alpha^4)}$ —— ②

其中 $\alpha=\frac{\frac{D}{2}}{D}=\frac{1}{2}$，且 —— ③

联立①②③式解得 $T_1=\frac{15}{16}T$

$\tau=\frac{T}{W_p}$ 实心圆 $W_p=\frac{\pi D^3}{16}$ $I_p=\frac{\pi D^4}{32}$

空心圆 $W_p=\frac{\pi D^3}{16}(1-\alpha^4)$

其 $\alpha=\frac{d}{D}$

5. 解：(a) 图动载系数 ~~$k_{d1}=1+\sqrt{1+\frac{2h}{\Delta st_1}}$~~ $=2$ $\sigma_a=\sigma_{st}$

(b) 图动载系数 $k_{d2}=1+\sqrt{1+\frac{2h}{\Delta st_2}}=1+\sqrt{1+\frac{\pi E d^2 h}{2Ql}}$ $\sigma_b=k_{d2}\sigma_{st}$

(c) 图动载系数 $k_{d3}=1+\sqrt{1+\frac{2h}{\Delta st_3}}$

~~(d) 图~~ 其中 $\Delta st_3=\Delta l+\frac{Ql}{E_2A_2}$ $\Delta st_3>\Delta st_2$ $\Rightarrow k_{d2}>k_{d3}$ $\sigma_c=k_{d3}\sigma_{st}$

(d)

$\Delta st_4<\Delta st_2$ $\Rightarrow$ $k_{d4}>k_{d2}>k_{d3}$ $\sigma_d=k_{d4}\sigma_{st}$

则可知 $\sigma_d>\sigma_b>\sigma_c>\sigma_a$.

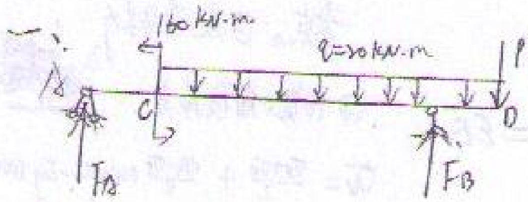
参考：

从 h 高自由下落：$k_d=1+\sqrt{1+\frac{2h}{\Delta st}}$

从 v 初速下落：$k_d=1+\sqrt{1+\frac{2H}{\Delta st}}$ 其中 $H=\frac{v^2}{2g}+h$

九九

一、



考点：① 作 F_s 图和 M 图

② $\sigma = \frac{My}{I_z}$，$\sigma_{max} = \frac{M_{max}}{W_z}$　　$\sigma_{max} < [\sigma]$

③ $\tau_{max} = \frac{3}{2}\frac{F_{s,max}}{A}$

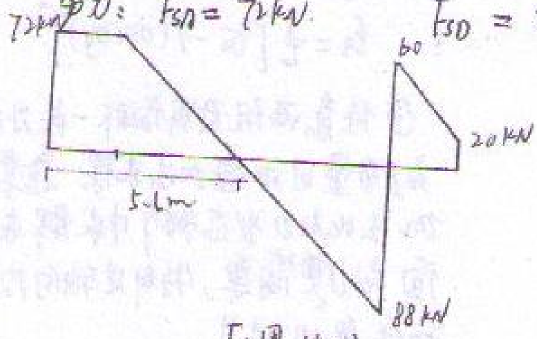
矩形截面

受力图如上所示，$\sum M_A = 0$：$10F_B + 160 - 20\times12 - 20\times10\times7 = 0$　解得 $F_B = 148$ kN

由 $\sum F_y = 0$：$F_A + F_B - 20\times10 - 20 = 0$　解得 $F_A = 72$.

节点弯矩：$M_{C左} = 2F_A = 144$ kN·m（下）；$M_{C右} = 144 - 160 = -16$ kN·m（上）；$M_B = 2F_B + \frac{1}{2}ql^2 = 8$

剪力：$F_{SA} = 72$ kN　　$F_{SD} = 20$ kN　　$F_{B右} = 60$ kN　$F_{B左} = -88$ kN.



F_s 图（kN）

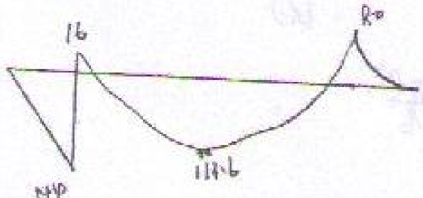
如图知：有 $M_{max} = 144$ kN·m，$F_{s max} = 88$ kN

则 $\sigma_{max} = \frac{M_{max}}{W_z} = \frac{6\times144\times10^3}{0.3^2\times0.5^2} = 11.52$ MPa.

故有 $\sigma_{max} < [\sigma]$ 安全.

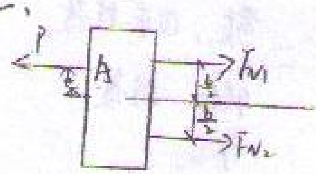
又 $\tau_{max} = \frac{3}{2}\frac{F_{s,max}}{A} = \frac{3}{2}\times\frac{88\times10^3}{0.3\times0.5} = 0.88$ MPa

有 $\tau_{max} < [\tau]$，安全.



M 图（kN·m）

二、



解：取左边部受力分析如图所示。

由几何变形条件可知两杆伸长一样，即 $\Delta l_1 = \Delta l_2$

由物理方面有 $\Delta l_1 = \frac{F_{N1}l}{E_1A}$，$\Delta l_2 = \frac{F_{N2}l}{E_2A}$

即 $\frac{F_{N1}l}{E_1A} = \frac{F_{N2}l}{E_2A}$　　— — — ①

考点：① $\Delta l = \frac{Fl}{EA}$

② 超静定 $\begin{cases}\text{几何方面}\\\text{物理方面}\end{cases}$

又由平衡关系 $M_A = 0$ 有 $F_{N1}\left(\frac{b}{2} - e\right) = F_{N2}\left(\frac{b}{2} + e\right)$ — ②

联立 ① ② 式解得 $e = \frac{E_1 - E_2}{2(E_1 + E_2)}b$

三、解：(1) 至少应布置 2 片电阻片，分别在轴向和与轴向成 45° 角方向

轴向为 ε_{0°，与轴向成 45° 方向应变为 ε_{45°

考点：① 应变分析

(2) 在轴向有 $\sigma = \frac{P}{A}$ 则 $P = \sigma A$ 又 $\sigma = E\varepsilon$

故 $P = \varepsilon_{0^\circ} E \frac{\pi D^3}{4} = \frac{\pi E D^3 \varepsilon_{0^\circ}}{4}$

② 任意角度应力

$$\sigma_\alpha = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\alpha - \tau_{xy}\sin 2\alpha$$

(3) 圆轴任一点，P 和 m 单独作用时，应力图如下所示

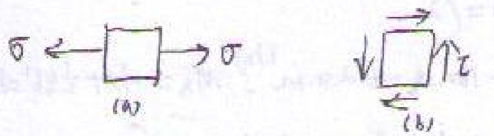
③ 任意角度应变

广义胡克定律：

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y+\sigma_z)]$$

$$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x+\sigma_z)]$$

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x+\sigma_y)]$$



(a) 中 45° 有 $\sigma_1' = \frac{\sigma}{2}$，$-45^\circ$ $\sigma_3' = \frac{\sigma}{2}$

(b) 中 45° $\sigma_1'' = \tau$ $\quad -45^\circ$ $\sigma_3'' = -\tau$

③ 任意两组载荷作用下一点处的应力分量可由叠加法求得。通常叠加法比应力状态分析中求解斜截面应力更简单，特别是轴向拉压和纯剪切时!!

则 P 单独作用时 $\varepsilon_{45}' = \frac{1}{E}(\sigma_1' - \nu\sigma_3') = \frac{1-\nu}{E}\frac{\sigma}{2}$

m 单独作用时 $\varepsilon_{45}'' = \frac{1}{E}(\sigma_1'' - \nu\sigma_3'') = \frac{1+\nu}{E}\tau$

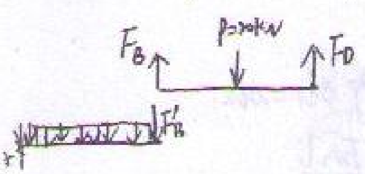
叠加有 $\varepsilon_{45} = \varepsilon_{45}' + \varepsilon_{45}'' = \frac{1}{E}\left[(1+\nu)\tau + (1-\nu)\frac{\sigma}{2}\right]$ ——(x)

又 $\tau = \frac{m}{W_p}$，$\sigma = \varepsilon_0 E$，$W_p = \frac{\pi D^3}{16}$

代入(x)式得 $m = \frac{E\pi D^3[2\varepsilon_{45^\circ} - (1-\nu)\varepsilon_{0^\circ}]}{32(1+\nu)}$

四、解 取 BCDE 为研究对象，受力如下所示。

考点：① 求挠度



由 $M_B = 0$ 有：$F_D \times 1 - 20 \times \frac{1}{2} = 0$ 解得 $F_D = 10$ kN

又由 $\sum F_y = 0$ 有 $F_B + F_D - 20 = 0$ 得 $F_B = 10$ kN.

将 F_B 反加在 AB 梁上，即 $F_B = 10$kN. 受力如图示.

$\therefore W_B = \frac{ql^4}{8EI} + \frac{F_B l^3}{3EI} = 2.29$ mm

转角	挠度
1	2
2	3
6	8
16	48
24	384

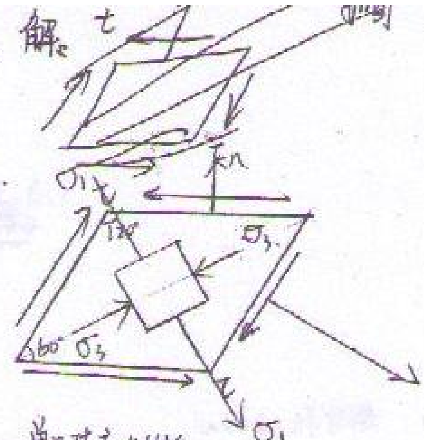
$W_C = \frac{1}{2}\Delta l_{DE} + \frac{1}{2}W_B + \frac{Pl^3}{48EI}$

$= \frac{F_D l}{2EA} + \frac{1}{2}W_0 + \frac{Pl^3}{48EI}$

$= 1.603$ mm

具体见第 2 次 (三.)

五、解：



解：如图示，假设x方向

则 $\sigma_x=0$，$\tau_x=\tau$，$\sigma_{120°}=0$

则 $\sigma_{120°}=\dfrac{\sigma_x+\sigma_y}{2}+\dfrac{\sigma_x-\sigma_y}{2}\cos 120°\times 2-\tau_{xy}\sin 120°\times 2=0$

解得 $\sigma_y=-\dfrac{2\sqrt{3}}{3}\tau$

则 $\sigma_1=\dfrac{\sigma_x+\sigma_y}{2}+\sqrt{\left(\dfrac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$

$=\dfrac{\sqrt{3}}{3}\tau$

$\sigma_3=\dfrac{\sigma_x+\sigma_y}{2}-\sqrt{\left(\dfrac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$

$=-\sqrt{3}\tau$

$\sigma_2=0$

$2\alpha_0=\arctan\dfrac{-2\tau_x}{\sigma_x-\sigma_y}=-\sqrt{3}$ 则 $\alpha=-30°$

考点：单元体应力分析

步骤：① 设x方向，写出各已知应力（$\sigma_x, \sigma_y, \tau_x, \sigma_\alpha$等），α等）

② 将上述力代入下列公式。

$$\begin{cases}\sigma_\alpha=\dfrac{\sigma_x+\sigma_y}{2}+\dfrac{\sigma_x-\sigma_y}{2}\cos 2\alpha-\tau_x\sin 2\alpha\\ \tau_\alpha=\dfrac{\sigma_x-\sigma_y}{2}\sin 2\alpha+\tau_x\cos 2\alpha\end{cases}$$

（注意，不是都代，选简单快速得答案的代）

③ 求主应力：$\sigma_{1,3}=\dfrac{\sigma_x+\sigma_y}{2}\pm\sqrt{\left(\dfrac{\sigma_x-\sigma_y}{2}\right)^2+\tau_x^2}$

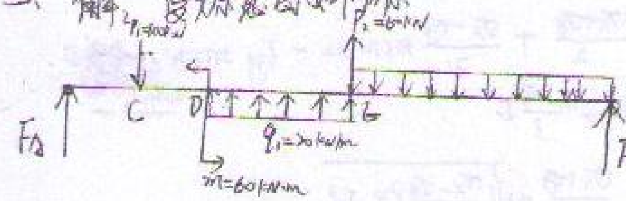
（注意：$\sigma_1, \sigma_2, \sigma_3$ 的值从大到小的）

④ 求角度：$2\alpha_0=\arctan\dfrac{-2\tau_x}{\sigma_x-\sigma_y}$

第④步中还应注意α角：为斜截面外法线与x轴正向夹角。从x轴到外法线方向逆时针为正

二000年

一、解：受力示意图如下所示



$\sum M_A=0$： $8F_B+60+20\times2\times3+60\times4-100-20\times4\times6=0$ 解得 $F_B=20kN$

$\sum F_y=0$： $F_A+F_B+2q_1+P_2-P_1-4q_2=0$ 即 $F_A+20+40+60-100-80=0$ 解得 $F_A=60kN$

各截面的弯矩：$M_C=F_A\times1=60kN\cdot m$（下侧受拉）

$M_{D左}=F_A\times2-P_1\times1=20kN\cdot m$（下侧受拉）

$M_{D右}=-40kN\cdot m$（上侧受拉）

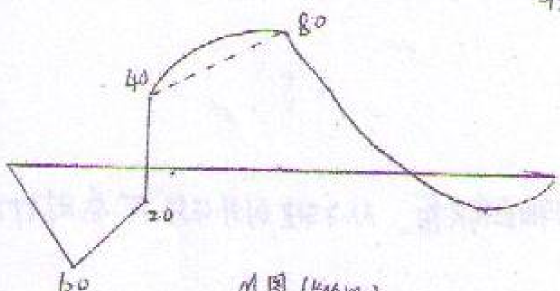
$M_E=4F_B-\frac{1}{2}q_2l_{E右}^2=-80kN\cdot m$（上侧受拉）

各截面剪力：$F_{SA}=60kN$

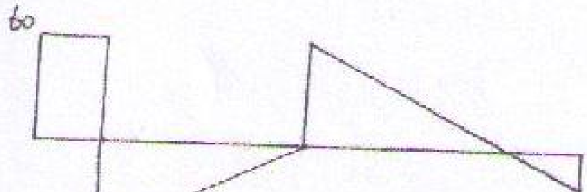
$F_{C左}=60kN$，$F_{C右}=-40kN$

$F_{D左}=-40kN$

$F_B=-20kN$，$F_{E右}=60kN$



M图（kN·m）

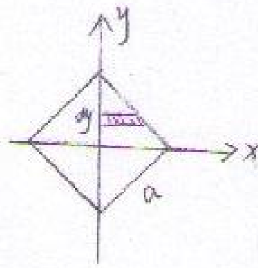


二、解：

(1) 对截面中轴的 I_z 为 $I_x=\int_A y^2dA$

$=4\int_0^{\frac{\sqrt{2}}{2}a}y^2(\frac{\sqrt{2}}{2}a-y)dy$

$=\frac{1}{12}a^4$



知识点：

① 边长为a的正方形对其对角线惯性矩 $I=\frac{a^4}{12}$

② $\sigma_x=\frac{My}{I_z}$，$\varepsilon_x=\frac{\sigma_x}{E}$.

$\Delta L=\int_0^L\varepsilon_x dx$.

③ $\tau_{max}=\frac{F_S S^*_{z\,max}}{I_z b}$

关键求：$\frac{S^*_{z\,max}}{b}$！

距端点为x入任意截面的弯矩 $M_x=Px$

则 $\sigma_x=\frac{M_xy}{I_x}=\frac{Px\frac{\sqrt{2}}{2}a}{\frac{1}{12}a^4}=\frac{6\sqrt{2}Px}{a^3}$ 又、$\varepsilon_x=\frac{\sigma_x}{E}$

则 $\varepsilon_x=\frac{6\sqrt{2}Px}{Ea^3}$ ∴ $\Delta l=\int_0^L\varepsilon_x dx=\int_0^L\frac{6\sqrt{2}Px}{Ea^3}dx=\frac{3\sqrt{2}PL}{Ea^3}$

(2) 梁横的最大弯矩为 $M_{max}=PL$.

故 $\sigma_{max}=\frac{M_{max}y}{I_x}=\frac{PL\frac{\sqrt{2}}{2}a}{\frac{1}{12}a^4}=\frac{6\sqrt{2}PL}{a^3}$

最大切应 $\tau_{max} = \frac{F_S S^*_{z,max}}{I_z b}$ 要使求 τ_{max} 则应求 $\frac{S^*_{z,max}}{b}$ 最大

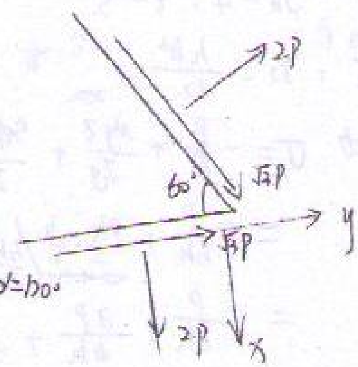
设最大切应力位置距中性轴距为 y，则 $\frac{S^*_{z,max}}{b} = \frac{\frac{1}{2}b \times (\frac{\sqrt{2}}{2}a - y)\left[\frac{1}{3}(\frac{\sqrt{2}}{2}a - y) + y\right]}{b}$

$= \frac{1}{2}(\frac{\sqrt{2}}{2}a - y)(\frac{\sqrt{2}}{6}a + \frac{2}{3}y)$

令上式为 $f(y)$ 则 $\frac{\partial f}{\partial y} = -\frac{1}{2}(\frac{\sqrt{2}}{6}a + \frac{2}{3}y) + \frac{1}{2}(\frac{\sqrt{2}}{2}a - y) \times \frac{2}{3} = 0$ 得 $y = \frac{\sqrt{2}}{8}a$.

即当 $y = \frac{\sqrt{2}}{8}a$ 时 $f(y)$ 取最大值 为 $f(y) = \frac{S^*_{z,max}}{b} = \frac{3a^2}{32}$

故 $\tau_{max} = \frac{P}{\frac{1}{3}a^4} \times \frac{3a^2}{32} = \frac{9P}{8a^2}$



三、

解：如图示 设 x 轴方向，则 原图可知

$\sigma_x = 2P$，$\tau_x = -\sqrt{3}P$，$\sigma_{120°} = 2P$，$\tau_{120°} = \sqrt{3}P$

$\sigma_{120°} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 120° \times 2 - \tau_x \sin 120° \times 2$

即 $2P = \frac{2P + \sigma_y}{2} + \frac{2P - \sigma_y}{2} \times (-\frac{1}{2}) - \sqrt{3}P \times \frac{\sqrt{3}}{2} = 0$ 解得 $\sigma_y = 4P$

故主应力为：$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_x^2} = 3P + 2P = 5P$

$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_x^2} = P$

故 $\sigma_1 = 5P$，$\sigma_2 = P$，$\sigma_3 = 0$

则主应变 $\varepsilon_1 = \frac{1}{E}[\sigma_1 - \nu(\sigma_2 + \sigma_3)] = \frac{P(5-\nu)}{E}$

$\varepsilon_2 = \frac{1}{E}[\sigma_2 - \nu(\sigma_1 + \sigma_3)] = \frac{P(1-5\nu)}{E}$

$\varepsilon_3 = \frac{1}{E}[\sigma_3 - \nu(\sigma_1 + \sigma_2)] = -\frac{6P\nu}{E}$

知识点：

① 体应变 $\theta = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z)$

$= \frac{1-2\nu}{E}(\sigma_1 + \sigma_2 + \sigma_3)$

(2)

② 体积改变能密度：

$v_V = \frac{1-2\nu}{6E}(\sigma_1 + \sigma_2 + \sigma_3)^2$

③ 形状改变能密度：

$v_d = \frac{1+\nu}{6E}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]$

四、解法一：

解：(1) 设 ε' 与轴线方向夹角为 α

则有 $\varepsilon' = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\alpha + \frac{\gamma_{xy}}{2}\sin 2\alpha$

$\varepsilon'' = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos(2\alpha + 90°) + \frac{\gamma_{xy}}{2}\sin(2\alpha + 90°)$

由于轴只有扭矩 m，则 ε_x，ε_y 均为 0，则 $\tan 2\alpha = \frac{\varepsilon'}{\varepsilon''} = \frac{3.25 \times 10^{-4}}{-5.63 \times 10^{-4}} = -0.577$

则 $\sin 2\alpha = -0.5$ 故有 $\gamma_{xy} = \frac{2\varepsilon'}{\sin 2\alpha} = \frac{2 \times 3.25 \times 10^{-4}}{-0.5} = -1.3 \times 10^{-3}$

又 $\tau = \frac{T}{W_P} = \frac{-m}{W_P}$，又 $\gamma_{xy} = \frac{\tau}{G}$ 则有 $m = \tau W_P = -\gamma_{xy} G W_P = -\gamma_{xy}\frac{E}{2(1+\nu)}\frac{\pi d^3}{16} = 19.63\,kN$.

解法二：[illegible]

的主应力为σ''，则由广义胡克定律有

$$\varepsilon'=\frac{1}{E}[\sigma'-\nu(-\sigma'')]=\frac{1+\nu}{E}\sigma' \qquad \varepsilon''=\frac{1+\nu}{E}\sigma''$$

代入已知量得 $\sigma'=\frac{E}{1+\nu}\varepsilon'=50MPa$，$\sigma''=\frac{E}{1+\nu}\varepsilon''=-86.62MPa$

$$\tau=\sqrt{(\sigma')^2+(\sigma'')^2}=\sqrt{50^2+103.9^2}=100MPa$$

由$\tau=\frac{T}{W_p}=\frac{M_e m}{W_p}$ 得 $m=\tau W_p=100\times10^6\times\frac{\pi\times(0.1)^3}{16}=19.63\,kN$，体积应变 $\theta=\frac{1-2\nu}{E}(\sigma_1+\sigma_2+\sigma_3)$
$=0$

五、解：依题图有 $M_y=\frac{Ph}{2}$，$M_z=\frac{Pb}{2}$，

组合变形：

$$\sigma=\frac{P}{A}+\frac{M_y z}{I_y}+\frac{M_z y}{I_z}$$

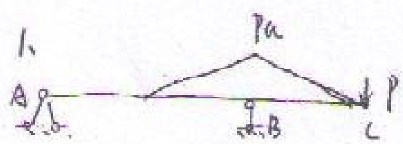
A点坐标 $y_A=\frac{b}{4}$，$z=\frac{h}{2}$

$I_y=\frac{bh^3}{12}$，$I_z=\frac{hb^3}{12}$

则A处应力
$$\sigma=-\frac{P}{A}+\frac{M_y z}{I_y}+\frac{M_z y}{I_z}$$
$$=-\frac{P}{bh}+\frac{Ph}{2}\cdot\frac{h}{2}\Big/\frac{bh^3}{12}+\frac{Pb}{2}\cdot\frac{b}{4}\Big/\frac{hb^3}{12}$$
$$=-\frac{P}{bh}+\frac{3P}{bh}+\frac{3P}{2bh}$$
$$=\frac{7P}{2bh}$$

则 $\varepsilon_x=\frac{\sigma}{E}=\frac{7P}{2Ebh}$

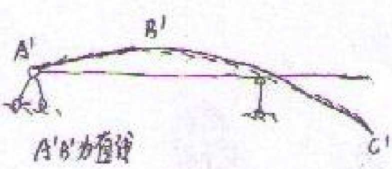
六、1、



挠曲线

口诀：上负凸，下正凹

零直



2、解：$\tau=\frac{P}{A}=\frac{P}{\pi\frac{d^2}{4}}<[\tau]$ 则 $P=\frac{\pi}{4}d^2[\tau]$

$\sigma_{bs}=\frac{P}{d\frac{l}{2}}=\frac{2P}{dl}<[\sigma_{bs}]$ 则 $P=\frac{[\sigma_{bs}]dl}{2}$

故有 $\frac{\pi}{4}d^2[\tau]=\frac{dl}{2}[\sigma_{bs}]$

则 $l/d=\frac{\pi[\tau]}{2[\sigma_{bs}]}$

3. 由 $K_d = 1+\sqrt{1+\dfrac{2h}{\Delta_{st}}}$

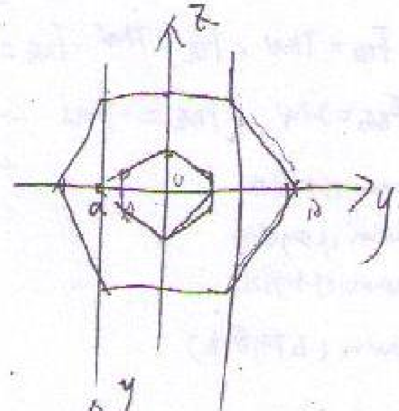
(a)中 $\Delta_{st_1} = \dfrac{QL^3}{48EI} + \dfrac{Q}{4k}$

(b)中 $\Delta_{st_2} = \dfrac{QL^3}{48EI} + \dfrac{Q}{k}$

则比较知 $\Delta_{st_1} < \Delta_{st_2}$

故 $k_{d_1} > k_{d_2}$ 则 $\sigma_{d_1} = k_{d_1}\sigma_{st} > \sigma_{d_2} = k_{d_2}\sigma_{st}$ 故图(a)中动应力大

4.

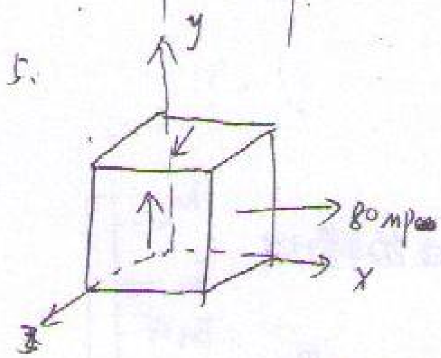


解：过a点作垂直于y轴直线①，并取其为中性轴，于是该中性轴在y、z两个形心主惯性轴的截距分别为 $a_{y_1} = -y_a$，$a_{z_1} = \infty$

则与其对应的截面核心边界点1的坐标为

$\rho_{y_1} = -\dfrac{i_z^2}{a_{y_1}} = \dfrac{i_z^2}{y_a}$，$\rho_{z_1} = -\dfrac{i_y^2}{a_{z_1}} = 0$

5.



解：如图知 $\sigma_x = 80$ mpa.

$\tau_{yx} = 50$，$\sigma_y = \sigma_z = 0$.

则有主应力 $\sigma = \dfrac{\sigma_y+\sigma_z}{2} \pm \sqrt{\left(\dfrac{\sigma_y-\sigma_z}{2}\right)^2 + \tau_{yz}^2}$

$= \pm 50$ mpa.

则 $\sigma_1 = 80$ mp，$\sigma_2 = 50$ mpa，$\sigma_3 = -50$ mpa.

故比能 $u = \dfrac{1}{2E}\left[\sigma_1^2+\sigma_2^2+\sigma_3^2 - 2\nu(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)\right]$

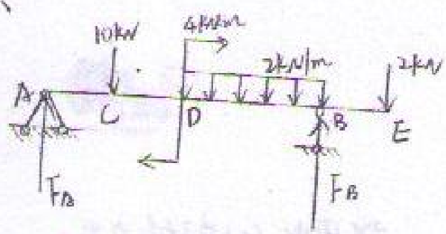
$= 32750$ pa

$\sigma_{r3} = \sigma_1 - \sigma_3 = 130$ mpa

6. 解：推导欧拉公式时，假定材料是在线弹性范围内工作，因此，压杆在失稳、变弯前的应力不得超过材料的比例极限 σ_p，否则，应力与应变不成正比，不符合假定条件，当然不能用欧拉公式计算压杆临界力。欧拉公式只适用细长压杆，即长细比 λ 大于 $\lambda_p = \pi\sqrt{E/\sigma_p}$ 的压杆。

二00一年

一、



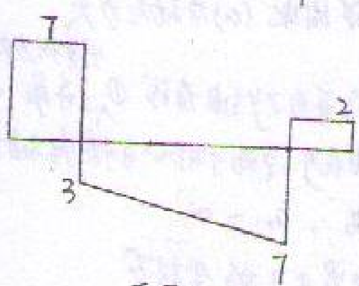
解：受力示意图如左图所示，

$\sum M_A=0$：$4F_B-10\times1-4-2\times2\times3-2\times5=0$

$F_B=9kN$

$\sum F_y=0$：$F_A+F_B-10-2\times2-2=0$

$F_A=7kN$



F_S图 (kN)

各截面剪力：$F_{SA}=7kN$，$F_{C左}=7kN$，$F_{C右}=-3kN$

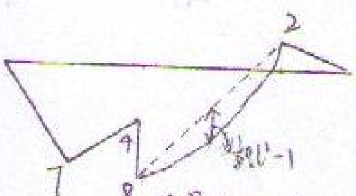
$F_{SE}=2kN$　$F_{B右}=2kN$，$F_{B左}=-7kN$

各截面弯矩：$M_C=7kN\cdot m$ (下侧受拉)

$M_{D左}=4kN\cdot m$ (下侧受拉)

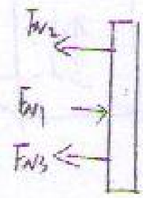
$M_{D右}=8kN\cdot m$ (下侧受拉)

$M_B=2kN\cdot m$ (上侧受拉)



M图 (kN·m)

二、



解：三平行杆的轴力(如右)均未知，但平衡力只有两个独立的平衡方程，

故为一次超静定

(1) 静力方面：$F_{N1}-F_{N2}-F_{N3}=0$ ①

(2) 物理方面：温度变化时 1杆伸长 $\delta=\alpha\Delta t l=\alpha(t-t_0)l$

在轴力作用下各杆长度变化：

$\Delta l_1=\frac{F_{N1}l}{EA}$，$\Delta l_2=\frac{F_{N2}l}{EA}$，$\Delta l_3=\frac{F_{N3}l}{EA}$ ②

(3) 几何方面：

$\Delta l_2=\Delta l_3$ ③

$\Delta l_2=\delta-\Delta l_1$ ④

由①②③可得 $F_{N1}=2F_{N2}=2F_{N3}$ ⑤

由②④⑤得 $\frac{F_{N1}l}{EA}+\frac{F_{N2}l}{EA}=\alpha\Delta t l$

得 $F_{N2}=F_{N3}=\frac{\alpha(t-t_0)EA}{3}$，$F_{N1}=\frac{2\alpha(t-t_0)EA}{3}$

故 $\sigma_1=\frac{F_{N1}}{A}=\frac{2\alpha(t-t_0)E}{3}$ (压应力)

$\sigma_2=\sigma_3=\frac{F_{N2}}{A}=\frac{\alpha(t-t_0)E}{3}$ (拉应力)

路截面变化的正应力为 $\sigma=\frac{My}{I_z}$，又 $I_z=\frac{bh^3}{12}$

则 $\sigma=\frac{My}{bh^3/12}=\frac{12P\frac{L}{2}y}{bh^3/12}=\frac{6PLy}{bh^3}$

故 $F_N^*=\int_A\sigma\,dA=\int_{\frac{h}{4}}^{\frac{h}{2}}\frac{6PLy}{bh^3}b\,dy=\frac{3PLy^2}{h^3}\Big|_{\frac{h}{4}}^{\frac{h}{2}}=\frac{9PL}{16h}$

(2) ~~路截面变化的切应力为~~ $\cancel{\tau=\frac{F_s S_z^*}{I_z b}=\frac{P}{\frac{bh^3}{12}\cdot b}\int_y^{\frac{h}{2}}y\,b\,dy=\frac{6Py^2}{bh^3}-\frac{12P}{32bh}}$

则 $\cancel{F_S^*=\int_A\tau\,dA=\int_{\frac{h}{4}}^{\frac{h}{2}}\left(\frac{6Py^2}{bh^3}-\frac{12P}{32bh}\right)b\,dy}$ $\cancel{=\frac{6P}{bh^3}\left(\frac{h^2}{4}-y^2\right)}$

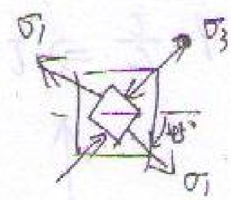
$\cancel{=\frac{2Py^3}{h^3}\Big|_{\frac{h}{4}}^{\frac{h}{2}}-\frac{12P}{32bh}y\Big|_{\frac{h}{4}}^{\frac{h}{2}}=}$

$\cancel{F_S^*=\int_A\tau\,dA=\int_{\frac{h}{4}}^{\frac{h}{2}}\frac{6P}{bh^3}\left(\frac{h^2}{4}-y^2\right)dy=}$

$\tau=\frac{F_S S_z^*}{I_z b}=\frac{P}{I_z b}\int_y^{\frac{h}{2}}y\,b\,dy=\frac{P}{I_z 2}\left(\frac{h^2}{4}-y^2\right)$

则 $F_S^*=\int_A\tau\,dA=\int_{\frac{h}{4}}^{\frac{h}{2}}\frac{P}{2I_z}\left(\frac{h^2}{4}-y^2\right)b\,dy=\frac{5}{32}P$

四、解 (1) 电阻片应沿与纵轴成45°方向粘贴；



(2) 当M作用在轴上时，任一点的应力图如下图所示

主应力为 $\sigma_{1,3}=\frac{\sigma_x+\sigma_y}{2}\pm\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau^2}$ 则 $\sigma_1=\tau,\ \sigma_2=0,\ \sigma_3=-\tau$

则 ~~材料与-45°方向应力为~~ $\cancel{\sigma_{-45°}=\frac{\sigma_x+\sigma_y}{2}+\frac{\sigma_x-\sigma_y}{2}\cos2\alpha-\tau_x\sin2\alpha}$

$\cancel{=-\tau}$

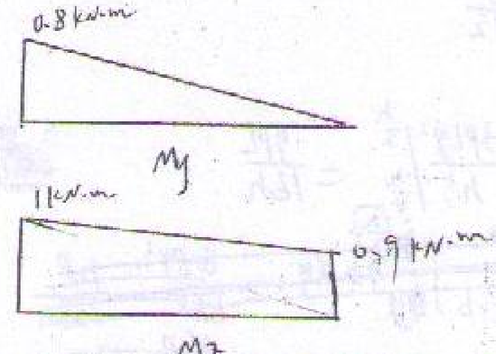
$\cancel{\sigma_{45°}=\tau}$

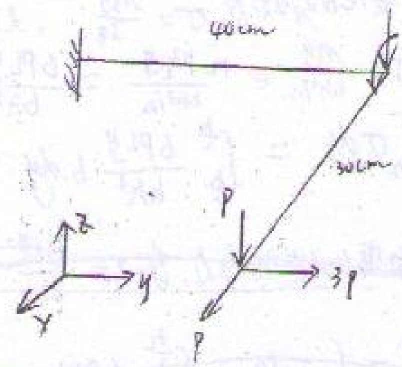
则 $\varepsilon_{45°}=\varepsilon_1=\frac{1}{E}(\sigma_1-\nu\sigma_3)=\frac{1+\nu}{E}\tau$

则 $\cancel{\varepsilon=\frac{1}{E}(\sigma_{45°}-\nu\sigma_{-45°})}$ ~~而~~ $\cancel{\varepsilon=\frac{1}{E}(-\tau-\nu\tau)}$ ~~则~~ $\cancel{\tau=\frac{-\varepsilon E}{1+\nu}}$

又 $\tau=\frac{T}{W_p}=\frac{M}{W_p}$ 则 $M=\tau W_p=\frac{\varepsilon E}{1+\nu}W_p=\frac{\pi D^3(1-\alpha^4)\varepsilon E}{16(1+\nu)}$

五、解





A截面的弯矩为 $M_A=\sqrt{M_y^2+M_z^2}=\sqrt{0.8^2+1^2}=1.28\,kN\cdot m$

~~$\tau=\frac{T}{W_p}$~~ 故最大正应力为 $\sigma_{max}=\frac{M_A}{W_z}+\frac{F_{NA}}{A}=\frac{1.28\times10^3}{\frac{\pi d^3}{32}}+\frac{3\times2\times10^3}{\frac{\pi d^2}{4}}$

$=62.51\,mpa$

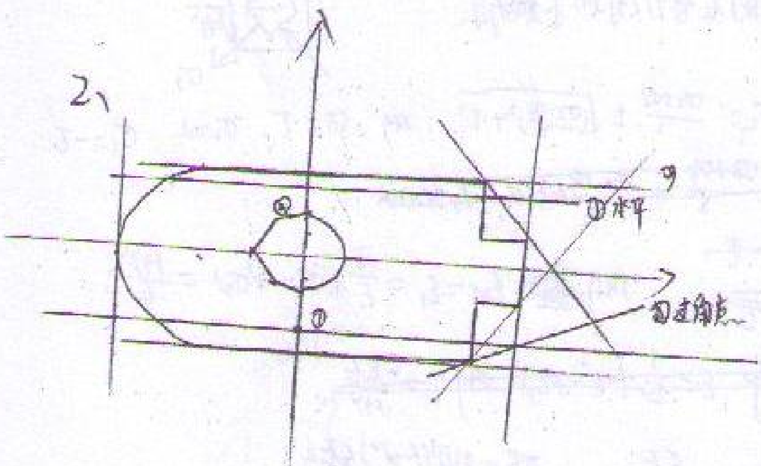
$\tau=\frac{T}{W_p}=\frac{P\times0.3}{\frac{\pi d^3}{16}}=14.15\,mpa$

则 $\sigma_{r3}=\sqrt{\sigma_{max}^2+4\tau^2}=68.63\,mpa$

故 $\sigma_{r3}<[\sigma]=120\,mpa$ 安全

六、1、$\gamma=\frac{\tau}{G}\Rightarrow\tau=\gamma G=\alpha G$

2、



3、$EI\theta(x)=EIy'(x)=\int_l M(x)\,dx+C$

当 $x=0$ 时 $\theta(0)=0$，则 $C=0$

而 $\int_l M(x)\,dx$ 为弯矩图面积，正负相抵消 故 $\theta=ma/EI$

4. $k_d = 1+\sqrt{1+\frac{2H}{\Delta_{st}}}$

$\Delta_{st} = \frac{Qa^3}{3EI_z}$，$I_z = \frac{\pi d^4}{64}$

故 $k_d = 1+\sqrt{1+\frac{6EHI_z}{Qa^3}} = 1+\sqrt{1+\frac{3\pi EHd^4}{32Qa^3}}$

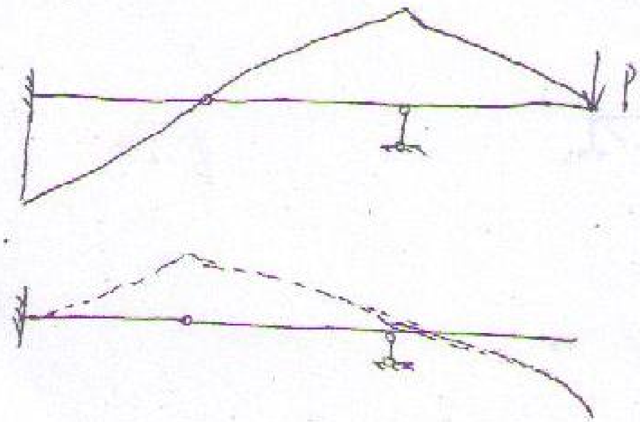
5. $F_{cr} = \frac{\pi^2 EI}{(\mu l)^2}$

$F_{cr}^a = \frac{\pi^2 EI}{(0.7\times 25d)^2} = \frac{\pi^2 EI}{306.25d^2}$

$F_{cr}^b = \frac{\pi^2 EI}{(1\times 20d)^2} = \frac{\pi^2 EI}{400d^2}$

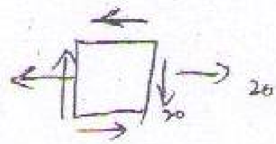
故 $F_{cr}^b < F_{cr}^a$，(b)易失稳，$F_{cr}^b = \frac{\pi^2 E}{400d^2}\times\frac{\pi d^4}{64} = \frac{\pi^3 E d^2}{256000}$

6. 解：



7.

(a)图 $\sigma_x = 20$ $\tau_x = 20$



$\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_x^2}$

$= 10+\sqrt{100+400}$

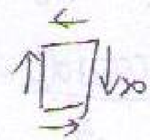
$= 10+10\sqrt{5}$

$\sigma_3 = \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_x^2}$

$= 10-10\sqrt{5}$

$\sigma_{r3} = \sigma_1-\sigma_3 = 20\sqrt{5}$ MPa

(b)图 $\cancel{\sigma=20}$ $\tau_x = 20$



$\sigma_{1,3} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_x^2}$

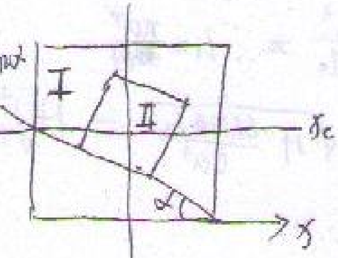
$= \pm 20$

$\sigma_1 = 20$ MPa，$\sigma_2 = 20$ MPa，$\sigma_3 = -20$ MPa

$\sigma_{r3} = \sigma_1-\sigma_3 = 40$ MPa　　故 $\sigma_{r3(a)} > \sigma_{r3(b)}$　a图危险

二00二年.

一、1、如图示II部分对x_c轴的惯性矩为



转轴公式 $I_{x_c II} = \frac{I_{xII}+I_{yII}}{2} + \frac{I_{xII}-I_{yII}}{2}\cos 2\alpha - I_{xyII}\sin 2\alpha$

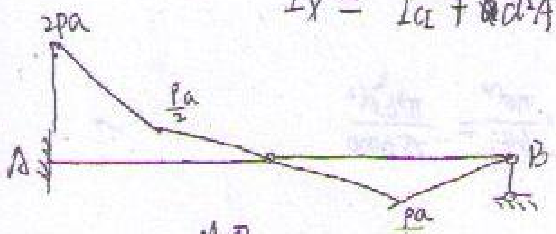
$= \frac{1}{12}(\frac{a}{2})^4 = \frac{a^4}{12\times16}$

故截面对x_c轴惯性矩为 $I_{cI} = I_{xc} - I_{IIc}$

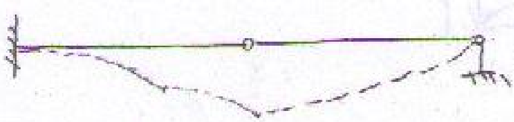
$= \frac{a^4}{12} - \frac{a^4}{12\times16} = \frac{15a^4}{12\times16}$

由平行移轴公式: $I_x = I_{cI} + d^2A = \frac{15a^4}{12\times16} + (\frac{a}{2})^2(a^2-\frac{a^2}{4}) = \frac{17a^4}{64}$

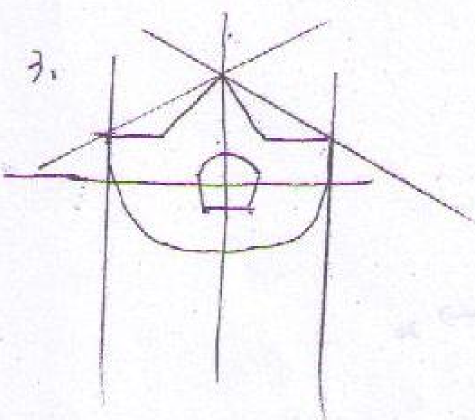
2、



M图



3、



4、 $\cancel{\tau=\frac{P}{A}}$ $\tau = \frac{F_s}{A} = \frac{P/2}{\pi d^2/4} = \frac{2P}{\pi d^2} \le [\tau] \Rightarrow P = \frac{[\tau]\pi d^2}{2}$

$\sigma_b = \frac{F_b}{A_b} = \frac{P}{d\frac{L}{3}} = \frac{3P}{dL} \le [\sigma_{bs}] \Rightarrow P = \frac{[\sigma_{bs}]dl}{3}$

$\Rightarrow \frac{[\tau]\pi d^2}{2} = \frac{[\sigma_{bs}]dl}{3}$

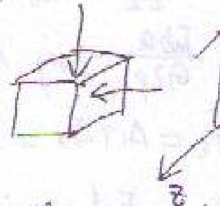
$\Rightarrow \frac{l}{d} = \frac{3\pi[\tau]}{2[\sigma_{bs}]}$

5、解：$K_d = 1 + \sqrt{1 + \frac{2H}{\Delta_{st}}}$ ……①

$H = \frac{v_0^2}{2g} + h$

$\Delta_{st} = \frac{Q}{k}$ 　　代入①式 $\Rightarrow K_d = 1 + \sqrt{1 + \frac{2k}{Q}\left(\frac{v_0^2}{2g} + h\right)}$

6、解：如图 $\sigma_y = \frac{F}{A} = -\frac{P}{A} = \frac{-6\times10^3}{0.01\times0.01} = -60mpa$.



又 $\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)$，由于刚体不变形，则 $\varepsilon_x = 0$

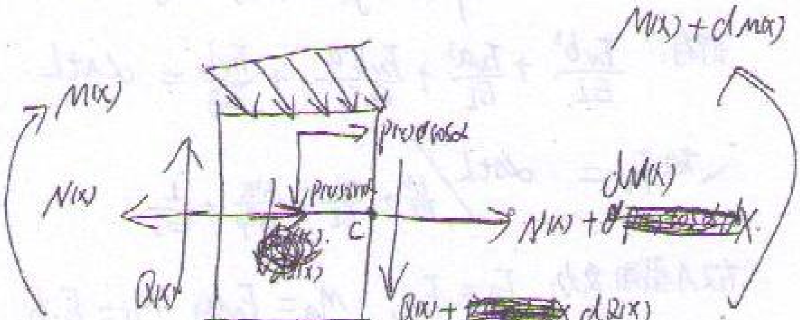
则 $\sigma_x = \nu\sigma_y = 0.33\times(-60) = -19.8mpa$

故可得 $\sigma_1 = 0$，$\sigma_2 = -19.8mpa$，$\sigma_3 = -60mpa$.

$\sigma_{r3} = \sigma_1 - \sigma_3 = 60mpa$

$\sigma_{r4} = \sqrt{\frac{1}{2}[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2]} = 52.95mpa$

7、解：$\Sigma F_x = 0$.



$N(x) = dN(x) + N(x) + p(x)\cos\alpha\, dx$

$\Rightarrow \frac{dN(x)}{dx} = -p(x)\cos\alpha$

$\Sigma F_y = 0$：

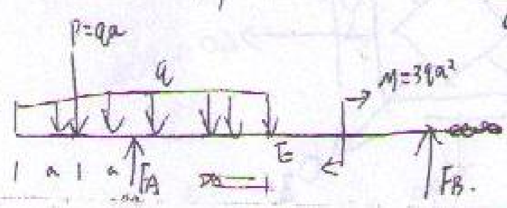
$Q(x) = Q(x) + p\sin\alpha\, dx + dQ(x)$

$\Rightarrow \frac{dQ(x)}{dx} = -p\sin\alpha$.

$\Sigma M_C = 0$：$M(x) + dM(x) - M(x) - Q(x)dx - \frac{h}{2}p(x)\cos\alpha\, dx + p(x)\sin\alpha\, dx\,\frac{dx}{2} = 0$.

略去二阶无穷小项，得 $\frac{dM(x)}{dx} = Q(x) + \frac{h}{2}p(x)\cos\alpha$.

二、解：

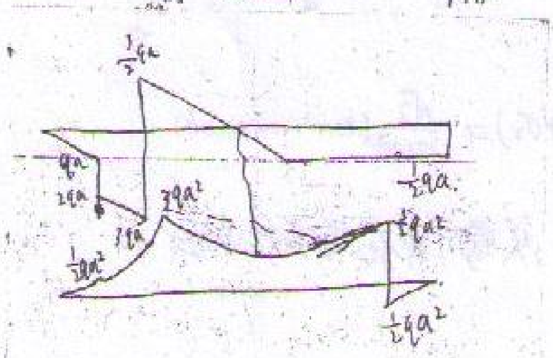


$\Sigma M_B = 0$：$qa\cdot5a + q\cdot4a\cdot4a - 3qa - 4aF_A =$

$F_A = \frac{9}{2}qa$

$\Sigma F_y = 0$：$F_A + F_B - qa - 4qa = 0$

$F_B = \frac{1}{2}qa$

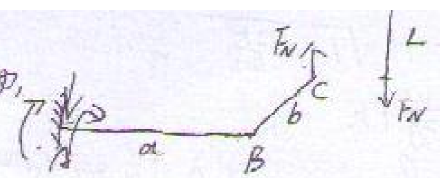


各截面剪力：$F_{BE} = -\frac{1}{2}qa$，$F_{DE} = qa$　$F_{DG} = 2qa$

$F_{A左} = 3qa$，$F_{A右} = -\frac{3}{2}qa$.

各截弯矩：

二、解：图结构为一次超静定，温度降低时钢丝将收缩，产生温度拉力由而产生温度挠力，受力分析如右图：



(1) 求在 F_N 作用下C点挠度。

将AB刚化，则有 $\Delta_1=\frac{F_N b^3}{EI}$

将BC刚化，则 $\Delta_2=\frac{F_N a^3}{EI}+\varphi_{AB}b$

又 $\varphi_{AB}=\frac{Ta}{GI_p}=\frac{F_N b a}{GI_p}$，则 $\Delta_2=\frac{F_N a^3}{EI}+\frac{F_N b^2 a}{GI_p}$

故C点挠度 $f_C=\Delta_1+\Delta_2=\frac{F_N b^3}{EI}+\frac{F_N a^3}{EI}+\frac{F_N b^2 a}{GI_p}$

CE钢丝拉长 $\Delta_L=\frac{F_N L}{EA}$

温度降低时缩短，$\Delta l_t=\alpha\Delta t L$

由几何变形协调条件：$f_C=\Delta l_t-\Delta_L$

即有：$\frac{F_N b^3}{EI}+\frac{F_N a^3}{EI}+\frac{F_N b^2 a}{GI_p}+\frac{F_N L}{EA}=\alpha\Delta t L$

故 $F_N=\alpha\Delta t L\Big/\left(\frac{b^3}{EI}+\frac{a^3}{EI}+\frac{b^2 a}{GI_p}+\frac{L}{EA}\right)$

故A截面反力：$F_A=F_N$，$M_A=F_N a$，$M_T=F_N b$

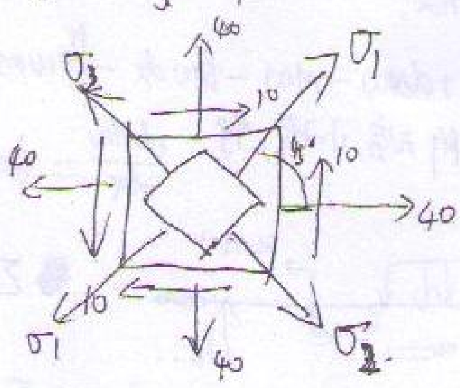
四、

解：(1) 如图示应力状态可知，$\sigma_x=40$MPa，$\sigma_y=40$，$\tau_{xy}=-10$MPa.

则 $\sigma_{1,2}=\frac{\sigma_x+\sigma_y}{2}\pm\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$

$=40\pm10$

故 $\sigma_1=50$MPa $\sigma_2=30$MPa, $\sigma_3=0$



$\alpha_0=\frac{1}{2}\arctan\frac{-2\tau_x}{\sigma_x-\sigma_y}=\frac{\pi}{4}=45°$

故主应力单元体如右图所示。

(2) 45°角方向的应变为 $\varepsilon_{45°}=\varepsilon_1=\frac{1}{E}(\sigma_1-\nu\sigma_2)$

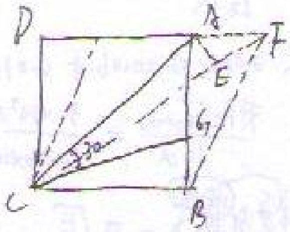
则对角线长度改变量 $\Delta l=\varepsilon_{45°}l=\frac{l}{E}(\sigma_1-\nu\sigma_2)=\frac{\sqrt{2}}{200\times10^3}(50\times10^6-0.3\times30\times10^6)$

$=2.9\times10^{-4}$

由 $\sigma_x=\sigma_y$，为均匀应力状态，故45°角无变化，只需计算在剪应力单独作用下30°角的改变量。

对角线AC的转角

$$\Delta\varphi_1=\frac{AE}{AC}=\frac{AF\sin45^\circ}{AC}=\frac{\gamma AB\sin45^\circ}{AB/\sin45^\circ}=\gamma\sin^2 45^\circ$$



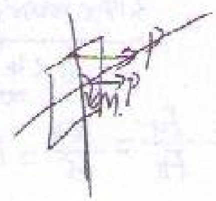
同理可得与BC边夹角15°的CG边转角

$$\Delta\varphi_2=\gamma\sin^2 15^\circ$$

故 30°角改变量 $\Delta\theta=\Delta\varphi_1-\Delta\varphi_2=0.56\times10^{-4}$

五、解:(1)钢杆在偏心力作用下,可等效为图示荷载作用

其中 $M=\frac{Ph}{2}$



故 $\sigma_{max}=\frac{P}{A}+\frac{M_{max}}{W_z}$

$$=\frac{P}{bh}+\frac{Ph/2}{bh^2/6}=\frac{4P}{bh}<[\sigma]$$

故 $P_{max}=\frac{bh[\sigma]}{4}$

(2) 在弯矩作用下D点竖向挠度为 $f_V=\frac{Ml^2}{2EI}=\frac{\frac{Ph}{2}\cdot(10h)^2}{2E\frac{bh^3}{12}}=\frac{300P}{Eb}$ (↓)

~~水平位移为~~0

~~在拉力作用下竖向~~截面转角 $\theta=\frac{Ml}{EI}=\frac{\frac{Ph}{2}\cdot10h}{E\frac{bh^3}{12}}=\frac{60P}{Ebh}$

水平位移为 $S_{H_1}=-\theta\cdot\frac{h}{2}=-\frac{30P}{Eb}$ (←)

在拉力作用竖向位移为0,水平位移 $S_{H_2}=\frac{Pl}{EA}=\frac{P10h}{Ebh}=\frac{10P}{Eb}$ (←)

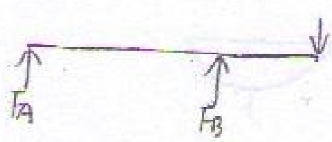
故 D点位移:竖向 $f_V=\frac{300P}{Eb}$ (↓),水平:~~$S_H=S_{H_1}+S_{H_2}$~~

$S_H=S_{H_1}+S_{H_2}=-\frac{20P}{Eb}$ (←)

六、解:(1)图示结构受力分析如右图所示

由 $\sum M_A=0$:$2F_B-30\times3=0$ 得 $F_B=45kN$

由 $\sum F_y=0$. $F_A+F_B-30=0$ $F_A=-15kN$



则弯矩与剪力图,如右所示

M图 (kN·m)

$\sigma_{max}=\frac{M_{max}y_{max}}{I_z}$

$$I_z=\frac{bh^3}{12}-\frac{b_1h_1^3}{12}=\frac{0.08\times0.14^3}{12}-\frac{0.07\times0.1^3}{12}=1.246\times10^{-5}$$

Fs图 (kN)

故 $\sigma_{max}=\frac{30\times10^6\times0.07}{1.246\times10^{-5}}mpa=168.5mpa$

$\tau_{max} = \frac{F_S S^*_{z,max}}{I_z d}$

又 $S^*_{z,max} = 0.08\times0.02\times0.06 + 0.01\times0.05\times0.025 = 1.085\times10^{-4}$

故 $\tau_{max} = \frac{F_S S^*_{z,max}}{I_z d} = \frac{30\times10^3\times1.085\times10^{-4}}{1.246\times10^{-5}\times0.01} = 26.1MPa$

(2) 压杆临界柔度 $\lambda_p = \pi\sqrt{\frac{E}{\sigma_p}} = 99.3$

而该杆的柔度 $\lambda = \frac{\mu l}{i} = \mu l/\sqrt{I/A} = \frac{4\mu l}{d} = \frac{4\times1\times4}{0.06} = 266.67 > \sigma_p$

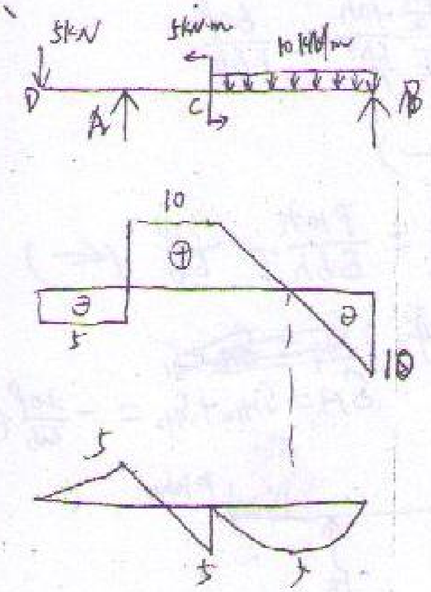
故该杆为大柔度压杆，适用于欧拉公式。

有 $F_{cr} = \frac{\pi^2 EI}{(\mu l)^2} = \frac{3.14^2\times200\times10^9\times\frac{\pi\times0.06^4}{64}}{(1\times4)^2} = 78.4kN$

则 $n_{st} = \frac{F_{cr}}{F_N} = \frac{F_{cr}}{F_B} = \frac{78.4}{45} = 1.74 < 3$ 故不安全。

二〇〇三年

一、



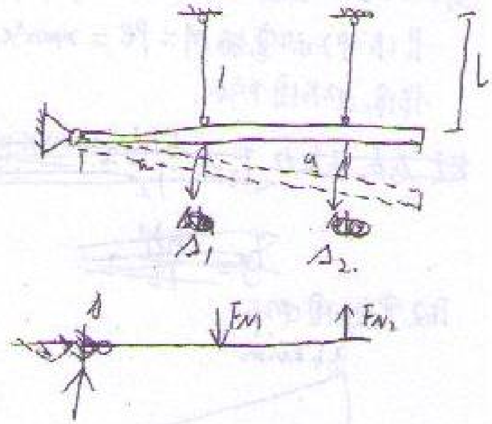
解 $\Sigma M_B = 0$: $5\times4+5+\frac{1}{2}\times10\times4 - F_A\cdot3 = 0$

得 $F_A = 15\ kN$

$\Sigma F_y = 0$: $F_A + F_B - 5 - 10\times2 = 0\ kN$

$\Rightarrow F_B = 10kN$

二、解：图示结构为一次超静定，其受力分析如右所示。



(1) 静力方面：$\sum M_A = 0$：$F_{N2}\cdot 2a - F_{N1}\cdot a = 0$

(2) 物理方面：轴力作用下 $\Delta l_1 = \frac{F_{N1} l}{EA}$，$\Delta l_2 = \frac{F_{N2} l}{EA}$

温度上升时 1、2 两杆伸长均为

$\Delta l_{t1} = \Delta l_{t2} = \alpha \Delta t l$

(3) 几何方面：$\Delta_1 = \Delta l_{t1} - \Delta l_1$

$\Delta_2 = \Delta l_{t2} + \Delta l_2$

$\frac{\Delta_1}{\Delta_2} = \frac{a}{2a}$

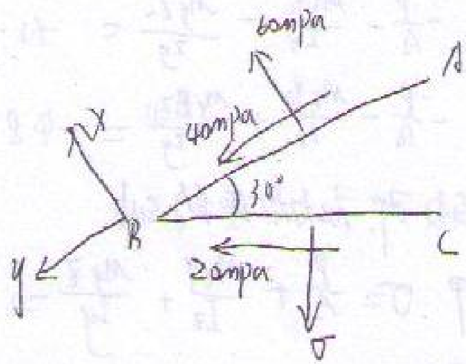
由上述几式可得 $F_{N1} = 2F_{N2}$

$$\left.\begin{array}{l} F_{N1} = 2F_{N2} \\ \dfrac{\alpha\Delta t l - \frac{F_{N1} l}{EA}}{\alpha\Delta t l + \frac{F_{N2} l}{EA}} = \dfrac{1}{2} \end{array}\right\}$$

解得 $F_{N2} = \frac{\alpha\Delta t EA}{5}$

$F_{N1} = \frac{2\alpha\Delta t EA}{5}$

三、解：如图示设 x、y 轴方向，则可知



$\sigma_x = 60mpa$，$\tau_x = -40mpa$，$\alpha = 150°$

$\tau_{-150°} = 20mpa$

则有 $\tau_{-150°} = \frac{\sigma_x - \sigma_y}{2}\sin(2\times 150°) + \tau_x \cos 300°$

即 $20 = \frac{60-\sigma_y}{2}\sin 300° - 40\cos 300°$

~~亦即 20 = 60~~ 解得 $\sigma_y = 152.38 mpa$

故 $\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 300° - \tau_x \sin 300°$

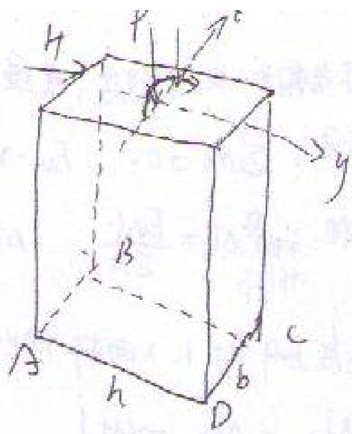
$= \frac{60+152.38}{2} + \frac{60-152.38}{2}\times\frac{1}{2} - 40\frac{\sqrt{3}}{2} = 48.45 mpa$

$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_x^2} = \frac{60+152.38}{2} \pm \sqrt{\left(\frac{60-152.38}{2}\right)^2 + (40)^2} = \begin{cases} 167.29 mpa \\ 45.09 mpa \end{cases}$

故主应力为 $\sigma_1 = 167.29 mpa$，$\sigma_2 = 45.09 mpa$，$\sigma_3 = 0$

则 $\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = 83.65 mpa$

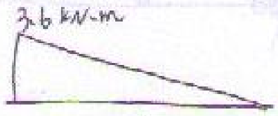
四、短柱在竖直方向受偏心力P作用，因等效为受轴力 $P_0\ (P_0=P)$ 和弯矩 $M = Pe = 24\times10^3\times0.025 = 600\ N\cdot m$ 作用，如右图所示。

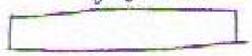


~~校核最大正应力~~ $\cancel{I_z = \frac{bh^3}{12} = \frac{0.1\times0.2^3}{12} = 6}$

$\cancel{I_y = \frac{hb^3}{12}}$

做弯矩图如下





故 $\sigma_A = -\frac{P}{A} + \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{24\times10^3}{0.2\times0.1} + \frac{3.6\times10^3\times0.1}{\frac{0.1\times0.2^3}{12}} + \frac{0.6\times10^3\times0.05}{\frac{0.2\times0.1^3}{12}}$

$= (-1.2+5.4+1.8)\times10^6 = 6\ MPa$

$\sigma_B = -\frac{P}{A} + \frac{M_z y_B}{I_z} - \frac{M_y z_B}{I_y} = -1.2+5.4-1.8 = 2.4\ MPa$

$\sigma_C = -\frac{P}{A} - \frac{M_z y_C}{I_z} - \frac{M_y z_C}{I_y} = -1.2-5.4-1.8 = -8.4\ MPa$

$\sigma_D = -\frac{P}{A} - \frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -4.8\ MPa$

中性轴即应力为零的轴。

即 $\sigma = -\frac{P}{A} + \frac{M_z y}{I_z} + \frac{M_y z}{I_y} = 0$ ~~亦即~~ $\cancel{1.2+5.4y+1.8z=0}$ ~~也即~~

亦即 $1.2+54y+36z=0$ 也即 $45y+30z+1=0$

五、解：最大正应力 $\sigma_{max} = \frac{M}{W_z} = \frac{20\times10^3}{\frac{0.15\times0.3^2}{6}} = 8.89\ MPa$

最大切应力 $\tau_{max} = \frac{3}{2}\frac{F}{A} = \frac{3}{2}\times\frac{10\times10^3}{0.15\times0.3} = 0.33\ MPa$

$I_z = \frac{bh^3}{12} = \frac{0.15\times0.3^3}{12} = 3.375\times10^{-4}$

$N^* = \int_A \sigma\, dA = \int_{0.075}^{0.15} \frac{My}{I_z}\frac{b}{2}\,dy = \frac{3M}{h^3}y^2\Big|_{0.075}^{0.15} = 37.5\ kN$

切应力沿y轴方向的分布规律

$$\tau = \frac{F_S S_z^*}{I_z b} = \frac{Q}{I_z b}\int_y^{\frac{h}{2}} y\, b\, dy = \frac{Q}{2I_z}\left(\frac{h^2}{4}-y^2\right)$$

故 $$Q^* = \int_A \tau\, dA = \int_{0.075}^{0.15} \frac{Q}{2I_z}\left(\frac{h^2}{4}-y^2\right)\frac{b}{2}\, dy = \frac{3Q}{h^3}\left(\frac{h^2}{4}y-\frac{1}{3}y^3\right)\Big|_{0.075}^{0.15} = 0.181$$

六、解：~~取结点~~

如图知 AD、CD 两杆为一端固定，一端铰接 故 $\mu_1 = 0.7$，BD 的 $\mu_2 = 1$

则 AD、CD 两杆临界压力为 $$F_{cr_1} = \frac{\pi^2 EI}{(\mu_1 l_{AD})^2} = \frac{\pi^2 EI}{\left(0.7\times\frac{l}{\cos 30^\circ}\right)^2} = \frac{1.53\pi^2 EI}{l^2}$$

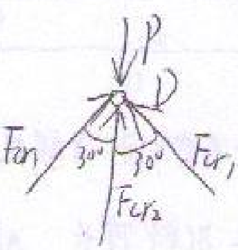
BD 杆临界力为 $$F_{cr_2} = \frac{\pi^2 EI}{(\mu_2 l)^2} = \frac{\pi^2 EI}{l^2}$$

故 $F_{cr_1} > F_{cr_2}$，可知 BD 杆先失稳，由于结构为超静定，能继续承载

直到 AD 或 DC 杆失稳时，结构才失稳，此时取结点 D 受力分析

如下：则 $\sum F_y = 0$ $2F_{cr_1}\cos 30^\circ + F_{cr_2} = P$

即 $$P_{cr} = 2\times\frac{1.53\pi^2 EI}{l^2}\times\frac{\sqrt{3}}{2} + \frac{\pi^2 EI}{l^2} = \frac{3.65\pi^2 EI}{l^2}$$

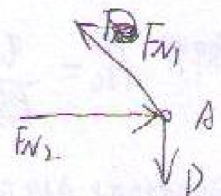


七、解：取结点A分析如下

如平衡条件：$\sum F_y = 0$：$F_{N_1}\cos 45^\circ - P = 0$，

$\sum F_x = 0$、$F_{N_1}\cos 45^\circ = F_{N_2}$

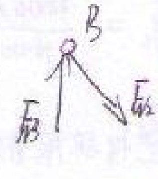
即 $F_1 = \sqrt{2}P$，$F_2 = P$



再取 B点 有

$\sum F_y = 0$、$F_{N_3} - F_{N_2}\cos 45^\circ = 0$

则 $F_{N_3} = P$



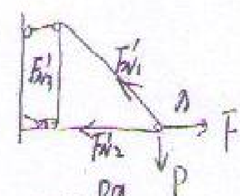
故体系应变能 $$U = \sum_{i=1}^{3}\frac{F_{N_i}^2 l_i}{2EA} = 2\times\frac{P^2 a}{2EA} + \frac{(\sqrt{2}P)^2\sqrt{2}a}{2EA} = \frac{P^2 a(1+\sqrt{2})}{EA}$$

故竖向位移 $$\Delta_{AV} = \frac{\partial V_\varepsilon}{\partial P} = \frac{2(1+\sqrt{2})Pa}{EA}$$

为求水平位移 有一水平力 F，作用 A点，如右图

A点、由平衡条件有 $\sum F_y = 0$ $F'_{N_1}\cos 45^\circ - P = 0 \Rightarrow F'_{N_1} = \sqrt{2}P$

$\sum F_x = 0$ $F'_{N_2} + F'_{N_1}\cos 45^\circ = F \Rightarrow F'_{N_2} = F - P$



$F'_{N_3} = P$，故 $U_1 = \sum \frac{F_{N_i}^2 l_i}{2EA} = \frac{(F-P)^2 a}{2EA} + \frac{(1+\sqrt{2})P^2 a}{2EA}$ $\therefore \Delta_{AV} = \frac{\partial U}{\partial F}\Big|_{F=0} = -\frac{Pa}{EA}$

八

1. 解：由图知 ~~τ~~ $\tau_{yz}=50\,mpa$，$\sigma_x=80\,mpa$，$\sigma_y=\sigma_z=0$

则 $\sigma=\frac{\sigma_y+\sigma_z}{2}\pm\sqrt{(\frac{\sigma_y-\sigma_z}{2})^2+\tau_{yz}^2}=\pm\sqrt{50^2}=\pm50$

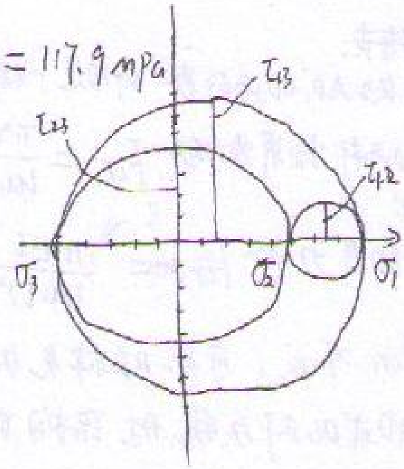
故主应力为 $\sigma_1=80\,mpa$，$\sigma_2=50\,mpa$，$\sigma_3=-50\,mpa$.

则 $\sigma_{r3}=\sigma_1-\sigma_3=130\,mpa$

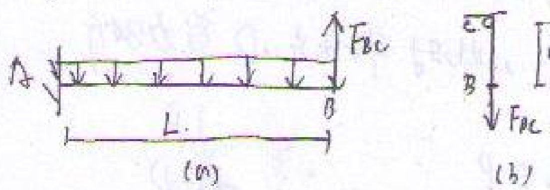
$\sigma_{r4}=\sqrt{\frac{1}{2}[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2]}=117.9\,mpa$

则 $\sigma_{rT}=\tau_{13}+\tau_{23}$

$=\frac{\sigma_1-\sigma_3}{2}+\frac{\sigma_2-\sigma_3}{2}=65+50=115\,MPa$



2、解：图示结构为一次超静定，梁AB和杆BC受力如下



(a)中均布作用下B端挠度 $w_B=\frac{qL^4}{8EI}-\frac{F_{BC}L^3}{3EI}$.

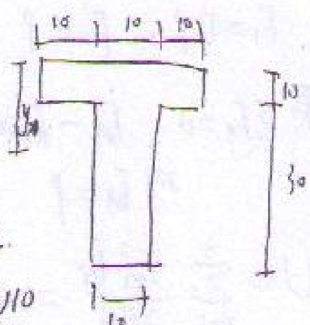
(b)中BC杆伸长 $\Delta l_{BC}=\frac{F_{BC}a}{EA}$

由变形条件有 $w_B=\Delta l_{BC}$ 即 $\frac{qL^4}{8EI}-\frac{F_{BC}L^3}{3EI}=\frac{F_{BC}a}{EA}$

解得 $F_{BC}=\frac{qL^4/8EI}{L^3/3EI+a/EA}$

3. 解：(1) 线弹性阶段时，中性轴在形心位置.

则 $\bar{x}_c=0$，$\bar{y}_c=\frac{30\times10\times5+30\times10\times25}{30\times10+30\times10}=15$



(2) 当出现塑性极限弯矩时，有 $S_I=S_{II}$.

设为y. 则有 $30\times10+(y-10)10=(40-y)10$

解得 $y=10$ 即 此时 $\bar{x}_c=0$，$\bar{y}_c=10$

4、解：在图示扭矩作用下，$\sigma_x=\sigma_y=0$，$\tau_x=\frac{T}{W_p}=\frac{16m}{\pi d^3}=\frac{16\times 2.5\times10^3}{3.14\times 0.06^3}=58.98$ mpa

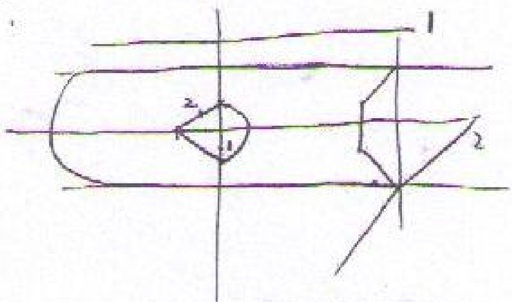
则 $\varepsilon_x=\varepsilon_y=0$，$\gamma_{xy}=\frac{\tau_x}{G}=\frac{\tau}{E/2(1+\nu)}=\frac{2(1+\nu)\tau}{E}=7.19\times10^{-4}$

$$\varepsilon_{30^\circ}=\frac{\varepsilon_x+\varepsilon_y}{2}+\frac{\varepsilon_x-\varepsilon_y}{2}\cos2\alpha+\frac{\gamma_{xy}}{2}\sin2\alpha$$
$$=\frac{\gamma_{xy}}{2}\sin(-60^\circ)=-3.11\times10^{-4}$$

设与纵轴成α方向剪应变为零，则

则 $-\frac{\gamma_\alpha}{2}=\frac{\varepsilon_x-\varepsilon_y}{2}\sin2\alpha-\frac{\gamma_{xy}}{2}\cos2\alpha$ 即 $\cos2\alpha=0$ 则 $\alpha=45^\circ$

5.



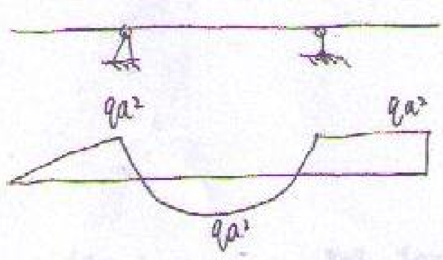
6、解：$K_d=1+\sqrt{1+\frac{2h}{\Delta_{st}}}$

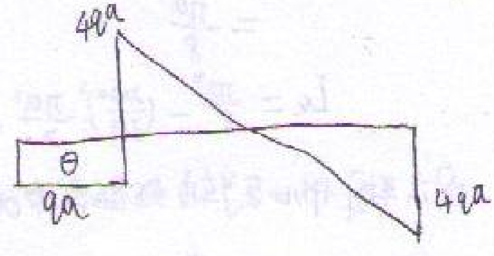
$h=H+\frac{v_0^2}{2g}$，$\Delta_{st}=\frac{QL^3}{48EI}$

则 $K_d=1+\sqrt{1+\frac{2H+v^2/g}{QL^3/48EI}}$

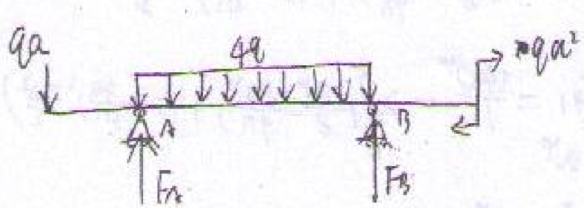
又 $\sigma_{st}=\frac{M_{max}}{W_z}=\frac{QL}{4W}$ 则 $\sigma_d=K_d\sigma_{st}=K_d\frac{QL}{4W}$

7、





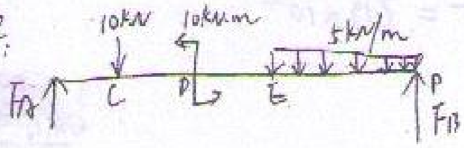
$\sum M_B=0$：$F_B\cdot2a+qa^2-qa^2-\frac{1}{2}4q(2a)^2=0$
$\Rightarrow F_B=4qa$



$\sum F_y=0\Rightarrow$ $F_A+F_B-qa-4q\cdot2a=0$
$\Rightarrow F_A=5qa$

二00四年

一、解:

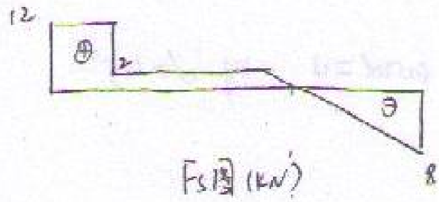


解: $\sum M_B=0$: $10\times4+10+\frac{1}{2}\times5\times2^2-5F_A=0$

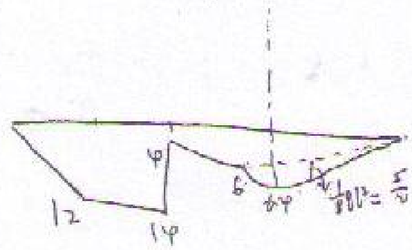
得 $F_A=12kN$

$\sum F_y=0$: $10+5\times2+F_A+F_B=0$

得 $F_B=8kN$



Fs图(kN)

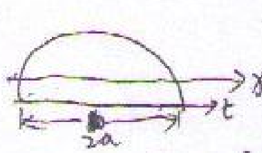


二、解: (1)设正方形对z轴惯性矩为I_{z1},半圆对z轴惯性矩为I_{z2}

则 $I_z=I_{z1}-2I_{z2}$

$=\frac{(4a)^4}{12}-\frac{\pi d^4}{64}=\frac{(4a)^4}{12}-\frac{\pi(2a)^4}{64}=(\frac{64}{3}-\frac{\pi}{4})a^4$

(2)求半圆图对其形心的惯性矩,如下图:



由平行移轴公式有 $I_t=I_{xc}+(\frac{2d}{3\pi})^2A$

则半圆对其形心轴惯性矩 $I_{xc}=I_t-(\frac{2d}{3\pi})^2A$

$I_t=\int_A y^2dA=2\int_0^{\frac{\pi}{2}}d\theta\int_0^a r^3\sin^2\theta\,dr$

$=\frac{\pi a^4}{8}$

$I_{xc}=\frac{\pi a^4}{8}-(\frac{2\times2a}{3\pi})^2\cdot\frac{\pi a^2}{2}=\frac{\pi a^4}{8}-\frac{8a^4}{9\pi}$

图示半圆形心与y轴距离为 $d=2a-\frac{4a}{3\pi}$

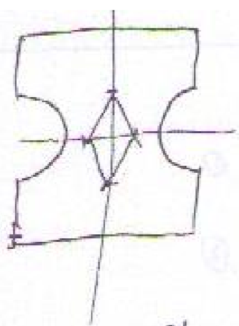
则半圆对y轴惯性矩 $I_{y1}=I_{xc}+d^2A=(\frac{\pi a^4}{8}-\frac{8a^4}{9\pi})+(2a-\frac{4a}{3\pi})^2\cdot\frac{\pi a^2}{2}$

故截面对y轴惯性矩为 $I_y=I_{y0}-2I_{y1}=\frac{(4a)^4}{12}-2[(\frac{\pi a^4}{8}-\frac{8a^4}{9\pi})+(2a-\frac{4a}{3\pi})^2\frac{\pi a^2}{2}]$

$=(\frac{8}{3}-\frac{17\pi}{4})a^4$

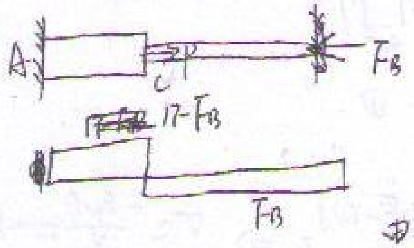
2b) $I_p=I_x+I_y=(\frac{140}{3}-\frac{18\pi}{4})a^4=(38-\frac{9}{2}\pi)a^4$

（1）截面形状如下



三、解：在力P作用下，AC杆伸长 $\Delta l_{AC} = \frac{P l_{AC}}{EA_1} = \frac{17\times10^3\times0.5}{200\times10^9\times10\times10\times10^{-6}} = 4.25\times10^{-4}m = 0.425mm > 0.2m$

故为一次超静定，其受力如下：



则得轴力图，此时有

$\Delta l_{AC_1} = \frac{F_{NAC} l_{AC}}{EA_1} = \frac{(17-F_B)\times10^3\times0.5}{200}$

$\Delta l_{BC_1} = \frac{F_{NBC} l_{BC}}{EA_2}$

由几何关系有：$\Delta l_{AC_1} - \Delta l_{BC_1} = \Delta$

则有 $\frac{F_{NAC} l_{AC}}{EA_1} - \frac{F_{NBC} l_{BC}}{EA_2} = \Delta$ 代入数据有

$$\frac{(17-F_B)\times10^3\times0.5}{200\times10^9\times10\times10\times10^{-6}} - \frac{F_B\times10^3\times1}{200\times10^9\times5\times5\times10^{-6}} = 0.2\times10^{-3}$$

解得 $F_B = 1kN$.

故此时 $F_{NAC} = 16kN$, $F_{NBC} = -1kN$

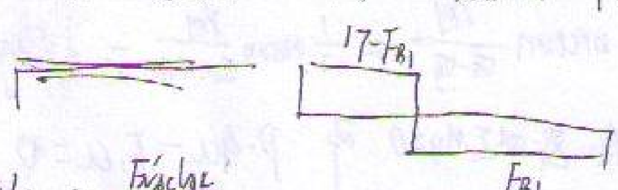
则 $\sigma_{AC} = \frac{F_{NAC}}{A_1} = 160mpa$, $\sigma_{BC} = \frac{F_{NBC}}{A_2} = \frac{-1\times10^3}{5\times5\times10^{-6}} = -40mpa$

C截面位移 $\Delta_C = \frac{F_{NAC} l_{AC}}{EA_1} = 4\times10^{-4}m = 0.4mm$

(2) 温度升高20°c 时阶梯杆伸长 $\Delta l_t = \alpha\Delta t l = \alpha\Delta t(l_{AC} + l_{CB})$

几何~~关系有：Δl_A~~

此时，B端支反力设为 F_{B1}，故轴力图如下.



$\Delta l_{AC_2} = \frac{F'_{NAC} l_{AC}}{EA_1}$ ， $\Delta l_{BC_2} = \frac{F'_{NBC} l_{BC}}{EA_2}$

几何关系有：~~$\frac{F'_{NAC} l_{AC}}{EA_1} + \Delta l_t = \Delta$~~ $\Delta l_{AC_2} + \Delta l_t - \Delta l_{BC_2} = \Delta$

即 $\frac{(17-F_{B1}) l_{AC}}{EA_1} + \alpha\Delta t(l_{AC} + l_{CB}) - \frac{F_{B1} l_{BC}}{EA_2} = \Delta$ 代入数据解得 $F_{B1} = 2.$

则 $F_{NAC1} = 14.4kN$, $F_{NBC1} = -2.6kN$

$\sigma_{AC} = \frac{F_{NAC1}}{A_1} = 144\ mpa$, $\sigma_{BC} = \frac{F_{NBC1}}{A_2} = \frac{-2.6\times10^3}{5\times5\times10^{-6}} = -104\ mpa$

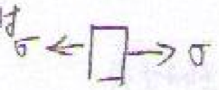
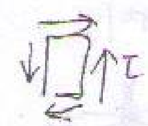
$\Delta_{C_1} = \frac{F_{NAC1} l_{AC}}{EA_1} + \alpha\Delta t l_{AB} = 0.72\ mm$

四、解：(1) 由广义胡克定律有，

$$\varepsilon_{45^\circ} = \frac{1}{E}(\sigma_{45^\circ} - \nu\sigma_{-45^\circ}) \quad \text{---- ①}$$

$$\varepsilon_{90^\circ} = \frac{1}{E}(\sigma_{90^\circ} - \nu\sigma_{0^\circ}) \quad \text{---- ②}$$

~~当~~ P、Me 单独作用时，单位应力如下，

只P作用时  只Me作用时 

故 $\left.\begin{array}{l}\sigma_{45^\circ} = \frac{\sigma}{2}+\tau, \quad \sigma_{-45^\circ} = \frac{\sigma}{2}-\tau \\ \sigma_{90^\circ} = 0, \quad \sigma_{0^\circ} = \sigma\end{array}\right\}$ ---- ③

将③代入①、②式有 $\left.\begin{array}{l}\varepsilon_{45^\circ} = \frac{1}{E}\left[\frac{\sigma}{2}+\tau - \nu(\frac{\sigma}{2}-\tau)\right] \\ \varepsilon_{90^\circ} = \frac{-\nu\sigma}{E}\end{array}\right\} \Rightarrow \begin{array}{l}\sigma = -\frac{\varepsilon_{90^\circ}E}{\nu} = -64\,\text{MPa} \\ \tau = 69.69\,\text{MPa}\end{array}$

又 $\sigma = \frac{P}{A}$，$\tau = \frac{Me}{W_p}$

则 $P = \sigma A = -\frac{\varepsilon_{90^\circ}E}{\nu}A = \frac{96\times10^{-6}\times200\times10^{9}}{0.3}\times3.14\times\frac{0.02^2}{4} = 20.1\,kN$

$Me = \tau W_p = \frac{\tau\pi d^3}{16} = \frac{69.69\times10^{6}\times3.14\times0.02^3}{16} = 109.4\,N\cdot m$

(2) 若0°片应变，由 $\nu = -\frac{\varepsilon_{90^\circ}}{\varepsilon_{0^\circ}}$ 时

$\varepsilon_{0^\circ} = -\frac{\varepsilon_{90^\circ}}{\nu} = \frac{96\times10^{-6}}{0.3} = 320\times10^{-6}$

(3) 切应变 $\gamma_{xy} = \frac{\tau}{G} = \frac{2\tau(1+\nu)}{E} = 9.06\times10^{-4}$

故 ε_1 的方向与轴线夹角 $\alpha = \frac{1}{2}\arctan\frac{\gamma_{xy}}{\sigma_x-\sigma_y} = \frac{1}{2}\arctan\frac{\gamma_{xy}}{\varepsilon_{0^\circ}-\varepsilon_{90^\circ}} = 32.67^\circ$

五、解：(1) 如图示 由平衡条件有，由 $\Sigma M_B = 0$ 即 $P\cdot4a - F_1a = 0$

所以 $P = F_1/4$

故 F_1 达到最大时 P 也最大，$\therefore F_{cr1} = \frac{\pi^2EI}{(\mu l)^2} = \frac{3.14^2\times200\times10^{9}\times\frac{\pi(0.05)^4}{64}}{(1\times3)^2}$

$= 67.19\,kN$

故 $P_{max} = \frac{F_{cr1}}{4} = 16.8\,kN$

(2) 在B处再加一根同样的杆时，结构为一次超静定，只有1、2杆同时失稳时才能丧失承载力，则 $\sum M_B = 0$：$P \cdot 4a - F_{cr2} \cdot 2a - F_{cr1} \cdot a = 0$.

此时1、2杆相同 故 $F_{cr1} = F_{cr2}$

故此时 $P_{max} = \dfrac{3F_{cr1}}{4} = 50.4kN$

六、解：(1) 图示结构为一次超静定，解除B端约束，代之支反力 F_R，如下图。

故梁B端的挠度为 $\Delta_B = \dfrac{(Q-F_R)L^3}{3EI}$

弹簧压缩长度 $\Delta_1 = \dfrac{F_R}{C}$

由变形条件有：$\Delta_B = \Delta_1$ 即 $\dfrac{(Q-F_R)L^3}{3EI} = \dfrac{F_R}{C}$ 解得 $F_R = \dfrac{QCL^3}{3EI+CL^3}$

故 $\Delta_{st} = \Delta_1 = \dfrac{F_R}{C} = \dfrac{QL^3}{3EI+CL^3}$

则动载系数 $k_d = 1+\sqrt{1+\dfrac{2h}{\Delta_{st}}} = 1+\sqrt{1+\dfrac{2h(3EI+CL^3)}{QL^3}}$

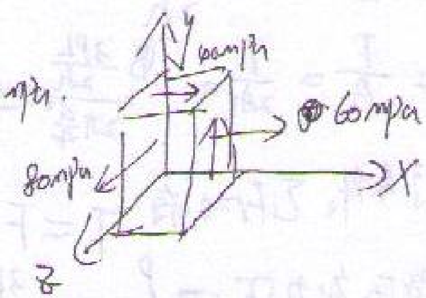
(2) A端弯矩最大为 $M_{max} = (Q-F_R)L$.

则 $\sigma_{st} = \dfrac{M_{max}}{W} = \dfrac{(Q-F_R)L}{W}$

$\therefore \sigma_{d,max} = k_d \sigma_{st} = \left(1+\sqrt{1+\dfrac{2h(3EI+CL^3)}{QL^3}}\right)\dfrac{(Q-F_R)L}{W}$

七、

1. 解：$\sigma_x = 60mpa$，$\tau_y = 40mpa$，$\sigma_z = 80mpa$.

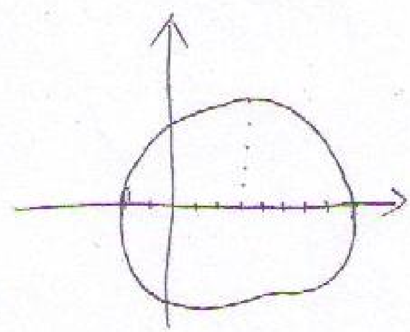


$\sigma = \dfrac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\dfrac{\sigma_x-\sigma_y}{2}\right)^2+\tau_y^2}$

$= \dfrac{60}{2} \pm \sqrt{30^2+40^2}$

$= 30 \pm 50 \begin{cases} -20\ mpa \\ 80\ mpa \end{cases}$

故 $\sigma_1 = 80mpa$，$\sigma_2 = 50mpa$，$\sigma_3 = -20mpa$

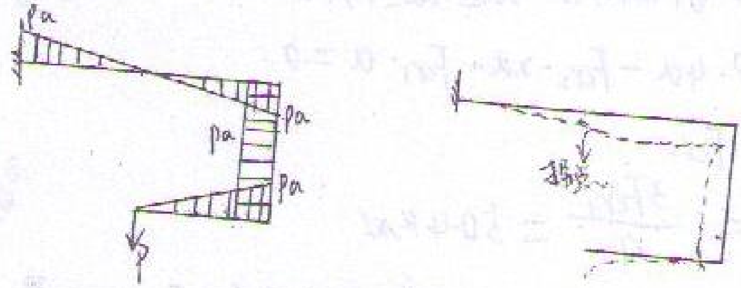


$\sigma_{r3} = \sigma_1 - \sigma_3 = 100\ mpa$.

$\sigma_{r4} = \sqrt{\frac{1}{2}[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2]} = 100\ mpa$

$\sigma_T = \tau_{13} + \max(\tau_{12}, \tau_{23}) = 50+50 = 100mpa$

2、解：



3、解：最大弯曲切应力 $\tau_{max}=\frac{3}{2}\frac{F_s}{A}=\frac{3F_s}{2bh}$

最大弯曲正应力 $\sigma_{max}=\frac{M_{max}}{W_z}=\frac{6M}{bh^2}$

平均切应力 $\tau_m=\frac{F_s}{A}=\frac{F_s}{bh}$

又 $\tau=\frac{F_sS_z^*}{I_zb}=\frac{F_s}{I_zb}\int_A y\,dA=\frac{F_s}{I_zb}\int_y^{\frac{h}{2}}y\,dy=\frac{F_s}{2I_z}\left(\frac{h^2}{4}-y^2\right)$

故令 $\tau=\tau_m$ 即 $\frac{F_s}{bh}=\frac{F_s}{2I_z}\left(\frac{h^2}{4}-y^2\right)$

解得 $y=\pm\frac{h}{\sqrt{12}}$

4、解：(1) $F_s=P$

两根梁作为一个整体时，连接缝即为中性层，由切应力互等定理有

$\tau'=\tau_{max}=\frac{3}{2}\frac{F_s}{A_0}=\frac{3P}{2bh}$

中性层上的剪切内力

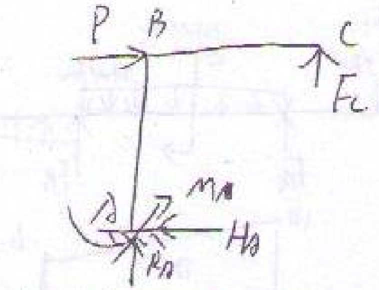
$T=\tau'bL=\frac{3PL}{2h}$

则 $\tau=\frac{T}{A}=\frac{T}{2A_1}=\frac{\frac{3PL}{2h}}{2\pi\frac{d^2}{4}}=\frac{3PL}{\pi d^2h}$

(2) 由平衡条件，$\Sigma F_x=0$ 有 $T=F$，则 $F=\frac{3PL}{2h}$

故挤压应力 $\sigma_{bs}=\frac{P}{A_{bs}}=\frac{3PL/2h}{dh}=\frac{3PL}{2h^2d}$

5、解：(1) 图示刚架为一次超静定，故如下图解除C端的约束代之于支反力 F_C.



P 单独作用下 C端挠度

$$\Delta_{C1} = \frac{PL^2}{2EI} \cdot L$$

F_C 作用于 C 端时挠度

$$\Delta_{C2} = \frac{F_C L^3}{3EI} + \frac{F_C L^2}{EI} \cdot L$$

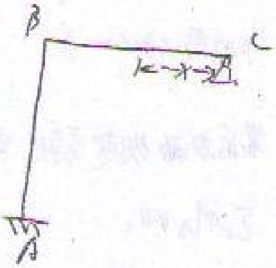
由变形条件有 $\Delta_{C1} = \Delta_{C2}$ 则有 $\frac{PL^3}{2EI} = \frac{F_C L^3}{3EI} + \frac{F_C L^3}{EI}$ 解得 $F_C = \frac{3}{8}P$

故 由整个体系 $\Sigma F_y = 0$：$R_A + F_C = 0$ 则 $R_A = \frac{3}{8}P$ (↓)

$\Sigma F_x = 0$：$P - H_A = 0$ 则 $H_A = P$

$\Sigma M_A = 0$：$F_C L - PL + M = 0$ $\quad M = \frac{5}{8}PL$

(2) BC杆弯矩 $M_1(x) = F_C x = \frac{3Px}{8}$ $\quad (0 \le x \le L)$



AB杆弯矩 $M_2(x) = F_C L - Px$ $\quad (0 \le x \le L)$

$= \frac{3PL}{8} - Px$

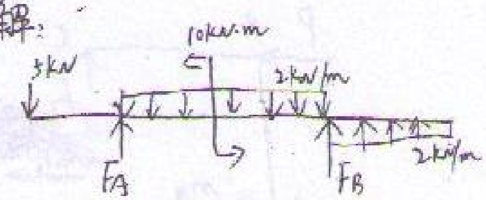
故弯曲应变能为

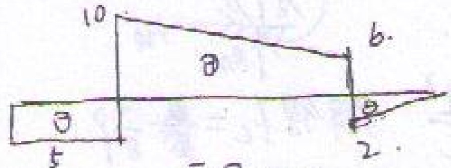
$$U = \int_0^L \frac{M_1^2(x)}{2EI} dx + \int_0^L \frac{M_2^2(x)}{2EI} dx$$

$$= \int_0^L \frac{9P^2x^2}{128EI} dx + \int_0^L \frac{1}{2EI} \left(\frac{3}{8}PL - Px\right)^2 dx$$

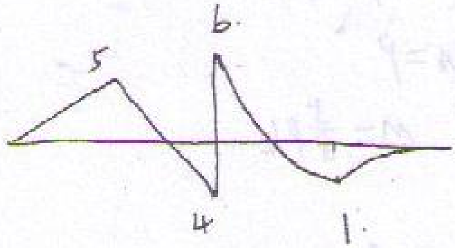
$$= \frac{7P^2L^3}{96EI}$$

1、解：





F_S图（kN）



M图（kN·m）

解：$\sum M_A=0$.

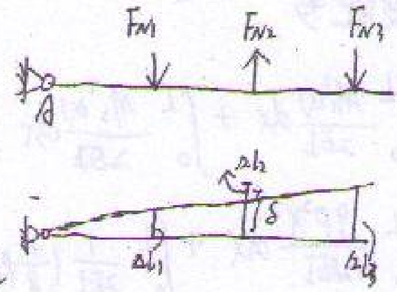
$5\times3+\frac{1}{2}\times2\times2^2+\frac{1}{2}\times2\times1+10-2F_A=0$

$F_A=15kN$.

$\sum F_y=0$. $F_A+F_B-5-2\times2+2\times1=0$

$F_B=-8kN$

2、解：设装后为两侧杆都受拉，受力如右图



由平衡条件：$\sum M_A=0$，

$2F_{N2}a-F_{N1}a-3F_{N3}a=0$

物理条件有 $\Delta l_1=\frac{F_{N1}l}{EA}$，$\Delta l_2=\frac{F_{N2}l}{EA}$，$\Delta l_3=\frac{F_{N3}l}{EA}$

几何条件有：$\frac{\Delta l_1}{\Delta l_3}=\frac{1}{3}$，$\Delta l_1+\Delta l_3=2(\delta-\Delta l_2)$

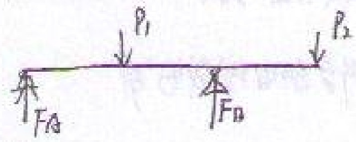
联立上述几式可解得 $F_{N1}=\frac{EA\delta}{7l}$，$F_{N2}=\frac{5EA\delta}{7l}$，$F_3=\frac{3EA\delta}{7l}$

故扣各杆应力：$\sigma_1=\frac{F_{N1}}{A}=\frac{E\delta}{7l}=30mpa$

$\sigma_2=\frac{F_{N2}}{A}=\frac{5E\delta}{7l}=150mpa$

$\sigma_3=\frac{F_{N3}}{A}=\frac{3E\delta}{7l}=90mpa$

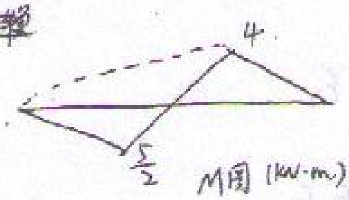
三、解：梁的受力图如下　　由平衡条件，$\sum M_A=0$.

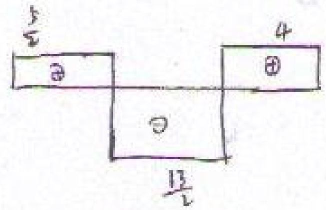


$2F_B-3P_2-P_1=0$　　$F_B=\frac{21}{2}$ kN.

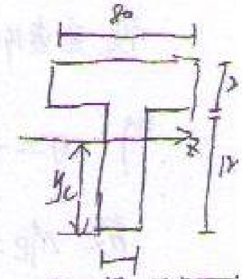
$\sum F_y=0$:　$F_A+F_B-9-4=0$，$F_A=\frac{5}{2}$ kN.

故作梁的M图和F_S图如左。





T型截面的中性轴如右图



$$y_C=\frac{80\times20\times130+120\times20\times60}{80\times20+120\times20}=88$$

$$I_z=\frac{0.08\times0.02^3}{12}+0.042^2\times0.08\times0.02+\frac{0.02\times0.12^3}{12}+0.028^2\times0.02\times0.12=7.64\times10^{-6}$$

故B端上侧 $\sigma_{tB}=\frac{M_By_1}{I_z}=\frac{4\times10^3\times0.052}{7.64\times10^{-6}}=27.23$ MPa

B端下侧 $\sigma_{cB}=\frac{M_By_2}{I_z}=\frac{4\times10^3\times0.088}{7.64\times10^{-6}}=46.07$ MPa

1-1截面下侧 $\sigma_{t1}=\frac{M_1y_2}{I_z}=\frac{2.5\times10^3\times0.088}{7.64\times10^{-6}}=28.81$ MPa

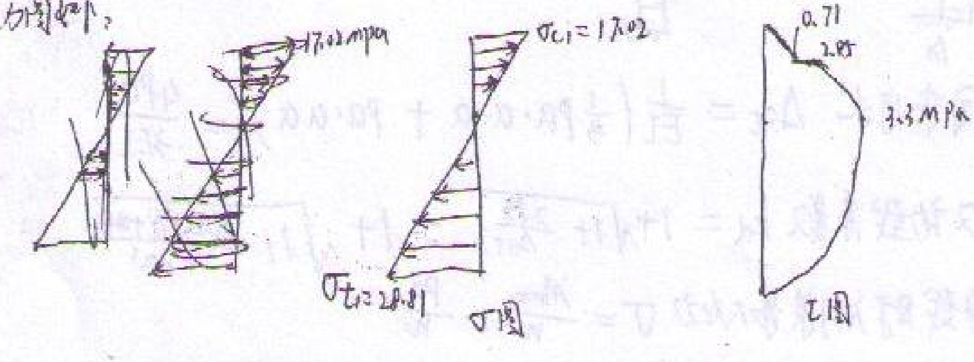
上侧 $\sigma_{c1}=\frac{M_1y_1}{I_z}=\frac{2.5\times10^3\times0.052}{7.64\times10^{-6}}=17.02$ MPa

故最大拉应力为28.81 MPa，$\sigma_{c\max}=46.09$ MPa

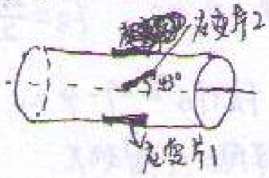
$$\tau_{max}=\frac{F_{S\max}S^*_{z\max}}{I_zb}=\frac{6.5\times10^3\times0.02\times0.088\times0.044}{7.637\times10^{-6}\times0.02}=3.3\text{ MPa}$$

(3) 由上计算可知 1-1截面下侧为拉力 $\sigma_{t1}=28.81$ MPa，上侧为压力 $\sigma_{c1}=17.02$ MPa

正应力图如下：



四、解、最少要在圆轴外表面贴两片电阻片，其贴法如图所示、



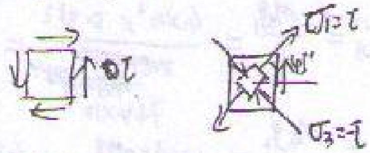
，即一片沿轴线方向，一片与轴线成45°角

设应变片1所测值为ε_1，应变片2所测值为ε_2

则 $\sigma = \frac{M_e}{W_t}$，$W_t = \frac{\pi D^3}{32}$ ~~故 $M_e =$~~，$\sigma = \varepsilon_1 E$

故 $M_e = \sigma W_t = \varepsilon_1 E \frac{\pi D^3}{32}$

当M_e作用时，应力图如下

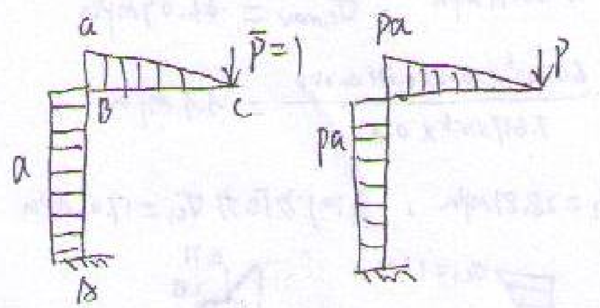


由广义胡克定律

$$\varepsilon_{45^\circ} = \varepsilon_2 = \frac{1}{E}(\sigma_1 - \nu\sigma_3) = \frac{1+\nu}{E}\tau$$

又 $\tau = \frac{M_t}{W_p}$ 故 $M_t = \tau W_p = \frac{\varepsilon_2 E}{1+\nu}\frac{\pi D^3}{16} = \frac{\pi \varepsilon_2 D^3 G}{8}$

五、解：当单位力作用于C端时，和重力P的物体作用C端时，钢架弯矩图如下



由图乘可求 $\Delta_{st} = \frac{1}{EI}\left(\frac{1}{3}Pa\cdot a\cdot a + Pa\cdot a a\right) = \frac{4Pa^3}{3EI}$

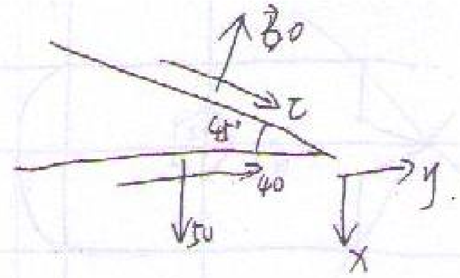
故动载系数 $k_d = 1+\sqrt{1+\frac{2h}{\Delta_{st}}} = 1+\sqrt{1+\frac{3EI(v^2/g+2h)}{4Pa^3}}$

静载时钢架最大应力 $\sigma = \frac{M_{max}}{W} = \frac{Pa}{W}$

故最大正应力 $\sigma_d = k_d\sigma = \left(1+\sqrt{1+\frac{3EI(v^2/g+2h)}{4Pa^3}}\right)\frac{Pa}{W}$

六、解：如图示设坐标、坐标方向，则有

$\sigma_x = 50mpa$， $\tau_x = -40mpa$ ， $\sigma_{135°} = 30mpa$



则有 $\sigma_{135°} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 270° - \tau_x \sin 270°$

即 $30 = \frac{50+\sigma_y}{2} + \frac{50-\sigma_y}{2}\cos 270° + 40\sin 270°$

解 $\sigma_y = 90mpa$

故 $\tau_\theta = \frac{\sigma_x-\sigma_y}{2}\sin 270° + \tau_x \cos 270° = 20mpa$

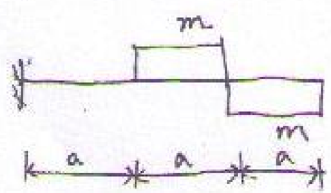
主应力 $\sigma = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_x^2} = 70 \pm 44.72 = \begin{cases} 114.72\ mpa \\ 25.28\ mpa \end{cases}$

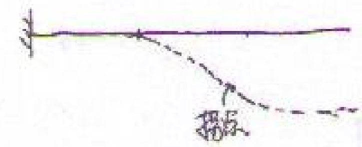
故主应力 $\sigma_1 = 114.72mpa$， $\sigma_2 = 25.28mpa$， $\sigma_3 = 0$.

最大剪应力 $\tau_{max} = \frac{\sigma_1-\sigma_3}{2} = 57.36mpa$

七、

1. 解：弯矩图如下 ，挠曲线大致形状如下





2、解：如图可知 $\sigma_y = 40mpa$， $\sigma_x = 20mpa$， $\tau_x = 30$.

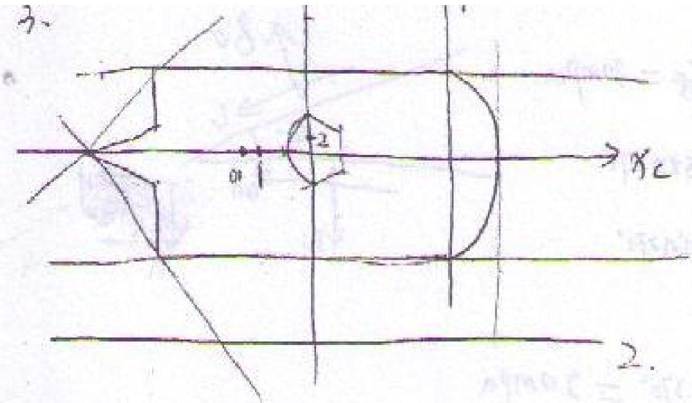
则 $\sigma = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2+\tau_x^2} = 10 \pm 31.62 = \begin{cases} 41.62\ mpa \\ -21.62\ mpa \end{cases}$

故主应力 $\sigma_1 = 41.62mpa$， $\sigma_2 = 40mpa$， $\sigma_3 = -21.62mpa$

$\sigma_{r3} = \sigma_1 - \sigma_3 = 63.24mpa$

$\sigma_{r4} = \sqrt{\frac{1}{2}[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2]} = 62.45mpa$

3.



4.

(a) $F_{cra} = \frac{\pi^2EI}{(\mu l)^2} = \frac{\pi^2EI}{25l^2}$

(b) $F_{crb} = \frac{\pi^2EI}{(\mu l)^2} = \frac{\pi^2EI}{(0.7\times7l)^2} = \frac{\pi^2EI}{24.01l^2}$

(c) $F_{crc} = \frac{\pi^2EI}{(\mu l)^2} = \frac{\pi^2EI}{(0.5\times9l)^2} = \frac{\pi^2EI}{20.25l^2}$

(d) $F_{crd} = \frac{\pi^2EI}{(2\times2l)^2} = \frac{\pi^2EI}{16l^2}$

故有 $F_{crd} > F_{crc} > F_{crb} > F_{cra}$

则第四杆不易失稳，第一杆最易失稳

5、解：由强度条件有

$\sigma_{bs} \le [\sigma_{bs}]$

则有 $\frac{P}{\pi(\frac{D^2}{4}-\frac{d^2}{4})} \le [\sigma_{bs}]$ 可得 $D \ge \sqrt{\frac{4P}{\pi[\sigma_{bs}]}+d^2} = \sqrt{\frac{4\times35\times10^3}{3.14\times300\times10^6}+0.014^2}$

$=0.0186m = 18.6mm$

又 $\tau \le [\tau]$

则 $\frac{P}{A}\le[\tau]$ 即 $\frac{P}{\pi d h}\le[\tau]$ 可得 $h \ge \frac{P}{\pi d[\tau]} = \frac{35\times10^3}{3.14\times0.014\times80\times10^6} = 0.00995m$

$= 9.95mm$

故端部圆头的尺寸D最小为18.6mm，h最小为9.95mm

二00八年

一、1. 解：(1) ~~τ_{max}~~ 空心圆轴 $I_p=\frac{\pi D^4}{32}-\frac{\pi d^4}{32}=\frac{\pi D^4(1-\alpha^4)}{32}$ 其 $\alpha=\frac{d}{D}=0.9$

$$W_p=\frac{\pi D^3(1-\alpha^4)}{16}$$

则 $\tau_{max}=\frac{T}{W_p} \Rightarrow T=\tau_{max}W_p$

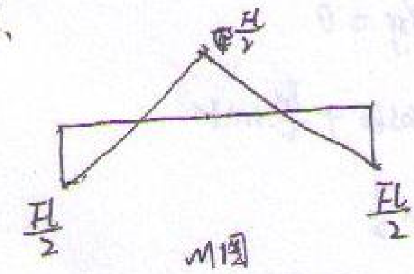
故 相对扭转角 $\varphi=\frac{Tl}{GI_p}=\frac{\tau_{max}W_p l}{GI_p}=\frac{2\tau_{max}l}{GD}=\frac{70\times10^6\times1}{80\times10^9\times0.1}=0.017$

(2) ~~设~~ 当为实心时 ~~I_p~~ $W_{p'}=\frac{\pi d_1^3}{16}$

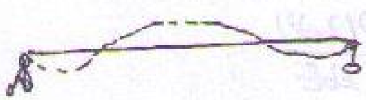
则 $\tau_{max}=\frac{T}{W_{p'}}=\frac{\tau_{max}\frac{\pi D^3}{16}(1-\alpha^4)}{\frac{\pi d_1^3}{16}}$ 即 $1=\frac{D^3}{d_1^3}(1-\alpha^4)$

则 $d_1=D\sqrt[3]{(1-\alpha^4)}=70.1\,mm$

2.



M图



3. 解：$\tau_s=\frac{F_s}{A_s}=\frac{P/2}{\pi\frac{d^2}{4}}=\frac{2P}{\pi d^2}=\frac{2\times8\times10^3}{3.14\times0.01^2}=50.96\,mpa$

$$\sigma_{bs}=\frac{F_{bs}}{A_{bs}}=\frac{P}{ld}=\frac{8\times10^3}{0.04\times0.01}=20\,mpa$$

4. 解：

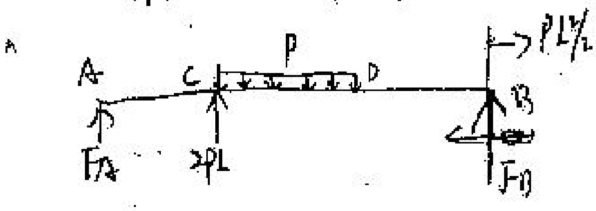
$$M=2\left(\int_{A_1}\sigma_y\, y\, dA_1+\int_{A_2}\sigma_s\, y\, dA_2\right)$$

$$=2\left(\int_0^{y_s}\frac{\sigma_s y}{y_s}\,y\,b\,dy+\int_{y_s}^{\frac{h}{2}}\sigma_s\,y\,b\,dy\right)$$

$$=2\left(\frac{\sigma_s b}{3y_s}y^3\Big|_0^{y_s}+\frac{1}{2}\sigma_s b y^2\Big|_{y_s}^{\frac{h}{2}}\right)=\frac{1}{4}\sigma_s bh^2-\frac{1}{3}\sigma_s b\,y_s^2$$

则 $y_s=\frac{1}{2}\sqrt{\frac{3(\sigma_s bh^2-4M)}{\sigma_s b}}$

一、解：反力如下

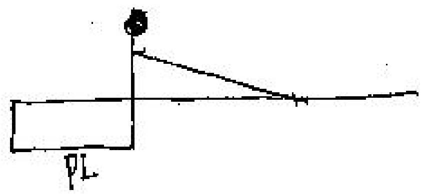


$\sum M_B=0$：~~$F_A\cdot 3L$~~

$pL\frac{3L}{2}-\frac{PL^2}{2}-2PL\cdot 2L-F_A\cdot 3L=0$

$\Rightarrow F_A=-PL\ (\downarrow)$

$\sum F_y=0$. $F_A+F_B+2PL-PL=0\Rightarrow F_B=0$



三、解：(1) $\sigma_x=\frac{P}{A}=\frac{P}{bh}$，则 $\varepsilon_x=\frac{\sigma_x}{E}=\frac{P}{bhE}$

则 $\nu=-\frac{\varepsilon_y}{\varepsilon_x}$ 有 $\varepsilon_y=-\nu\varepsilon_x=\frac{-P\nu}{bhE}$，无切应力则 $\gamma_{xy}=0$

故BC的应变 $\varepsilon_{BC}=\varepsilon_{30°}=\frac{\varepsilon_x+\varepsilon_y}{2}+\frac{\varepsilon_x-\varepsilon_y}{2}\cos 60°+\frac{\gamma_{xy}}{2}\sin 60°$

$=\frac{P-P\nu}{2bhE}+\frac{P+P\nu}{2bhE}\times\frac{1}{2}$

$=\frac{2P-2P\nu+P+P\nu}{4bhE}=\frac{3P-P\nu}{4bhE}$

则 $\Delta BC=\varepsilon_{BC}l_{BC}=\frac{P(3-\nu)}{4bhE}\cdot 2h=\frac{P(3-\nu)}{2bE}$

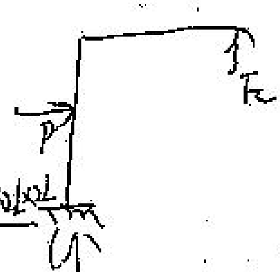
(2) 设∠ABC的改变量Δθ，则为直角改变量

$\frac{\Delta\theta}{2}=\frac{(\varepsilon_x-\varepsilon_y)}{2}\sin 60°+\frac{\gamma_{xy}}{2}\cos 60°$

~~$=\frac{P+P\nu}{bhE}\times\frac{\sqrt{3}}{2}=\frac{\sqrt{3}P(1+\nu)}{2bhE}$~~

$\Rightarrow\Delta\theta=\frac{\sqrt{3}P(1+\nu)}{2bhE}$

四、解：图示钢架为一次超静定，现去除C端支承代之以反力F_C，如下图



只在力P作用下C端挠度

$w_{C_1}=\frac{PL^2}{2EI}\cdot 2L=\frac{PL^3}{EI}\ (\downarrow)$

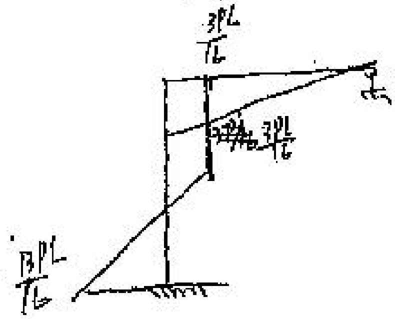
只在力F_C作用下 $w_{C_2}=\frac{F_C(2L)^3}{3EI}+\frac{F_C\cdot 2L\times 2L\times 2L}{EI}\ (\uparrow)$

在 P与Fc各自单独作用 $w_c=0$ 则 $w_{c1}-w_{c2}=0$

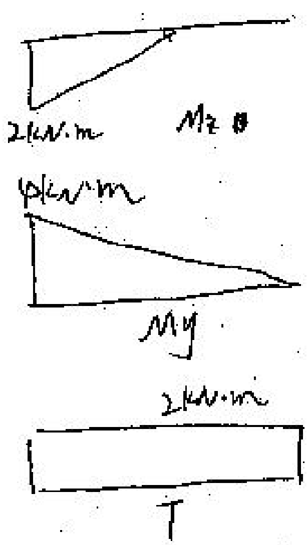
即有 $\frac{PL^3}{EI}=\frac{F_c(2L)^3}{3EI}+\frac{F_c(2L)^3}{EI}$ 解得 $F_c=\frac{3P}{32}$

作刚架弯矩图如下

故在A处弯矩最大 $M_{max}=\frac{13PL}{16}$



五、解：作构件的弯矩图和扭矩图如下



则固定端弯矩最大，为危险截面

$M=\sqrt{M_z^2+M_y^2}=\sqrt{2^2+4^2}=4.472\ kN\cdot m$

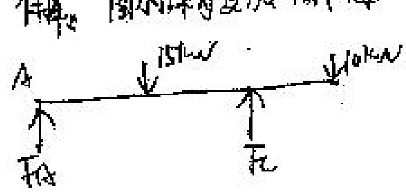
$\sigma_{r3}=\frac{\sqrt{M^2+T^2}}{W_z}$

$=\frac{\sqrt{(4.473\times10^3)^2+(2\times10^3)^2}}{\frac{\pi d^3}{32}}$

$=49.93\ MPa<[\sigma]$

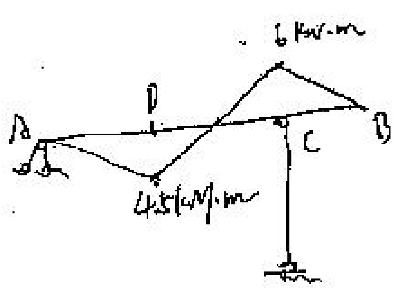
合格.

六、解：图示结构受力如下图



$\sum M_A=0.\quad F_c\times2-15\times1-10\times2.6=0.$

$\Rightarrow F_c=20.5\ kN$



则C截面 $\sigma_{ct}=\frac{M_c y_1}{I_z}=\frac{6\times10^3\times0.075}{5314\times10^{-8}}=8.469\ MPa.$

$\sigma_{cc}=\frac{M_c y_2}{I_z}=\frac{6\times10^3\times0.135}{5314\times10^{-8}}=14.11\ MPa$

D截面，$\sigma_{Dt}=\frac{M_D y_2}{I_z}=\frac{4500\times0.135}{5314\times10^{-8}}=10.59\ MPa$

$\sigma_{Dc}=\sigma_{max}=\frac{4500\times0.075}{5314\times10^{-8}}=6.35\ MPa$

故 $\sigma_{tmax}=10.59\ MPa,\quad \sigma_{cmax}=14.11\ MPa$

~~压杆CD的临界压力~~ $\cancel{F_{cr}=\frac{\pi^2 EI}{(\mu l)^2}}$

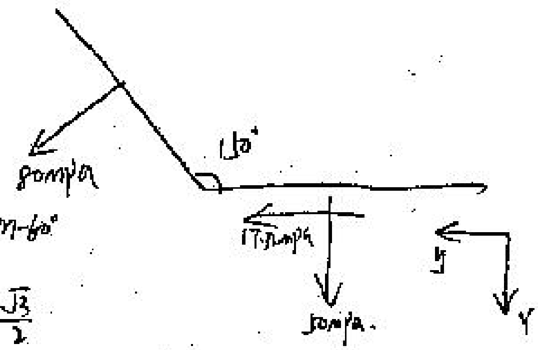
CD杆的 $I_z=\frac{\pi d^4}{64}=\frac{3.14\times0.04^4}{64}=1.256\times10^{-7}$

则CD杆临界压力 $F_{cr}=\frac{\pi^2EI}{(\mu l)^2}=\frac{3.14^2\times200\times10^9\times1.256\times10^{-7}}{(1\times1.5)^2}=110.08\,kN$.

则 $n=\frac{F_{cr}}{F_{CD}}=5.5>n_{st}=2.5$ 故安全.

七、解、

如图，设x、y轴方向，则



$\sigma_x=50\,mpa$，$\tau_x=17.32\,mpa$

$\sigma_{-30^\circ}=80\,mpa$

则有 $\sigma_{-30^\circ}=\frac{\sigma_x+\sigma_y}{2}+\frac{\sigma_x-\sigma_y}{2}\cos(-60^\circ)-\tau_x\sin(-60^\circ)$

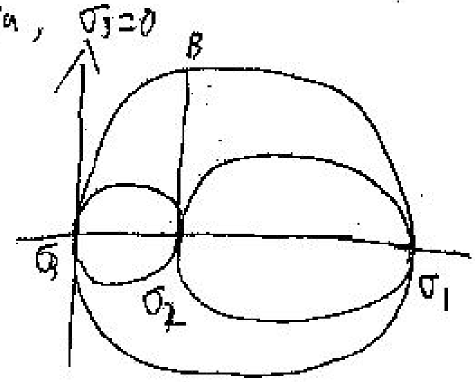
即 $80=\frac{50+\sigma_y}{2}+\frac{50-\sigma_y}{2}\times\frac{1}{2}+17.32\times\frac{\sqrt3}{2}$

解得 $\sigma_y=110\,mpa$.

则主应力 $\sigma=\frac{\sigma_x+\sigma_y}{2}\pm\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_x^2}=80\pm34.64=\begin{cases}114.64\,mpa\\45.36\,mpa\end{cases}$

故主应力为 $\sigma_1=114.64\,mpa$，$\sigma_2=45.36\,mpa$，$\sigma_3=0$

(2) 作应力圆如下所示、



故最大切应力 $\tau_{max}=\frac{\sigma_1-\sigma_3}{2}=57.32$

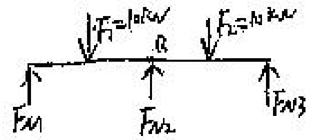
如应力圆可知最大切应力方向与σ_1,σ_3都成45°角.

又主应力方向 $\alpha_0=\frac{1}{2}\arctan\frac{-2\tau_x}{\sigma_x-\sigma_y}=15^\circ$

故最大切应力方向为60°

(3)

八、解：受力分析如下

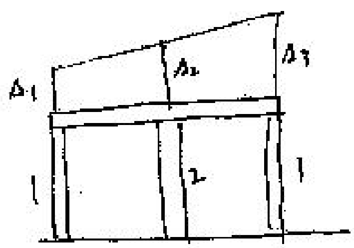


(1) 静力方面：$\Sigma F_y=0$，$F_{N1}+F_{N2}+F_{N3}-20=0$。

$\Sigma M_B=0$，$F_{N1}l=F_{N3}l$ 即 $F_{N1}=F_{N3}$。

(2) 物理方面：$\Delta l_1=\frac{F_{N1}l}{EA_1}$，$\Delta l_2=\frac{F_{N2}l}{EA_2}$，$\Delta l_3=\frac{F_{N3}l}{EA_2}$

$\Delta l_t=\alpha\Delta t l$



(3) 几何方面：$\Delta_1=\Delta l_1+\Delta l_t$，$\Delta_2=\Delta l_2+\Delta l_t$，$\Delta_3=\Delta l_3$

$2\Delta_2=\Delta_1+\Delta_3$

则得相容方程 $2\Delta l_2+2\Delta l_t=\Delta l_1+\Delta l_3+2\Delta l_t$

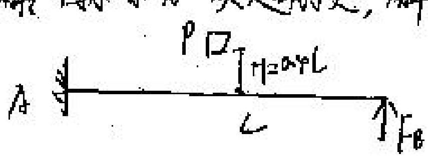
即 $\frac{2F_{N2}l}{EA_2}=\frac{F_{N1}l}{EA_1}+\frac{F_{N3}l}{EA_3}$

又 $A_1=A_3=\frac{1}{4}A_2$

故 $F_{N1}=F_{N3}=\frac{1}{4}F_{N2}$

则 $F_{N1}=F_{N3}=\frac{10}{3}kN$，$F_{N2}=\frac{40}{3}kN$。

九、解 图示梁为一次超静定，解除B端约束代之以反力 F_B，如下所示。



由约束条件知 B端挠度 $w_B=0$。

静载时

则 $\frac{P(\frac{l}{2})^2}{2EI}\frac{l}{2}+\frac{P(\frac{l}{2})^3}{3EI}-\frac{F_Bl^3}{3EI}=0$

解得 $F_B=\frac{5P}{16}$

则各段弯矩 BC段 $M(x)=\frac{5P}{16}x$ $(0\le x\le\frac{l}{2})$

$\cancel{AC段\ M(x)=\frac{5Px}{16}-P(x-\frac{l}{2})\ (\frac{l}{2}\le x\le l)}$

$\frac{\partial M(x)}{\partial P}=\frac{5x}{16}$

AC段 $M(x)=\frac{5Px}{16}-P(x-\frac{l}{2})$ $\frac{l}{2}\le x\le l$

$\frac{\partial M(x)}{\partial P}=\frac{5x}{16}-x+\frac{l}{2}=\frac{l}{2}-\frac{11x}{16}$

C处位移

则 $\Delta_{st}=\frac{1}{EI}\left[\int_0^{\frac{l}{2}}\frac{5Px}{16}\cdot\frac{5x}{16}dx+\int_{\frac{l}{2}}^{l}P(\frac{l}{2}-\frac{11x}{16})(\frac{l}{2}-\frac{11x}{16})dx\right]$

$=\frac{7Pl^3}{768EI}$

则 $K_d=1+\sqrt{1+\frac{2H}{\Delta_{st}}}=1+\sqrt{1+\frac{6144H}{7mg}}$

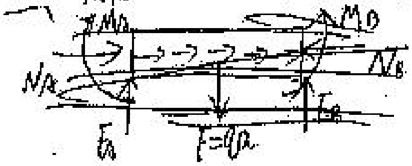
(1) 若A为铰支时 $\Delta_{st1}=\frac{PL^3}{48EI}$，则 $\Delta_{st1}>\Delta_{st}$，故 $K_{d1}<K_d$

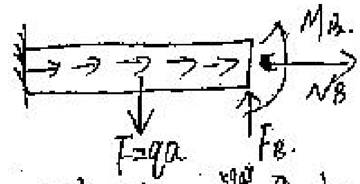
4

二00七年

一、作F_S，M图

二、解：图示为3次超静定，解除B端约束，受力如下所示。





在F作用下B端挠度 $w_1=\frac{Fa^3}{3EI}+\frac{Fa^2}{2EI}\cdot a=\frac{5qa^4}{6EI}$ (↓)，转角 $\theta_1=\frac{Fa^2}{2EI}=\frac{qa^3}{2EI}$

F_B作用下B端挠度 $w_2=\frac{F_B(2a)^3}{3EI}=\frac{8F_Ba^3}{3EI}$ (↑)，转角 $\theta_2=\frac{F_B\cdot(2a)^2}{2EI}=\frac{2F_Ba^2}{EI}$

M_B作用下B端挠度：$w_3=\frac{M_B(2a)^2}{2EI}=\frac{2M_Ba^2}{EI}$ (↑) 转角 $\theta_3=\frac{M_B 2a}{EI}$

由于B端为固定端：$w_1+w_2+w_3=0$，$\theta_1+\theta_2+\theta_3=0$

即 $\frac{5qa^4}{6EI}-\frac{8F_Ba^3}{3EI}-\frac{2M_Ba^2}{EI}=0$，$\frac{qa^3}{2EI}-\frac{2F_Ba^2}{EI}-\frac{2M_Ba}{EI}=0$

解得 $F_B=\frac{qa}{2}$，$M_B=\frac{qa^2}{4}$

B端水平位移 $\Delta_{BH}=\int_0^{2a}\frac{qx\,dx}{EA}+\frac{N_B\cdot 2a}{EA}-\alpha\Delta t\cdot 2a=0$

解得 $N_B=\alpha\Delta t EA-qa$

则由平衡条件可求：
$$\begin{cases} F_A=\frac{qa}{2} \\ M_A=M_B+F_B\cdot 2a-F\cdot a=\frac{qa^2}{4} \\ N_A=N_B+q\cdot 2a=\alpha\Delta tEA+qa \end{cases}$$

三、解：(1) 纯弹性时，中性轴即为形心轴。

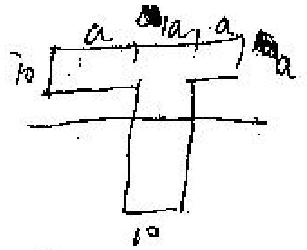
$y_c=\frac{30\times10\times5+30\times10\times25}{30\times10+30\times10}=15$

当出现塑性时，中性轴将上下部分面积均分，设为 y_{c2}

则 $30y_{c2}=(10-y_{c2})\times30+30\times10$

解得 $y_{c2}=10$.

~~$M_S = \sigma_s W_{z}$~~



~~$M_u = \sigma_s (S_t + S_c)$~~

~~仍弹性时~~

~~$I_z = \frac{bh^3}{12} - \frac{\cdots a^3}{12}$~~

$$I_z = \frac{30\times10^3}{12} + 10^2\times30\times10 + \frac{10\times30^3}{12} + 10^2\times30\times10 = 8.5\times10^4\ mm^4$$

$$W_z = \frac{I_z}{y_c} = 5.67\times10^3\ mm^3$$

$$M_s = \sigma_s W_z$$

~~$M_u = \sigma_s (S_t + S_c)$~~ $M_u = \sigma_s W_s$, $W_s = S_t + S_c$

$S_t = 10\times30\times$ ~~…~~ , $S_c = 10\times30\times15$

四、解：$H = \frac{v^2}{2g} + h$

$$\Delta_{st} = \frac{Qa^3}{3EI} + \frac{Qa\cdot a}{GI_p}\cdot a + \frac{Qa^3}{3EI} = \frac{2Qa^3}{3EI} + \frac{Qa^3}{GI_p}$$

则 $K_d = 1+\sqrt{1+\frac{2H}{\Delta_{st}}} = 1+\sqrt{1+\frac{(2h+v^2/g)}{\frac{2Qa^3}{3EI}+\frac{Qa^3}{GI_p}}}$

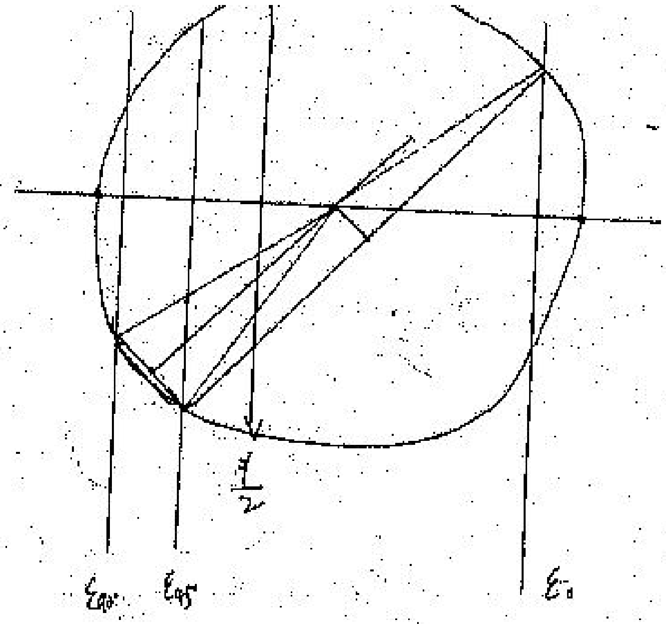
五、解：不静

(a)图 $\tau_a = \frac{F_s S_{z1}^*}{I_z b_1}$　其中S_{z1}^* = ~~…~~　$b_1 = 2t$

(b)图 $\tau_b = \frac{F_s S_{z2}^*}{I_z b_2}$　其 S_{z2}^*　$b_1 = 2t$

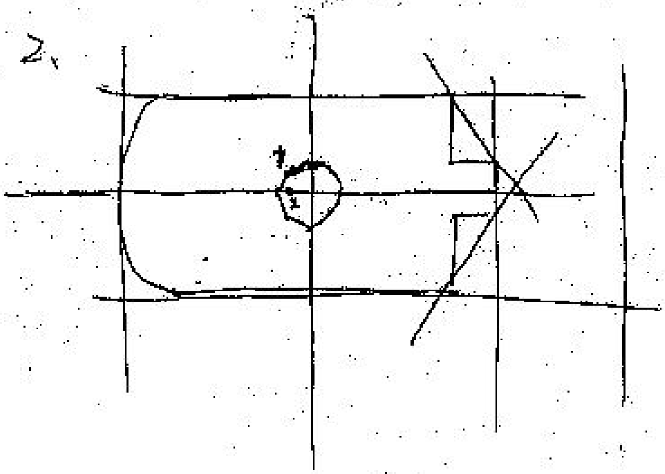
$S_{z1}^* > S_{z2}^* \Rightarrow \tau_a > \tau_b$

六、

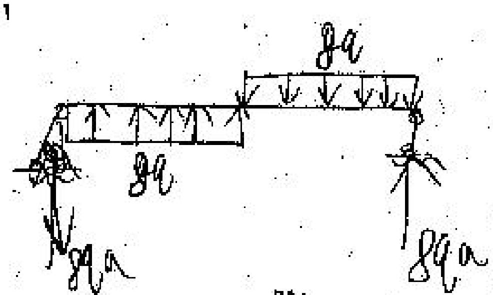


$\frac{\tau}{2}$
$\varepsilon_{90°}$
$\varepsilon_{45°}$
ε_0

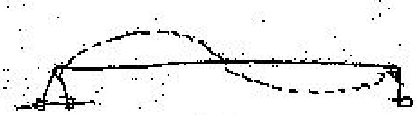
七、



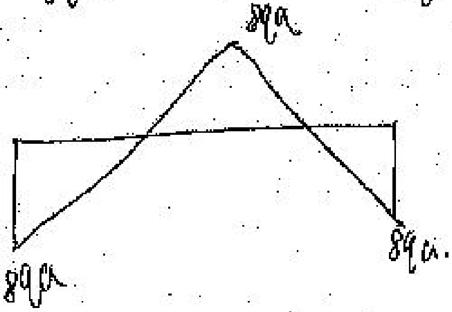
八、



8a
8a
8qa
8qa



挠曲线.



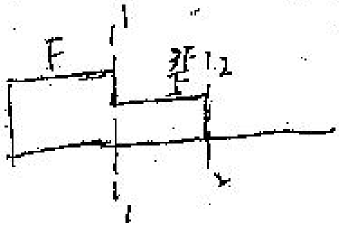
8qa
8qa.
8qa

4. 解：每个铆钉有2个剪切面

则 $\tau = \frac{F_s}{A_s} = \frac{F_N/2n}{\pi \frac{d^2}{4}} = \frac{200\times10^3\times4}{2\times5\times3.14\times0.016^2} = 99.5\,mPa < [\tau]$ 安全

挤压应力：$\sigma_{b1} = \frac{F_b}{l_1 d} = \frac{200\times10^3}{0.014\times0.016\times5} = 178.6\,mPa < [\sigma_{bs}]$ 安全

l_1 为 min(14, 10)



1-1截面：$\sigma_1 = \frac{F}{A} = \frac{\cancel{200\times10^3}}{\cancel{0.12-0}} = \frac{F}{(b-2d)\,l_1}$

$= \frac{200\times10^3}{(0.12-0.032)\times0.014}$

$= 162.3\ MPa$

$\frac{\sigma_1-[\sigma]}{[\sigma]} = 0.014 < 5\%$ 故安全.